Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2009

Test II

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Name (please print):____

NET ID (please print):__________

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Notes

- You will be turning in each problem in a separate pile. **Make sure that you do not put work for more than any one problem on any one piece of paper.** For this test, you will be turning in four different sets of work.
- Be sure your name *and* NET ID show up on *every page* of the test. If you are including work on extra sheets of paper, put your name and NET ID on each and be sure to staple them to the appropriate problem. **Problems without names will incur at least a 25% penalty for the problem**.
- This first page should have your name, NET ID, and signature on it. **It should be stapled on top of and turned in with your submission for Problem I.**
- You may use your calculator to perform calculations or to make graphs. **You may not use your calculator as an information storage and retrieval system.**
- You will be asked to write several lines of code on this test. **Make sure what you write is MATLAB code and not mathematics**. Also be clear if you are writing .* or ./ or .ˆ versus just * or / or ˆ for those operations where there is a distinction.

Name (please print):
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Problem I: [20 pts.] Finding Roots

(1) Given some function and its derivative:

$$y(x) = e^x - 3x^2 + 2 \qquad\qquad \left.\frac{dy}{dx}\right|_x = e^x - 6x$$

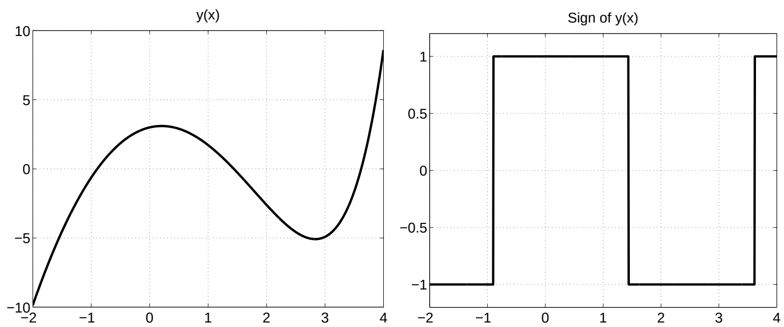
and assume an initial guess for a root x_1=0,

- Show the process of determining the *next* two approximations for a root of $y(x)$ using the Newton-Raphson method (that is, determine x_2 and x_3). Keep at least four significant figures.

k	x_i	$f(x_i)$	$f'(x_i)$	$\Delta x = -\frac{f}{f'}$	x_{i+1}
1	0	3	1	-3	-3
2	-3	-24.95	18.05	1.382	-1.618
3	-1.618	-5.655	N/R	N/R	N/R

- What is the calculated x-tolerance and function tolerance after having calculated x_3?
  x-tolerance: $|\Delta x|$=$|x_3 - x_2|$=1.382
  Function tolerance: $|y(x_3)| = 5.655$

(2) Given the graphs of the function $y(x)$ and the sign of $y(x)$:



Write the MATLAB code you would need to find all the roots of the function $y(x)$. At the end of the code, they should be stored in a vector called `TheRoots` which has as many entries as there are real roots.

```
clear; format short e

y = @(x) exp(x)-3*x.^2+2

TheRoots(1) = fzero(@(xD) y(xD), [-2 0])
TheRoots(2) = fzero(@(xD) y(xD), [ 0 2])
TheRoots(3) = fzero(@(xD) y(xD), [ 2 4])
```

Note - any valid brackets would be accepted, as would initial guessses significantly close to a root. Since this is a function in one variable, the shortcut version would be accepted as well; that is:

```
clear; format short e

y = @(x) exp(x)-3*x.^2+2

TheRoots(1) = fzero(y, [-2 0])
TheRoots(2) = fzero(y, [ 0 2])
TheRoots(3) = fzero(y, [ 2 4])
```

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Problem II: [30 pts.] Cool It!

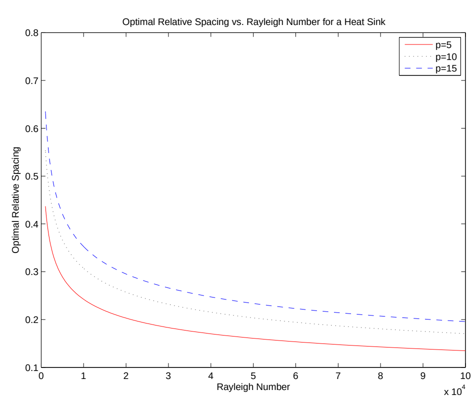
Professor Adrian Bejan in the Mechanical Engineering and Materials Science Department published a relationship for the optimal relative spacing r of pin fins on a heat sink as a function of the Rayleigh number, Ra.[1] For a particular set of pins, the equation can be simplified to:

$$(r^2 + 2r)\ \mathrm{Ra}^{1/4} = 2.75\ p^{1/3}\ (1 + r)^{2/3}$$

where p is the ratio between the total height of the heat sink to the diameter of each pin. Your job is to plot the relationship between r and Ra for three sets of pins - one where $p = 5$, one where $p = 10$, and one where $p = 15$. Write all the MATLAB code you need to solve r for 200 logarithmically spaced values of the Rayleigh number between 1000 and 100000 for each of the p values. Note that the solutions will always be between 0 and 1. Store these values in vectors called `rvals5`, `rvals10`, and `rvals15` respectively, then make a plot of these values versus the Rayleigh numbers. Use a red solid line for the $p = 5$ set, a black dotted line for the $p = 10$ set, and a blue dashed line for the $p = 15$ set. Add an ideally located and informative legend, proper axis labels, and a title. Note that p, r, and Ra are dimensionless so you do not need to worry about units. Save your plot as a color encapsulated postscript file called `PinFinWin.eps`.

```
clear; format short e
figure(1); clf

eqn = @(p, r, Ra) (r.^2+2*r).*(Ra.^(1/4))-2.75*(p.^(1/3)).*(1+r).^(2/3);

Ra = logspace(3, 5, 200);

for k=1:length(Ra)
    rvals5(k)  = fzero(@(rD) eqn( 5, rD, Ra(k)), [0 1]);
    rvals10(k) = fzero(@(rD) eqn(10, rD, Ra(k)), [0 1]);
    rvals15(k) = fzero(@(rD) eqn(15, rD, Ra(k)), [0 1]);
end

plot(Ra, rvals5,  'r-', ...
     Ra, rvals10, 'k:', ...
     Ra, rvals15, 'b--')

xlabel('Rayleigh Number')
ylabel('Optimal Relative Spacing')
legend('p=5', 'p=10', 'p=15', 0)
title('Optimal Relative Spacing vs. Rayleigh Number for a Heat Sink')

print -depsc PinFinWin
```

Note - the graph is:



[1] Adrian Bejan, "Geometric optimization of cooling techniques." *Air Cooling Technology for Electronic Equipment,* S. J. Kim and J. S. Woo, editors. CRC Press, 1996. pp 1-45. Adapted here from form relayed in Gerald Recktenwald, *Numerical Methods with MATLAB: Implementations and Applications.* Prentice Hall, 2000. p. 287.

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Problem III: [25 pts.] I'm On A Boat!

- If you are at sea but close enough to shore to see landmarks, one way to determine where you are is to pick out two of those landmarks and figure out the direction of a line between yourself and those landmarks. You can then figure out where you are by finding the intersection of those two lines. Using your map, you determine that the slope between a landmark at (2, 1) and you is $\frac{1}{4}$ while the slope between a landmark at (3, 5) and you is $-\frac{1}{5}$. Using your handy point-slope equations, you determine that the system you must solve is therefore:

$$(y-1) = \frac{1}{4}(x-2)$$
$$(y-5) = -\frac{1}{5}(x-3)$$

(1) Clearly show the equations above organized as a linear algebra equation.
(2) Clearly show the steps required to determine the x and y coordinate of your boat and present those values.
(3) What is the condition number of the linear system using the 1-norm?
(4) Your map and your angle measurement device are correct to six significant figures. Approximately how many significant figures are there in your calculated coordinates?

(1) Equations:

$$\begin{bmatrix} 1 & -4 \\ 1 & 5 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} -2 \\ 28 \end{Bmatrix}$$

(2) Solving

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \text{inv}\left(\begin{bmatrix} 1 & -4 \\ 1 & 5 \end{bmatrix}\right) \begin{Bmatrix} -2 \\ 28 \end{Bmatrix} = \frac{\begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix}}{(1)(5)-(1)(-4)} \begin{Bmatrix} -2 \\ 28 \end{Bmatrix} = \frac{\begin{Bmatrix} (5)(-2)+(4)(28) \\ (-1)(-2)+(1)(28) \end{Bmatrix}}{9} = \begin{Bmatrix} \frac{102}{9} \\ \frac{30}{9} \end{Bmatrix} = \begin{Bmatrix} \frac{34}{3} \\ \frac{10}{3} \end{Bmatrix} = \begin{Bmatrix} 11.33 \\ 3.333 \end{Bmatrix}$$

(3) Condition Number:

$$\text{cond}(A,1) = \|A\|_1 \; \|A^{-1}\|_1$$
$$\|A\|_1 = \max(2,9) = 9 \qquad \|A^{-1}\|_1 = \max(6/9, 5/9) = 6/9$$
$$\text{cond}(A,1) = 9 * (6/9) = 6$$

(4) Meaning: Since $\log_{10}(cond)$ is 0.778, between 0 and 1 digits will be lsot so we expect between 5 and 6 significant figures.

- Before going on your trip, you perform some calculations to figure out how much food, water, and reading material you want to bring aboard. You determine some constraints - first, assuming that each of the variables is measured in pounds, you determine that you will load up your boat with exactly 20 pounds. That is:

$$f + w + r = 20$$

Second, you decide that you need to bring exactly twice as much water as you bring food:

$$w = 2f$$

Finally, you only have \$50 to spend on your trip (and you plan to spend it all). Turns out that water costs \$2 per pound, food costs \$3 per pound, and reading material is \$7 per pound, so

$$2w + 3f + 7r = 50$$

(1) Clearly show the equations above organized as a linear algebra equation.
(2) By hand, clearly show the determinant of the coefficient matrix. Merely producing a number will give you no credit.
(3) Clearly show all the MATLAB code to calculate the amount of food, water, and reading material you will bring with you on the trip. At the end of your script, there must be a variable called `Food`, a variable called `Water`, and a variable called `Reading` that are holding on to their respective values.
(4) Show the MATLAB code required to calculate the condition number of the linear system, using the 2 norm.

You will want to put the solution to this part of the problem on its own piece of paper. Please be sure to *staple it* to this page and *put your name on it.*[2]

[2] You know who I'm talking about....

(1) Equations: Each should first be re-written with unknowns on one side, and in order:

$$\begin{aligned} f + w + r &= 20 \\ 2f - w &= 0 \\ 3f + 2w + 7r &= 50 \end{aligned}$$

Then write in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 3 & 2 & 7 \end{bmatrix} \begin{Bmatrix} f \\ w \\ r \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 50 \end{Bmatrix}$$

(2) Determinant:

$$\det(A) = (1)(-1)(7) + (1)(0)(3) + (1)(2)(2) - (3)(-1)(1) - (2)(0)(1) - (7)(2)(1) = -14$$

(3) Solving:

```
clear; format short e
A = [1 1 1; 2 -1 0; 3 2 7];
b = [20; 0; 50];
Vals = A\b
Food    = Vals(1)
Water   = Vals(2)
Reading = Vals(3)
```

(4) Condition number:

```
cond(A, 2)
```

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Problem IV: [25 pts.] America's Next Top Model

Two Pratt Fellows working on similar projects have performed an experiment whereby they measure the electrical current passing through a device as a function of some voltage applied across it. They save their data in a file called *SoMuchPotential.data*. They want you to determine which of three possible models is better for their data set. The first four data points are shown below - *however* - the file consists of two columns and an unknown number of rows.

```
 16   3.3
 36   3.7
 52   3.8
 67   3.9
 .     .
 .     .
etc   etc
```

The three models they are considering are:

$$\hat{y}_1(x) = 3.2e^{0.0030x} \qquad \hat{y}_2(x) = 2.6x^{0.10} \qquad \hat{y}_3(x) = 4.1\frac{x}{4.0+x}$$

(1) By hand, determine which model has the best mathematical fit for the *first four data points*. Clearly show all work and calculations used to defend your conclusion. Simply picking a model, or providing insufficient proof, will yield no credit. The following table may help:

x	y	$\hat{y}_1(x)$	$\hat{y}_2(x)$	$\hat{y}_3(x)$	$\|y-\hat{y}_1\|^2$	$\|y-\hat{y}_2\|^2$	$\|y-\hat{y}_2\|^2$
16	3.3	3.36	3.43	3.28	0.0036	0.0169	0.0004
36	3.7	3.56	3.72	3.69	0.0196	0.0004	0.0001
52	3.8	3.74	3.86	3.81	0.0036	0.0036	0.0001
67	3.9	3.91	3.96	3.87	0.0001	0.0036	0.0009

$$\bar{y} = \frac{1}{4}(3.3+3.7+3.8+3.9) = 3.675$$

$$S_t = \sum |y-\bar{y}|^2 = (0.375)^2 + (0.025)^2 + (0.125)^2 + (0.225)^2 = 0.2075$$

$$S_{r1} = \sum |y-\hat{y}_1|^2 = 0.0036 + 0.0196 + 0.0036 + 0.0001 = 0.0269 \qquad r_1^2 = \frac{S_t - S_{r1}}{S_t} = 0.870$$

$$S_{r2} = \sum |y-\hat{y}_2|^2 = 0.0169 + 0.0004 + 0.0036 + 0.0036 = 0.0245 \qquad r_2^2 = \frac{S_t - S_{r2}}{S_t} = 0.882$$

$$S_{r2} = \sum |y-\hat{y}_3|^2 = 0.0004 + 0.00010.0001 + 0.0009 = 0.0015 \qquad r_3^2 = \frac{S_t - S_{r3}}{S_t} = 0.993$$

The third model is mathematically best for the first four points as it has the r^2 value which is closest to 1. *Note* - you could also determine which is best by seeing which had the smallest S_r without finding $\bar{y}$ or the r^2 values.

(2) Write a program that will load the data, calculate the estimates for each of the models using *all* the data (i.e. not just the first four points), and calculate the appropriate statistics for each model. *Note* - you are *not* trying to find the coefficients for each model - those are already given above!

Your script should print out the values of the coefficients of determination for each model using four digits after the decimal point. It should also state which model is best. Example output for a different data set might look like:

```
r2 for Model 1: 0.5984
r2 for Model 2: 0.9425
r2 for Model 3: 0.6339
Model 2 is mathematically superior
```

You will want to put the solution to this part of the problem on its own piece of paper. Please be sure to *staple it* to this page and *put your name on it*.[3]

[3] Yes, you...

```
load SoMuchPotential.data
x = SoMuchPotential(:,1);
y = SoMuchPotential(:,2);

yhat1 = 3.2*exp(0.003*x);
yhat2 = 2.6*x.^(0.10);
yhat3 = 4.1*x./(4.0+x);

St  = sum((y-mean(y)).^2)
Sr1 = sum((y-yhat1).^2)
Sr2 = sum((y-yhat2).^2)
Sr3 = sum((y-yhat3).^2)

r21 = (St - Sr1) / St
r22 = (St - Sr2) / St
r23 = (St - Sr3) / St

fprintf('r2 for Model 1: %0.4f\n', r21)
fprintf('r2 for Model 2: %0.4f\n', r22)
fprintf('r2 for Model 3: %0.4f\n', r23)

if r21>r22 & r21>r23
    fprintf('Model 1 is mathematically superior\n')
elseif r22>r23
    fprintf('Model 2 is mathematically superior\n')
else
    fprintf('Model 3 is mathematically superior\n')
end
```

Admittedly, the above *if* tree does not check for ties. Code that might do that would be:

```
r2s = [r21 r22 r23];
r2m = max(r2s);
BestModel = find(r2s==r2m);
if length(BestModel)==1
     fprintf('Model %d is mathematically superior\n', BestModel)
elseif length(BestModel)==2
     fprintf('Models %d and %d are mathematically superior\n', BestModel)
else
     fprintf('All three models are identically mathematically valid\n')
end
```