Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2008

Test II

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Problem I: [15 pts.] Basic Mathematics and Statistics

By hand:

(a) Convert the number 18.4_{10} to binary. Show five digits after the binary point.
(b) Convert the number 1011.1001_2 to decimal.
(c) Determine $\|x\|_1$ if x=`[1 -4 3]'`
(d) Determine $\|A\|_\infty$ if A=`[1 -8; -3 2]`
(e) Determine S_t if y =`[1 -1 3 -7]'`

(a)

Division	Result	Remainder	Multiplication	Ones	Decimal
18/2	9	0	$0.4 * 2$	0	.8
9/2	4	1	$0.8 * 2$	1	.6
4/2	2	0	$0.6 * 2$	1	.2
2/2	1	0	$0.2 * 2$	0	.4
1/2	0	1	$0.4 * 2$	0	.8

$18.4_{10} = 10010.01100_2$

(b) $(1)(2^4)+(0)(2^2)+(1)(2^1)+(1)(2^0)+(1)(2^{-1})+(0)(2^{-2})+(0)(2^{-3})+(1)(2^{-4}) = 8+2+1+\frac{1}{2}+\frac{1}{16} = 11\frac{9}{16} = 11.5625$
(c) $\sum_{k=1}^{3} |x_i| = 1+4+3 = 8$
(d) max row 1-norm = max(1+8, 3+2) = max(9, 5) = 9
(e)

$$\bar{y} = \frac{1-1+3-7}{4} = -1$$

$$S_t = \sum_{k=1}^{4} (y_i - \bar{y})^2$$

$$S_t = (1-(-1))^2 + (-1-(-1))^2 + (3-(-1))^2 + (-7-(-1))^2 = 56$$

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Problem II: [20 pts.] Finding Roots

(1) Given some function and its derivative:

$$f(x) = xe^x - 1 \qquad\qquad f'(x) = xe^x + e^x$$

and assume an initial guess for a root x_1=-0.1, determine the *next* two approximations for a root of $f(x)$ using the Newton-Raphson method (that is, determine x_2 and x_3). Keep at least four significant figures. Also, what is a particularly bad "first guess"? State why you believe this to be true.

k	$x(k)$	$f(x(k))$	$f'(x(k))$	$-f/f'$	$x(k+1)$
1	$-1.0000e-01$	$-1.0905e+00$	$8.1435e-01$	$1.3391e+00$	$1.2391e+00$
2	$1.2391e+00$	$3.2778e+00$	$7.7303e+00$	$-4.2403e-01$	$8.1505e-01$
3	$8.1505e-01$	N/R	N/R	N/R	N/R

Bad first guess: anywhere $f'(x) = 0$, which in this case is $x = -1$. Results in an undefined value for the next guess due to division by zero.

(2) Given some function and its derivative:

$$f(x) = xe^x - 1 \qquad\qquad f'(x) = xe^x + e^x$$

and assuming an initial bracket of -1 to 1 (meaning an initial guess $x_1 = 0$), determine the *next* three approximations for a root of $f(x)$ using bisection (that is, determine x_2, x_3, and x_4). Keep four significant figures.

k	$x_l(k)$	$f(x_l(k))$	sign	$x_m(k)$	$f(x_m(k))$	sign	$x_u(k)$	$f(x_u(k))$	sign
1	-1.0000	-1.3679	$-$	0.0000	-1.0000	$-$	1.0000	1.7183	$+$
2	0.0000	-1.0000	$-$	0.5000	-0.1756	$-$	1.0000	1.7183	$+$
3	0.5000	-0.1756	$-$	0.7500	0.5878	$+$	1.0000	1.7183	$+$
4	0.5000	-0.1756	$-$	0.6250	0.1677	$+$	0.7500	0.5878	$+$

(3) Show all the MATLAB code to find the root of $f(x)$ that lies between $x = -1$ and $x = 1$. The code is started for you:

```
clear; format short e

% long version
f = @(x) x*exp(x)-1;
xVal = fzero(@(xD) f(xD), [-1 1]);

% medium version
f = @(x) x*exp(x)-1;
xVal = fzero(f, [-1 1]);

% one line version
xVal = fzero( @(x) x*exp(x)-1, [-1 1]);
```

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Problem III: [25 pts.] Quick Fluid Density Checker

One of the Grand Challenges for Engineering involves providing access to clean water. One quick check on water quality is to determine its density. A rapid density tester can be built based on Archimedes' principle - specifically the idea that the buoyancy force exerted on a submerged object is equal to the weight of the fluid displaced by the object. Given that, a sphere made from a material that is less dense than water should float. For example, if the sphere is half as dense as water, half the sphere will be below the surface and half will be above. If the sphere is less dense, less of the sphere will be below the surface.

If the sphere is less than half the density of the fluid, the location of the bottom of the sphere (the depth below the surface) can be found as an equation of the radius of the sphere and the ratio of the density of the sphere to the density of fluid[1]

$$d^3 + 4sr^3 = 3rd^2$$

where d is how far under the surface of the fluid the bottom of the sphere is, r is the radius of the sphere, and s is the *specific gravity* of the sphere - the ratio of the density of the sphere to the density of the water.

Your job is to find the relationship between d and s for some constant r value. Write a script file that asks the user to input a positive value for the radius and then checks to make sure the user gave some positive number; keep asking until the user gives a positive value for the radius. Then, calculate an array of d values for s values between 0.01 and 0.45 with an increment of 0.01. Make a plot of the ~~height~~ depth as a function of the specific gravity using a black straight line. For this problem, include a proper title and axis labels. To get full credit, your program must be written in such a way that only physically realizable ~~heights~~ depths are found.

```
% Get and validate input
r = input('Radius (larger than 0): ');
while r<=0
    r = input('Radius (No, really...LARGER THAN 0): ');
end

% Define array of s values
s = .01:.01:.5

% Solve for zeros of equation - pick an option
%% Option 1 - using fzero
%% NOTE!  Initial bracket is required using 0 will produce negative results!
f = @(d, s, r) d.^3 + 4 .* s .* r.^3 - 3 .* r .* d .^2
for k=1:length(s)
    d(k) = fzero(@(dD) f(dD, s(k), r), [0 r]);
end

%% Option 2 - using roots
%% NOTE!  roots gives all three possible roots use logic to find
%% the physically realizable one.  Problem statement indicates for 0<s<.5, 0<d<r
for k=1:length(s)
    AllRoots = roots([1 -3*r 0 4*s(k)*r.^3]);
    RightRootLoc = find(AllRoots>0 & AllRoots<r)
    d(k) = AllRoots(RightRootLoc)
end

% Make plot with labels and title
plot(s, d, 'k-')
xlabel('Specific Gravity');
ylabel('Depth');
title('Depth vs. Specific Gravity for a Density Tester')
```

[1]Numerical Methods with MATLAB: Implementations and Applications, pp. 281-283. Gerald Recktenwald. Prentice Hall, 2000

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Problem IV: [20 pts.] Virtual Reality

- A virtual reality game to simulate fencing would need to calculate when and where the blades cross. Sensors can be placed at the top and base of the blade to determine an equation for the straight line connecting the two points. Assuming "infinitely long blades," the blades will cross at the intersection of the two lines. In a 2-D system, the equations of the lines will be given as functions of x and y. Assuming the equations for each blade are as follows:

$$x + y = 6 \qquad\qquad 4x + 7y = 10$$

(1) Clearly show the equations above organized as a linear algebra equation.
(2) Clearly show the steps required to determine the x and y coordinate of the intersection and present those values.
(3) What is the condition number of the linear system using the Frobenius norm?
(4) Assuming the sensors are capable of measuring values to five significant figures, how many significant figures will you depend on in your answer above? Why do you think that?

$$\begin{bmatrix} 1 & 1 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \text{inv}\left(\begin{bmatrix} 1 & 1 \\ 4 & 7 \end{bmatrix}\right) \begin{bmatrix} 6 \\ 10 \end{bmatrix} = \frac{\begin{bmatrix} 7 & -1 \\ -4 & 1 \end{bmatrix}}{7-4} \begin{bmatrix} 6 \\ 10 \end{bmatrix} = \frac{\begin{bmatrix} (7)(6) + (-1)(10) \\ (-4)(6) + (1)(10) \end{bmatrix}}{3} = \begin{bmatrix} +\frac{32}{3} \\ -\frac{14}{3} \end{bmatrix}$$

cond(A,'fro') = norm(A, 'fro')*norm(inv(A), 'fro')

$$\text{cond(A,'fro')} = \sqrt{1^2 + 4^2 + 1^2 + 7^2} * \sqrt{\left(\frac{7}{3}\right)^2 + \left(-\frac{4}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{67}{3} = 2.233e+01$$

Since log10(A) of the condition number is between 1 and 2, you will lose between 1 and 2 digits, leaving between 3 and 4 significant figures.

- In another virtual reality game, the corner of a room is defined as the point at which the wall, the floor, and the ceiling intersect. The program can determine the equation for each of these by placing three sensors on each surface and determining the equation of a plane from those three sensors. Assuming the equations for the three planes are as follows:

$$x + z = 4 \qquad\qquad -3x + 2y + 2z = 5 \qquad\qquad -3x - 2y + z = -2$$

(1) Clearly show the equations above organized as a linear algebra equation.
(2) By hand, clearly show the determinant of the coefficient matrix. Merely producing a number will give you no credit.
(3) Clearly show all the MATLAB code to determine the x, y, and z coordinate of the intersection and present those values. At the end of your program, there must be a 3x1 column matrix with x as its first element, y as its second element, and z as its third element. Call this matrix `MyCorner`.
(4) Show the MATLAB code required to calculate the condition number of the linear system, using the 2 norm.

$$\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & 2 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$$

$$\text{det} = (1)(2)(1) + (0)(2)(-3) + (1)(-3)(-2) - (-3)(2)(1) - (-2)(2)(1) - (1)(-3)(0) = 18$$

```
A = [1 0 1; -3 2 2; -3 -2 1];
b = [4; 5; -2]
MyCorner = A\b
cond(A)
```

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Problem V: [20 pts.] Pipe Flow Equations

A couple researchers take data of fluid flow in a concrete pipe given various pipe diameters and slopes and come up with the following data[2]

Experiment	Diameter (D)	Slope (S)	Flow (Q)
1	0.6	0.01	0.82
2	0.6	0.05	1.95
3	0.9	0.01	2.38
4	0.9	0.05	5.66

One of their graduate students comes up with the following model for the flow:

$$\hat{Q}_{\text{powers}} = 36\ D^{2.7}\ S^{0.5}$$

while another comes up with the model

$$\hat{Q}_{\text{plane}} = -2.3 + 4.6\ D + 45\ S$$

Mathematically, which is the better model? Clearly show all work and calculations used to defend your conclusion. Simply picking a model, or providing insufficient proof, will yield no credit.

Experiment	Q	$\hat{Q}_{\text{powers}}$	$(Q - \hat{Q}_{\text{powers}})^2$	$\hat{Q}_{\text{plane}}$	$(Q - \hat{Q}_{\text{plane}})^2$
1	0.82	$9.0638e-01$	$7.4617e-03$	$9.1000e-01$	$8.1000e-03$
2	1.95	$2.0267e+00$	$5.8875e-03$	$2.7100e+00$	$5.7760e-01$
3	2.38	$2.7087e+00$	$1.0803e-01$	$2.2900e+00$	$8.1000e-03$
4	5.66	$6.0568e+00$	$1.5744e-01$	$4.0900e+00$	$2.4649e+00$

$$S_{\text{r,powers}} = \sum_{k=1}^{4}(Q - \hat{Q}_{\text{powers}})^2 = 0.27882$$

$$S_{\text{r,plane}} = \sum_{k=1}^{4}(Q - \hat{Q}_{\text{plane}})^2 = 3.0587$$

Since the sum of the squares of the estimate residuals for the powers model is smaller, it must have a larger r^2 value and is thus the better model. Specifically:

$$\bar{Q} = \frac{0.82 + 1.95 + 2.38 + 5.66}{4} = 2.7025$$

$$S_{\text{t}} = \sum_{k=1}^{4}(Q - \bar{Q})^2 = 12.961$$

$$r^2_{\text{powers}} = \frac{12.961 - 0.27882}{12.961} = 0.97849$$

$$r^2_{\text{plane}} = \frac{12.961 - 3.0587}{12.961} = 0.76401$$

[2] Applied Numerical Methods with MATLAB, p. 233. Steven Chapra. McGraw-Hill, 2005