Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2007

Test III

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Name (please print)______________________________

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:______________________________

Important Notes

(1) For this test, efficiency of code *does matter*. In some cases, there may be several different ways to approach a problem. If your method is quite a bit more complicated or longer than it needs to be, there will be a deduction. For example, if a problem were to ask that you calculate the sum of the variables in some 1x12 `a` vector, and you write `a(1)+a(2)+...` when the right answer is `sum(a)`, you will not receive full credit.

(2) For the calculations in Problem III, you do not need to reduce your interpolating functions, but you will need to calculate some single final value for the estimate of the speed.

(3) For the calculations in Problem IV, as long as you clearly set up the calculations you would perform, you do *not* actually have to reduce everything to a final answer. Conversely, if merely a final answer appears with no supporting work, you will receive no credit.

Problem I: [25 pts.] General Linear Models and Interpolation

For the given data set of bearing life as a function of temperature[1]:

Temp (T, °F)	100	120	140	160	180	200	220
Bearing life (kilohours)	28	21	15	11	8	6	4

and assuming the data already exists in MATLAB as column vectors called `Temp` and `BLife`, respectively, write the MATLAB code (probably on a new sheet of paper...with your name and NET ID on it...that you end up stapling to this sheet of paper) that will:

(a) Determine the coefficients for a 2nd-order polynomial fit of the data and call them `MyCoefsA`

(b) Determine the coefficients for a polynomial interpolation of the data and call them `MyCoefsB`

(c) Determine the coefficients for the linear model:

$$\text{Bearing Life} = \text{MyCoefsC}(1) + \text{MyCoefsC}(2) * \frac{1}{T + 459.69} + \text{MyCoefsC}(3) * \frac{1}{(T + 459.69)^2}$$

and call them `MyCoefsC`

(d) Determine estimates for the bearing life when T is 150°F using each of the models - call the estimates `BLA`, `BLB`, and `BLC`, respectively

(e) Calculate the coefficient of determination for the 2nd-order polynomial fit of the data.

[1] Problem and data adapted from Palm, Chapter 5, Problem 34, p. 349

(a)

```
MyCoefsA = polyfit(Temp, BLife, 2)
```

(b)

```
MyCoefsB = polyfit(Temp, BLife, 6)
% or
MyCoefsB = polyfit(Temp, BLife, length(BLife)-1)
```

(c)

```
A = [Temp.^0 1./(Temp+459.69) 1./(Temp+459.69).^2]
MyCoefsC = A\BLife
% or
MyCoefsX = polyfit(1./(Temp+459.69), BLife, 2)
MyCoefsC = MyCoefsX(3:-1:1)
```

(d)

```
BLA = polyval(MyCoefsA, 150)
BLB = polyval(MyCoefsB, 150)
% method below uses dot product; multiple methods available
BLC = [1 1./(150+459/69)  1./(150+459/69).^2]*MyCoefsC'
```

(e)

```
BLifeHat = polyval(MyCoefsA, Temp)
St = sum((BLife - mean(BLife)).^2)
Sr = sum((BLife - BLifeHat).^2)
r2 = (St - Sr) / St
```

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Problem II: [25 pts.] Nonlinear Models

For the given data set of metabolism as a function of mass[2]:

Mass (kg)	0.16	0.3	2	45	70	400
Metabolism (W)	0.97	1.45	4.8	50	82	270

and assuming the data already exists in MATLAB as column vectors called `Mass` and `Meta`, respectively,

```
% Assumptions:
Mass = [0.16 0.3  2   45 70 400]'
Meta = [0.97 1.45 4.8 50 82 270]'
```

write the MATLAB code that will:

(1) Determine the coefficients of the power-law model:

$$\text{Meta} = A * \text{Mass}^B$$

using linearization and transformed variables. Be sure your code clearly indicates the final values for the parameters `A` and `B`

```
P = polyfit(log(Mass), log(Meta), 1)
A = exp(P(2))
B = P(1)
% alternately:
P = polyfit(log10(Mass), log10(Meta), 1)
A = 10^(P(2))
B = P(1)
```

(2) Determine the coefficients of the saturation-growth-rate model:

$$\text{Meta} = C * \frac{\text{Mass}}{D + \text{Mass}}$$

using linearization and transformed variables. Be sure your code clearly indicates the final values for the parameters `C` and `D`

```
P = polyfit(1./Mass, 1./Meta, 1)
C = 1./P(2)
D = P(1)./P(2)
```

(3) Determine the coefficients of the nonlinear model:

$$\text{Meta} = E * \text{Mass} * e^{-(F*\text{Mass})}$$

using nonlinear regression. Be sure your code clearly indicates the final values for the parameters `E` and `F`.

```
MetaEqn = @(coefs, Mass) coefs(1)*Mass.*exp(-coefs(2)*Mass)
fSSR = @(coefs, Mass, Meta) sum((Meta - MetaEqn(coefs, Mass)).^2)
MyCoefs = fminsearch(@(MyCoefsD) fSSR(MyCoefsD, Mass, Meta), [1 1])
E = MyCoefs(1)
F = MyCoefs(2)
```

[2]Problem and data adapted from Chapra, Chapter 13, Problem 13.12, p. 313

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Problem III: [25 pts.] Interpolation and Integration

For the given data set taken from a motor initially at rest[3]:

Time (sec)	0	3	6	9	12	15	18	21	24	27	30
Ang. Speed (ω, rad/sec)	0	126	195	240	268	285	301	301	305	312	315

(1) Determine, by hand, the piecewise linear interpolating function that could be used to estimate the speed for times between 6 and 9 seconds. Use the function to estimate the speed when t=8 sec.

$$\omega = \omega_6 + \frac{\omega_9 - \omega_6}{t_9 - t_6}(t - t_6) = 195 + \frac{240 - 195}{9 - 6}(t - 6) = 195 + 15(t - 6) = 105 + 15t$$
$$\omega_8 = 105 + 15 * 8 = 225$$

(2) Determine, by hand, the quadratic polynomial interpolating function that could be used to estimate the speed for times between 3 and 9 seconds. Then use the function to estimate the speed when t=8 sec.

$$\omega = \omega_3 + \frac{\omega_6 - \omega_3}{t_6 - t_3}(t - t_3) + \left(\frac{\frac{\omega_9 - \omega_6}{t_9 - t_6} - \frac{\omega_6 - \omega_3}{t_6 - t_3}}{t_9 - t_3}\right)(t - t_3)(t - t_6)$$
$$\omega = 126 + \frac{195 - 126}{6 - 3}(t - 3) + \left(\frac{\frac{240 - 195}{9 - 6} - \frac{195 - 126}{6 - 3}}{9 - 3}\right)(t - 3)(t - 6)$$
$$\omega = 126 + 23(t - 3) - 1.33(t - 3)(t - 6)$$
$$\omega_8 = 126 + 23(5) - 1.33(5)(2) = 227.66$$

(3) Assuming variables `t` and `omega` exist as column vectors of the data above, write the MATLAB code that will generate a graph containing four different interpolating functions - plotted using 100 evenly spaced points between 0 and 30 - represented using the line styles given below:

(a) Piecewise linear interpolation (solid black line)
(b) Polynomial interpolation (dotted black line)
(c) Cubic spline interpolation with not-a-knot conditions (dashed black line)
(d) Cubic spline interpolation assuming the angular acceleration (first derivative of the angular speed) of the motor is *known* to be 20 rad/s^2 at time 0 sec and 0 rad/s^2 at time 30 sec (dash-dot black line)

You do not need to include a legend, a title, or axis labels, nor do you need to save the plot.

```
tmodel = linspace(min(t), max(t), 100);
PWLinear        = interp1(t, omega, tmodel, 'linear');
PolyInterp      = polyval(polyfit(t, omega, length(t)-1), tmodel);
NAKSplineInterp = spline(t, omega, tmodel)
% or interp1(t, omega, tmodel, 'spline');
ClampedSpline   = spline(t, [20; omega; 0], tmodel)

plot(...
    tmodel, PWLinear,        'k-',...
    tmodel, PolyInterp,      'k:',...
    tmodel, NAKSplineInterp, 'k--',...
    tmodel, ClampedSpline,   'k-.')
```

[3] Problem and data adapted from Palm, Chapter 5, Problem 21, pp. 344-345

(4) Assuming variables `t` and `omega` exist as column vectors of the data above, write the MATLAB code that will:

(a) Determine the *total integral* from t=0 sec to t=30 sec using the trapezoidal rule

(b) Graph the cumulative integral using the trapezoidal rule from t=0 sec to all the times between 0 sec and 30 sec using black squares. You do not need to include a legend, a title, or axis labels, nor do you need to save the plot.

```
% a
TotalTrapz = trapz(t, omega)
% b
plot(t, cumtrapz(t, omega), 'ks')
```

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Problem IV: [25 pts.] Integrals and Derivatives (Hand Calculations)

Again using the data from the previous problem:

Time (t, sec)	0	3	6	9	12	15	18	21	24	27	30
Ang. Speed (ω, rad/sec)	0	126	195	240	268	285	301	301	305	312	315

Note: $\Delta\ t$ is a constant 3 sec
By hand:

(1) Determine the angular acceleration (α, the derivative of angular speed) of the motor at t=9 sec using 2-point forward differences. That is, calculate:

$\alpha(9) = \frac{d\omega}{dt}\Big|_9 = \frac{\omega_{12}-\omega_9}{3} = \frac{268-240}{3} = \frac{28}{3} = 9.33$

(2) Determine the angular acceleration of the motor at t=18 sec using 2-point backward differences.

$\alpha(18) = \frac{d\omega}{dt}\Big|_{18} = \frac{\omega_{18}-\omega_{15}}{3} = \frac{301-285}{3} = \frac{16}{3} = 5.33$

(3) Determine the angular acceleration of the motor at t=6 sec using 3-point differences.

$\alpha(6) = \frac{d\omega}{dt}\Big|_6 = \frac{\omega_9-\omega_3}{2*3} = \frac{240-126}{6} = \frac{114}{6} = 19$

(4) Determine the angular acceleration of the motor at t=30 sec using 3-point differences.

$\alpha(30) = \frac{d\omega}{dt}\Big|_{30} = \frac{3\omega_{30}-4\omega_{27}+\omega_{24}}{2*3} = \frac{3(315)-4(312)+(1)(305)}{6} = \frac{2}{6} = 0.33$

(5) Determine the angular jerk (j_{ang}, the second derivative of angular speed) of the motor at t=12 sec using 3-point differences. That is, calculate:

$j_{\text{ang}}(12) = \frac{d^2\omega}{dt^2}\Big|_{12} = \frac{\omega_{15}-2\omega_{12}+\omega_9}{3^2} = \frac{285-2(268)+240}{9} = -\frac{11}{9} = -1.22$

(6) Determine the angular jerk of the motor at t=0 sec using 3-point differences.

$j_{\text{ang}}(0) = \frac{d^2\omega}{dt^2}\Big|_{0} = \frac{\omega_6-2\omega_3+\omega_0}{3^2} = \frac{195-2(126)+0}{9} = -\frac{57}{9} = -6.33$

(7) Determine the angle turned by the motor (θ, the integral of speed) for times between 0 and 9 sec using the most accurate numerical integral discussed in class for that number of points. That is, calculate:

$\theta_{0\to 9} = \int_0^9 \omega(t)\ dt = \frac{3}{8}(3)\left(\omega_0 + 3\omega_3 + 3\omega_6 + \omega_9\right) = \frac{9}{8}(0 + 3(126) + 3(195) + 240) = 1353.4$

(8) Determine the angle turned by the motor for times between 9 and 24 sec using the most accurate numerical integral discussed in class for that number of points.

$\theta_{9\to 24} = \int_9^{24} \omega(t)\ dt = \frac{1}{3}(3)\left(\omega_9 + 4\omega_{12} + \omega_{15}\right) + \frac{3}{8}(3)\left(\omega_{15} + 3\omega_{18} + 3\omega_{21} + \omega_{24}\right) =$
$(1)(240 + (4)(268) + 285) + \frac{9}{8}(285 + 3(301) + 3(301) + 305) = 4292.5$

(9) Determine the angle turned by the motor for times between 24 and 30 sec using the most accurate numerical integral discussed in class for that number of points.

$\theta_{24\to 30} = \int_{24}^{30} \omega(t)\ dt = \frac{1}{3}(3)\left(\omega_{24} + 4\omega_{27} + \omega_{30}\right)\ (1)(305 + (4)(312) + 315) = 1868$