Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2006

Test III

Rebecca A. Simmons
Michael R. Gustafson II

Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Problem I: [20 pts.] Linear Models

Palm Problem 5.40 had the following problem setup: "The number of twists T required to break a certain rod is a function of the percentage x and y of each of two alloying elements present in the rod." (variable names changed from original). Given the data from that problem:

y, %	x=1%	x=2%	x=3%
1	40	38	31
2	51	46	39
3	65	53	48
4	72	67	56

and assuming the data fits a plane; that is:

$$\hat{z} = A + Bx + Cy$$

write the Matlab code that will:

(1) Determine the values of `A`, `B`, and `C` for the model,

(2) Determine the coefficient of determination for the model, and

(3) Generate and plot the model as a surface with contour lines. Use 20 points in each direction for your model points. You must label the axes, but do not need a title.

You may assume the following line of code already exists:

```
z = [40 38 31; 51 46 39; 65 53 48; 72 67 56]
```

Name (please print):
Community Standard (print ACPUB ID):

Problem II: [20 pts.] Nonlinear Models

The following data set was obtained in an experiment:

x	1	2	3	4	5
y	1.0489	1.1910	1.4122	1.6910	2.0000

where x represents the independent data and y represents the dependent data. Assuming the following lines of code are already in your script:

```
clear
x = (1:5)';
y = [1.0489 1.1910 1.4122 1.6910 2.0000]'
```

(1) Write the code you would use to find the coefficients `alpha1` and `beta1` for an exponential model, `alpha2` and `beta2` for a power-law model, and `alpha3` and `beta3` for a saturation-growth model, all using untransformed variables. The models are given below:

$$\hat{y}_1 = \alpha_1 e^{\beta_1 x} \qquad \hat{y}_2 = \alpha_2 x^{\beta_2} \qquad \hat{y}_3 = \frac{\alpha_3 x}{\beta_3 + x}$$

(2) Using Matlab, the following estimates are obtained:

x	1	2	3	4	5	S_t	r^2
y	1.0489	1.1910	1.4122	1.6910	2.0000		N/A
Exponential Model, $\hat{y}_1$	1.0232	1.2089	1.4283	1.6875	1.9937	N/A	
Power-Law Model, $\hat{y}_2$	0.9307	1.2655	1.5147	1.7207	1.8996	N/A	0.9303
Saturation-Growth Model, $\hat{y}_3$	0.8702	1.3007	1.5576	1.7282	1.8498	N/A	0.8486

By hand, calculate S_t for the data and r^2 for the exponential model, and based on all the information state which of the three models provides the best *numerical* fit for the data and why.

Name (please print):
Community Standard (print ACPUB ID):

Problem III: [20 pts.] Linearized Models

In chemistry, the rate constant (k, 1/mol·sec) for a reaction can be expressed as a function of activation energy and temperature using the Arrhenius Equation:

$$k = Ae^{(-E_a/RT)}$$

where Ea is the activation energy in J/mol, R is the gas constant (use 8.314 J/(K·mol)), T is temperature in K, and A is a constant called the frequency factor, in 1/mol·sec. The following data set was collected (in the book **Chemistry**, by Chang) for the decomposition of acetaldehyde:

T (K)	730	760	790	810
k (1/mol·sec)	0.035	0.105	0.343	0.789

(1) Determine the linearized model you could use to determine E_a and A. Note that R is known, T is the independent variable, and k is the dependent variable. *Note:* if you cannot get this step, perform the rest of the problem using the transformed model:

$$\cos(k) = \cos(A) + \frac{E_a}{R}\sin(T)$$

(2) Set up, but do not solve, the linear system you would need in order to determine E_a and A based on transformed variables.

(3) Write the code that would allow you to

(a) solve for E_a and A using transformed variables,
(b) generate estimates of the rate constant, and
(c) determine the goodness of fit.

Name (please print):
Community Standard (print ACPUB ID):

Problem IV: [25 pts.] Interpolation

The book **Fundamentals of Thermodynamics**, by Moran, uses a fourth-order polynomial in temperature (T, in K) to approximate values for the specific heat of carbon monoxide (c_p, in kJ/kmol·K):

$$c_p = \alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4$$

Using the following data:

T (K)	400	450	500	550	600
c_p (kJ/kmol·K)	31.96	33.35	35.15	37.36	40.05

(1) Using nearest-neighbor interpolation by hand, what is an estimate of c_p when T=480 K?

(2) Using linear interpolation by hand, what is an estimate of c_p at 480 K?

(3) What is the first-order piecewise linear equation for estimates where T is between 500 K and 550 K?

(4) Show the Matlab code you could use to get the coefficients in the fourth-order model from the data in the table. Create variables called `alpha`, `beta`, `gamma`, `delta`, and `epsilon` that contain the proper values from the above model.

(5) By hand, estimate the first derivative of c_p as a function of T at $T = 400$ K and at $T = 600$ K using the most precise method we have discussed.

(6) Write the code that would allow you to:

(a) Calculate the values for the first derivative at the first and last points using the most precise method we have discussed, and store the values in arrays called `dcdTFirst` and `dcdTLast`,

(b) generate an array of model T values from 400 K to 600 K with 50 points, and store the values in an array called `Tm`,

(c) interpolate the data with clamped cubic splines using your derivative estimates and the model T values, and store the values in an array called `cpC`, and

(d) interpolate the data with cubic splines using the model T values and not-a-knot conditions at the end, and store the values in an array called `cpNK`.

Name (please print):
Community Standard (print ACPUB ID):

Problem V: [15 pts.] Integration

An accelerometer is a device that is used to ~~meter acceleros~~ measure acceleration. The data set below was obtained from a vehicle traveling in a straight line down a closed section of interstate highway to test both acceleration and braking. The vehicle was initially at rest at mile marker 0. Note therefore that:

$$v(t) = \int_0^t a(\tau)\ d\tau \qquad\qquad r(t) = \int_0^t v(\tau)\ d\tau$$

where a is the magnitude of acceleration, v is the speed, r is location on the road, t is time, and τ is a dummy variable for integrating time. Note also that the units for acceleration are given in mph/sec (miles per hour per second), and that time is given in seconds.

(1) Estimate the speed at all times using the most accurate method available at each point.

Time	Acc.	Speed
t (sec)	a (mph / sec)	v (mph)
0	0.75	
4	1.75	
8	3.00	
12	6.50	
16	8.00	
20	1.50	
24	-5.50	
28	-8.00	
32	-6.00	
36	-2.50	

(2) Using your speeds above, estimate the total distance traveled using the most accurate method available. Show your work, and be sure to include units.