Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2006

Test I

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Name (please print)_____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:_____

Problem I: [10 pts.] Binary

Convert the following numbers either into binary or into decimal notation. Be sure to clearly show your work in doing so, as merely reporting the correct answer will receive little credit.

(1) 11001.01101_2 to decimal

(2) 228_{10} to binary

Name (please print):
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Problem II: [20 pts.] Finding Roots

(1) Given some function and its derivative:

$$f(x) = x^2 + \sin(x) - 1 \qquad\qquad f'(x) = 2x + \cos(x)$$

and assume an initial guess for a root $x(1)$=1, determine the *next* three approximations for a root of $f(x)$ using the Newton-Raphson method. Note that the trigonometric terms should take arguments in radians. Keep four significant figures. Also indicate the final values for the x tolerance and the f tolerance.

(2) Given some function and its derivative:

$$f(x) = x^2 + \sin(x) - 1 \qquad\qquad f'(x) = 2x + \cos(x)$$

and assuming an initial bracket of -1 to 1 (meaning an initial guess $x(1) = 0$), determine the *next* three approximations for a root of $f(x)$ using bisection. Note that the trigonometric terms should take arguments in radians.Keep four significant figures. Also indicate the final values for the x tolerance and the f tolerance.

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Problem III: [15 pts.] Finding Roots II

(1) Given some function $g(x) = x^4 - 3x^2 + x - 1$ write the Matlab code you would use to find all roots - including the complex and imaginary ones - for $g(x)$.

(2) Given some function $f(a, x) = x\ \sin(ax) - 1$, write the Matlab code you would use to find the x value for which $f(0.35, x) = 0$. Assume an initial guess of $x = 1$.

(3) Given some function $f(a, x) = x\ \sin(ax) - 1$, write the Matlab code you would use to generate and plot an array of x values for which $f(a, x) = 0$ if a=0:1:10. Assume an initial guess of $x = 1$. Plot x as a function of a using black circles. You do not need to label or title this plot.

Name (please print):
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Problem IV: [20 pts.] Norms, Conditions, and Statistics

Given:

```
r = [1 -2 3 -4]';
s = [3 1 4 -5]';
t = [7 8 0 0]';
v = [5 4; 6 12]
```

determine the following quantities by hand **and** then write the Matlab code you would use to calculate them.

(1) `a`=$\|[r\ s]\|_1$

(2) `b`=$\|v(:,1)\|_1$

(3) `c`=$\|t * s'\|_\infty$

(4) `d`=$\|r\|_2$

(5) `e`=$\bar{s}$

(6) `f`=$\|v\|_1$

(7) `g`=$\|v\|_e$

(8) `h`=$\|v\|_\infty$

(9) `i`=condition number of v, based on 1-norm

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Problem V: [20 pts.] Linear Algebra

A paint store was overzealous last year and bought a surplus of two popular paint colors, but this year these colors are very unpopular. So the paint store has decided to recycle these unpopular paint colors by mixing them together to form this year's popular paint color. Each paint color is composed of 2 basic pigments: A and B. The unpopular colors have the following pigment composition:

Paint number	Pigment A	Pigment B
1	70.00	30.00
2	10.00	90.00

This year's desirable color has a pigment composition of: 33% pigment A and 67% pigment B.

(1) How much of each unpopular paint should be mixed together to form 1 gallon of the popular color? Let x_j be the fraction of unpopular colored paint j that is used to form the new popular color. Clearly set up the linear system and then solve by hand for x_1 and x_2. You must show your work - merely producing the correct answer will receive little credit. Also, you must solve using the inverse of the matrix - back-substitution may be used to check your work, but will also receive little credit.

(2) Calculate the condition number of this system by hand using the Frobenius norm.

(3) Given that the paint pigment percentages are known to four significant figures, what does the condition number say about the precision of your answer above?

(4) Write the Matlab code to solve the first two parts of this problem.

(Based on a problem from Numerical Methods with Matlab by Rechtenwald).

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Problem VI: [15 pts.] Linear Algebra II

(1) Assuming you have determined the following equations to be true:

$$\begin{aligned} x + 4y + 9z &= 10 \\ -x + 5z &= 3 \\ 3x + 6y - z &= 4 \end{aligned}$$

show the Matlab code you would use to solve for x, y, and z. In other words, at the end of your code, the variables `x`, `y`, and `z` should exist as 1x1 matrices containing the appropriate values. Note that you should **not** try to solve this by hand.

(2) Does a unique solution exist to this set of equations? You must clearly show your proof for your answer.

(3) Given the over-determined system:

$$\begin{aligned} x + y + z &= 6 \\ 2x - z &= 3 \\ 3y - 2z &= x + 5 \\ 6x - 2y &= 6z \end{aligned}$$

Show the Matlab commands to solve for x, y, and z. At the end of your code x, y, and z should exist as 1x1 matrices containing the appropriate values.