Duke University
Edmund T. Pratt, Jr. School of Engineering

IKV 97P Jepp644:
XiwxM
Vifigge E2Wmqqsrw
Qmgleipv2Kywxejwsr MM

Reqi,tpiewitvmrx:

Mr oiitmrk {mxl xli Gsqqyrmx} Wxerhevh0Mlezi rimkliv tvszmhih rsv viginzih er} ewwmxergi sr xlmw xiwx2 Myrhiwxerh mj mx mw pexiv hixivq mrih xlex Mkezi sv viginzih ewwmxergi0M{mpofi fvsyklx fijsvi xli Yrhivkvehyexi Nyhmgmep Fsevh erh0mj jsyrh viwtsrwfpi jsv egehiqmg hmwlsriwx} sv egehiqmg gsrxiq tx0jempxli gpeww2 Mepws yrhiwxerh xlex Meq rsx epps{ih xs wtieo xs er}sri i|gitx xli mrwxvygxsv efsyx er} ewtigx sj xlmw xiwx yrxmpxli mrwxvygxsv errsyrgiwmx mw epps{ih2 Myrhiwxerh mj mx mw pexiv hixivq mrih xlex Mhmh wtieo xs ersxliv tiwsr efsyx xli xiwx fijsvi xli mrwxvygxsv wemh mx{ewepps{ih0M{mpofi fvsyklx fijsvi xli Yrhivkvehyexi Nyhmgmep Fsevh erh0mj jsyrh viwtsrwfpi jsv egehiqmg hmwlsriwx} sv egehiqmg gsrxiq tx0jempxli gpeww2

Wmkrexyvi:

Tvsfpiq M>_54 txw2a Fmrev}

Gsrzivx xli jspps{mrk ryq fiw imkliv mrxs fmrev} sv mrxs higmqep rsxexmsr2 Fi wyvi xs gpievp} wls{ }syv{svo mr hsmrk ws0ewqivip} vitsvxmrk xli gsvvigx erw{iv{mpviginzi pmxpigvihmx2

,5- 55445245545$_6$ xs higmqep

,6- 66<$_{54}$ xs fmrev}

(1) 1 1 0 0 1 . 0 1 1 0 1
2^4 2^3 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4} 2^{-5}

16 + 8 + 0 + 0 + 1 + 0 + .25 + .125 + 0 + .03125

= 25.40625$_{10}$

(2) 228/2 = 114 R 0 LSB
114/2 = 57 R 0
57/2 = 28 R 1
28/2 = 14 R 0
14/2 = 7 R 0
7/2 = 3 R 1
3/2 = 1 R 1
1/2 = 0 R 1 MSB

= 11100100$_2$

Reqi,tpiewitvmrx->
Gsqqyrmx} Wxerhevh,tvmrxEGTYF MH->

Tvsfpiq MM>_64 txw2aJmrhmrk Vssxw

,5- Kmzir wsqijyrgxmsr erh mxwhivmzexmzi>

j,|-A |^6/ wmr,|--5 j^4,|-A 6|/ gsw,|-

erh ewwyq ier mrmxmepkyiww jsve vssx|,5-A50hixivq mri xli ri|xxlvii ettvs|mq exmsrw jsve vssx sjj,|-ywmrk xli Ri{xsr1Vetlwsr q ixlsh2R sxi xlex xli xvmksrsq ixvmg xivq wwlsyph xeoi evkyq irxwmr vehmerw2O iit jsyv significanx figures. Epws mrhmgexi xli final zepyiw jsv xli | xspivergi erh xli j xspivergi2

k	x(k)	f(x(k))	f'(x(k))	-f/f'	x(k+1)
1	1	0.8415	2.540	-.3313	.6687
2	.6687	$6.713\cdot10^{-2}$	2.122	$-3.164\cdot10^{-2}$	.6371
3	.6371	$7.633\cdot10^{-4}$	2.078	$-3.673\cdot10^{-4}$	.6367
4	.6367	$-6.783\cdot10^{-5}$			

Final $f_{TOL} = 6.783\cdot10^{-5}$ | $x_{TOL} = 3.673\cdot10^{-4}$ |

,6- Kmzir wsqijyrgxmsr erh mxwhivmzexmzi>

j,|-A |^6/ wmr,|--5 j^4,|-A 6|/ gsw,|-

erh ewwyq mrk er mrmxmepfvegoix sj 15 xs 5 ,qiermrk er mrmxmepkyiww|,5-A 4-0hixivq mri xli ri|xxlvii ettvs|mq exmsrw jsve vssx sjj,|-ywmrk fmwigxmsr2R sxi xlex xli xvmksrsq ixvmg xivq wwlsyph xeoi evkyq irxwmr vehmerw2O iit jsyv significanx figures. Epws mrhmgexi xli final zepyiw jsv xli | xspivergi erh xli j xspivergi2

sign(f(x_L))	x_L		x_M	f(x_M)	x_R	sign(f(x_R))
-	-1	⇙	0	-1	1	+
-	0	⇙	.5	-.2706	1	+
-	.5		.75	.2441 ⇘	1	+
-	.5		.625	-.02428	.75	+

Final $f_{TOL} = .02428$ | $x_{TOL} = \frac{.75-.5}{2} = 0.125$ |

Reqi,tpiewitvmrx->
Gsqqyrmx}Wxerhevh,tvmrxEGTYF MH->

Tvsfpiq MMM>_59 txw2aJmrhmrk Vssxw MM

,5- Kmzir wsqijyrgxmsrk,|-A |^{8} — 7|^{6} / | — 5{vmxixliQexqefgshi}sy{syph ywixs find epgpvssxw1mrgpyhmrk xligsqtpi|erhmqekmrev}sriw1jsvk,|-2

```
MyRoots = roots([1 0 -3 1 -1])
```

,6- Kmzir wsqijyrgxmsrj,e?|-A | wmr,e|-- 50{vmxixliQexqefgshi}sy{syph ywixs find xli | zepyijsv{lmgl j,4>79?|-A 42Ewwyqierrmxmepkyiwwsj|A 52

```
f = inline('x.*sin(a.*x)-1', 'a', 'x');
xloc = fzero(@(xi)f(0.35, xi), 1)
```

,7- Kmzir wsqijyrgxmsrj,e?|-A | wmr,e|-- 50{vmxixliQexqefgshi}sy{syph ywixs kirivexierh tpsxer evve}sj|zepyiwjsv{lmgl j,e?|-A 4mjeA4>5>542Ewwyqierrmxmepkyiwwsj|A 52Tpsx|ewejyrgxmsrsje ywmrkfpegogmvgpiw2]syhsrsxriihxspefipsvxmxpixlmwtpsx2

```
f = inline('x.*sin(a.*x)-1', 'a', 'x');
a = 0:1:10;
k = 1:length(a)
     xloc(k) = fzero(@(xi)f(a(k), xi), 1);
end
plot(a, xloc, 'ko')
```

Reqi,tpiewitvmrx->
Gsqqyrmx} Wxerhevh,tvmrxEGTYF MH->

Tvsfpiq MZ>_64 txw2aRsvqw0Gsrhmxmsrw0erh Wxexmwxmgw

Kmzir>

```
v A _5 16 7 18a+?
w A _7 5 8 19a+?
x A _; < 4 4a+?
z A _9 8? : 56a
```

hixivqmrixlijsgps{mrkuyerxmxmiwf}lerherhxlir{vmxixliQexpefgshi}sy{syphywixsgepgypexixliq2

,5-eAo_vwao5

a=norm([r s],1)

$[r\ s]=\begin{bmatrix}1&3\\-2&1\\3&4\\-4&-5\end{bmatrix}$ $\|[r\ s]\|_1$ = max 1 norm of columns = max(10, 18) = 13

,6-fAoz,>25-o5

b=norm(v(:,1),1)

$v(:,1)=\begin{bmatrix}5\\6\end{bmatrix}$ $\|v(:,1)\|$ = 11

,7-gAox*w4o5

c=norm(t*s', inf)

$t*s'=\begin{bmatrix}7\\8\\0\\0\end{bmatrix}\begin{bmatrix}3&1&4&-5\end{bmatrix}=\begin{bmatrix}21&7&28&-35\\24&8&32&-40\\0&0&0&0\\0&0&0&0\end{bmatrix}$

$\|t*s'\|_\infty$ = max 1-norm of rows = max(91, 104, 0, 0) = 104

,8-hAovo6

d=norm(r)

$\|r\|_2=\sqrt{1^2+2^2+3^2+4^2}=\sqrt{30}=5.477$

,9-iAw̄

e=mean(s)

$\bar{s}=\frac{1}{4}(3+1+4-5)=0.75$

,:-jAozo5

f=norm(v,1)

$\|v\|_1$ = max 1 norm of cols of $\begin{bmatrix}5&4\\6&12\end{bmatrix}$ = 16

,;-kAozoi

g=norm(v,'fro')

$\|v\|_F=\sqrt{5^2+6^2+4^2+12^2}=\sqrt{221}=14.87$

,<-lAozo5

h=norm(v,inf)

$\|v\|_\infty$ = max 1 norm of rows of $\begin{bmatrix}5&4\\6&12\end{bmatrix}$ = 18

,=-mAgsrhmxmsr ryq fivsjz0fewih sr 51rsvq

i=cond(v,1) or

i=norm(v,1)*norm(inv(v),1)

cond(v) = $\|v\|_1 \cdot \|inv(v)\|_1$ = (16)(1/2) = 8

$inv(v)=\frac{\begin{bmatrix}12&-4\\-6&5\end{bmatrix}}{60-24}=\frac{\begin{bmatrix}12&-4\\-6&5\end{bmatrix}}{36}$ $\|inv(v)\|_1$ = max 1 norm of cols of inv(v) = 1/2 (from 1st col)

Reqi ,tpiewi tvmrx->
Gsqqyrmx} Wxerhevh ,tvmrx EGTYF MH ->

Tvsfpiq Z>_64 txw2a Pmriev Epkifve

E temrx wxsvi {ewsziv~iepsywpewx}iev erh fsyklxe wyvtpyw sjx{s tstypev temrx gspsvw0fyx xlmw}iev xliwi gspsvw evi ziv} yrtstypev2 Ws xli temrx wxsvi lew high hih xs vig}gpi xliwi yrtstypev temrx gspsvw f} qm|mrk xliq xskixliv xs jsvq xlmw}iev+wtstypev temrx gspsv2 Iegl temrx gspsv mw gsq tswih sj 6 fewmg tmkq irxw> E erh F 2X li yrtstypev gspsvw lezi xli jspps{mrk tmkq irx gsq tswmxmsr>

Temrx ryq fiv	Tmkq irx E	Tmkq irx F
5	;4244	7424 4
6	5424 4	=424 4

X lmw}iev+whiwmvefpi gspsv lew e tmkq irx gsq tswmxmsr sj>77) tmkq irx E erh :;) tmkq irx F 2

,5- L s{ qygl sjiegl yrtstypev temrx wlsyph fi qm|ih xskixliv xs jsvq 5 kepssr sjxli tstypev gspsvC Pix |$_n$ fi xli jvegxmsr sjyrtstypev gspsvih temrx n xlex mw ywih xs jsvq xli ri{ tstypev gspsv2 Gpievp} wix yt xli priev w}wxiq erh xlir wspzi f} lerh jsv |$_5$ erh |$_6$ 2] sy qywx wls{ }syv{svo 1q ivip} tvshygmrk xli gsvvigxerw{iv {mpvigimzi pmxpi gvihmx2 E pws0}sy qywx wspzi ywmrk xli mrziwi sjxli qexvm| 1 fegolwyfwxmxyxmsr qe} fi ywih xs gligo }syv{svo0fyx{mppepws vigimzi pmxpi gvihmx2

,6- Gepgypexi xli gsrhmxmsr ryq fiv sjxlmw w}wxiq f} lerh ywmrk xli Jvsfirmyw rsvq 2

,7- Kmzir xlex xli temrx tmkq irx tivgirxekiwevi ors{r xs jsyv significanx figures, {lexhsiwxli gsrhmxmsr ryq fiv we} efsyx xli tvigmxmsr sj }syv erw{ivefsziC

,8- [vmxi xli Q exqef gshi xs wspzi xli first x{s tevxw sjxlmw tvsfpiq 2

,Fewih sr e tvsfpiq jvsq Ryq ivmgepQ ixlshw{mxl Q exqef f} Viglxir{eph-2

1) Pigment A = $0.70x_1 + 0.10x_2$

Pigment B = $0.30x_1 + 0.90x_2$

$$\begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} .33 \\ .67 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \text{inv}\left(\begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix}\right)\begin{bmatrix} .33 \\ .67 \end{bmatrix} = \frac{\begin{bmatrix} .9 & -.1 \\ -.3 & .7 \end{bmatrix}\begin{bmatrix} .33 \\ .67 \end{bmatrix}}{.63-.3} = \frac{\begin{bmatrix} .230 \\ .370 \end{bmatrix}}{.6} = \begin{bmatrix} .3833 \\ .6167 \end{bmatrix}$$

2) $\text{cond}(A,'fro') = \|A\|_e \cdot \|A^{-1}\|_e$

note: $A^{-1} = \begin{bmatrix} 1.5 & -.1667 \\ -.5 & 1.1667 \end{bmatrix}$

$\|A\|_e = \sqrt{.7^2+.3^2+.1^2+.9^2} = 1.1832$

$\|A^{-1}\|_e = \sqrt{1.5^2+.5^2+.1667^2+1.1667^2} = 1.972$

$\text{cond}(A,'fro') = 2.3316$

3) since $\log_{10}(\text{cond}(A,'fro')) = .3676$, the answers in part 1 lose between 0 and 1 digit of precision.

4)

```
A = [.7 .1; .3 .9]
b = [.33 .67]'
x = A\b
x1 = x(1)
x2 = x(2)
cond(A, 'fro')
```

Req i ,tpiewi tvmrx->
Gsq q yrmx} Wxerhevh ,tvmrx EGTYF MH ->

Tvsfpiq Z M>_59 txw2a Pmriev Epkifve MM

,5- Ewwyq mrk }sy lezi hixivq mrih xli jsgps{ mrk iuyexmsrwxs fi xvyi>

| / 8} / =~ A 54
−| / 9~ A 7
7| / :} − ~ A 8

(1)(x) +(4)(y) +(9)(z) = 10
(-1)(x) +(0)(y) +(5)(z) = 3
(3)(x) +(6)(y) +(-1)(z) = 4

wls{ xli Q exqef gshi }sy {syph ywi xs wspzi jsv |0}0erh ~2 Mr sxliv {svhw0 ex xli irh sj }syv gshi0 xli zevmefpiw |0}0erh ~ wlsyph i|mxew5|5 q exvmgiw gsrxemrmrk xli ettvstvmexi zepyiw2 R sxi xlex }sy wlsyph rsx xv} xs wspzi xlmwf} lerh2

```
A = [1 4 9; -1 0 5; 3 6 -1]
b = [10 3 4]'
MyVars = A\b
x = MyVars(1)
y = MyVars(2)
z = MyVars(3)
```

,6- Hsiwe yrmuyi wspyxmsr i|mxxs xlmwwix sj iuyexmsrwC] sy q ywx gpievp} wls{ }syv tvssj jsv }syv erw{ iv2

$$\det\left(\begin{bmatrix} 1 & 4 & 9 \\ -1 & 0 & 5 \\ 3 & 6 & -1 \end{bmatrix}\right) = \begin{vmatrix} 1 & 4 & 9 \\ -1 & 0 & 5 \\ 3 & 6 & -1 \end{vmatrix} \begin{matrix} 1 & 4 \\ -1 & 0 \\ 3 & 6 \end{matrix} = 0+60-54-0-30-4$$

$$= -28 \neq 0 \quad \underline{\text{so}}$$

unique soln exists!

,7- Kmzir xli szivhixivq mrih w}wxiq >

| / } / ~ A :
6| − ~ A 7
7} − 6~ A | / 9
:| − 6} A :~

(1)(x) +(1)(y) +(1)(z) = 6
(1)(x) +(0)(y) +(-1)(z) = 3
(-1)(x) +(3)(y) +(-2)(z) = 5
(6)(x) +(-2)(y) +(-6)(z) = 0

Wls{ xli Q exqef gsq q erhw xs wspzi jsv |0}0erh ~2 Ex xli irh sj }syv gshi |0}0erh ~ wlsyph i|mxew5|5 q exvmgiw gsrxemrmrk xli ettvstvmexi zepyiw2

```
A = [1 1 1; 1 0 -1; -1 3 -2; 6 -2 -6]
b = [6 3 5 0]'
MyVars = A\b
x = MyVars(1)
y = MyVars(2)
z = MyVars(3)
```