Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 53L Fall 2005

Test III

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Name (please print) SOLUTIONS

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

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Problem I: Nonlinear Regression [20 pts.]

Given the following data for the conductivity k of copper as a function of temperature T, from **Numerical Methods with Matlab** by Gerald Recktenwald:

T (K)	20.52	22.68	25.15	27.72	30.24	33.21
k (W/m/K)	32.47	30.33	31.14	27.46	23.29	20.72

and a hypothetical model of:

$$k(T) = \frac{1}{\frac{\alpha}{T} + \beta T^2}$$

show the Matlab code you would use in order to determine the coefficients α and β. Also show the code you would use to determine the r^2 value of the fit. Assume the following lines of code have already been entered:

```
T = [20.52 22.68 25.15 27.72 30.24 33.21]
D = [32.47 30.33 31.14 27.46 23.29 20.72]
```

note: D and k are the same

At the end of your code, there must be 1x1 variables called `alpha`, `beta`, and `r2`.

```
fSSR = inline('sum((Dd - 1 ./ (x(1) ./ Td + x(2)*Td.^2)).^2)', 'Td', 'Dd', 'x');
[MyCoefs, Sr] = fminsearch(@(x)fSSR(T, D, x), [1 1])
alpha = MyCoefs(1)
beta = MyCoefs(2)
St = sum((D - mean(D)).^2)
r2 = (St-Sr)/St
```

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Problem II: Linearization [30 pts.]

In a compression process in a piston-cylinder device, air pressure and temperature are measured as follows:

Pressure (P, psia)	Temperature (T, R)
20.0	44.9
80.2	164.8
141.1	228.4

In a compression process of this type, it is known from thermodynamics that the temperature and the pressure generally follow the equation:

$$T = kP^r$$

power law
Bring in the cows!

where T is absolute temperature in Rankine, P is absolute pressure in pounds per square inch, and k and r are constants determined by the compression type and units of measurement.

(1) By hand and using a linearized equation, find the coefficients for the model that make it best fit the data. Then use those values to determine the temperature at a pressure of 50 psia. Clearly show your work.

(2) Using a linearized equation, show the Matlab commands you would use to find the coefficients for the model that make it best fit the data. Then show how you would use those values to determine the temperature at a pressure of 50 psia. Be sure to write the code exactly as you would type it in Matlab.

Note: you can use any logarithm for linearizing power law!

$$\ln(T) = \ln(kP^r) = r\ln(P) + \ln(k)$$

$$M = [\ln(P)\ \ P.\hat{}0] = \begin{bmatrix} 2.996 & 1 \\ 4.385 & 1 \\ 4.949 & 1 \end{bmatrix} \qquad y = \ln(T) = \begin{bmatrix} 3.804 \\ 5.105 \\ 5.431 \end{bmatrix}$$

$$a = \mathrm{inv}(M'M)M'y\ ;\ M'M = \begin{bmatrix} 2.996 & 4.385 & 4.949 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2.996 & 1 \\ 4.385 & 1 \\ 4.949 & 1 \end{bmatrix} = \begin{bmatrix} 52.70 & 12.33 \\ 12.33 & 3 \end{bmatrix}$$

$$\mathrm{inv}(M'M) = \frac{\begin{bmatrix} 3 & -12.33 \\ -12.33 & 52.70 \end{bmatrix}}{(3)(52.70) - (12.33)^2} = \begin{bmatrix} .4949 & -2.034 \\ -2.034 & 8.694 \end{bmatrix}$$

$$M'y = \begin{bmatrix} 2.996 & 4.385 & 4.949 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3.804 \\ 5.105 \\ 5.431 \end{bmatrix} = \begin{bmatrix} 60.66 \\ 14.34 \end{bmatrix} \qquad a = \begin{bmatrix} .8527 \\ 1.276 \end{bmatrix}$$

$$r = a(1) = .8527$$

$$k = e^{a(2)} = 3.582$$

$$T = 3.582(P)^{.8527}$$

$$T50 = 3.582(50)^{.8527}$$

$$\boxed{T50 = 100.6}$$

must use inverse of the log you used.

```
Pdata = [20, 80.2, 141.1]'; % must be a column
Tdata = [44.9, 164.8, 228.4]'; %must be a column
Plog = log(Pdata)
Tlog = log(Tdata)
LinCoefs = polyfit(Tlog, Plog, 1);
r = LinCoefs(1);
k = exp(LinCoefs(2));
T50a = k*50^r %or T50b = exp(polyval(LinCoefs, log
(50)))

%LinCoefs =
%    8.521502214633887e-01
%    1.277830433746241e+00
%r = 8.521502214633887e-01
%k = 3.588845035998960e+00
%T50a = 1.006311351376041e+02
%T50b = 1.006311351376041e+02
```

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Problem III: Interpolation [20 pts.]

Temperature and density data for saturated water is given in the following table from **Introduction to Engineering Experimentation** by Wheeler and Ganji:

Temp (T, ^{o}C)	Density(ρ, kg/m^3)
0	999.9
20	998.2
40	992.2
60	983.2
80	971.8
100	958.5

Assume for Matlab that the following lines of code have already been entered:

```
T = 0:20:100;
rho = [999.9 998.2 992.2 983.2 971.8 958.5];
```

(1) By hand, determine the density of saturated water at T=35^{o}C using:

(a) Nearest Neighbor

35 is closest to 40, so rho35 = 992.2

(b) Linear Interpolation

rho35 = (rho40-rho20)/(T40-T20) * (T-T20) + rho(20)
rho35 = (992.2-998.2)/(20) * 15 + 998.2 = 993.7

(2) Show the Matlab commands to determine the density at T=70^{o}C using:

(a) Polynomial Interpolation

```
P = polyfit(T, rho, length(T)-1) % or polyfit(T, rho, 5)
rho70 = polyval(P, 70)
```

(b) Piecewise Cubic Hermite Interpolation

```
rho70 = interp1(T, rho, 70, 'pchip') %or rho70 = interp1(T, rho, 70, 'cubic')
```

(3) Show the Matlab commands to:

(a) Determine the density at T=85^{o}C using cubic spline interpolation using clamped end conditions, where the derivative of density at 0^{o}C is 0.068 $\frac{\text{kg}}{\text{m}^3 \cdot {}^o\text{C}}$ and the derivative of density at 100^{o}C is -0.731$\frac{\text{kg}}{\text{m}^3 \cdot {}^o\text{C}}$.

```
rho85 = spline(T, [.068 rho -.731], 85)
```

(b) Generate a vector of 50 interpolated points between 0^{o}C and 100^{o}C with not-a-knot conditions at the end points Plot the original data using black squares connected by a solid line and the interpolated data using green circles connected by a dotted line on the same graph. You do not need to label the graph.

```
Tmodel = linspace(0, 100, 50);
rhomodel = interp1(T, rho, Tmodel, 'spline') % or rhomodel = spline(T, rho, Tmodel)
plot(T, rho, 'ks-', Tmodel, rhomodel, 'go:');
% or
%
% plot(T, rho, 'ks-');
% hold on
% plot(Tmodel, rhomodel, 'go:');
% Note - order of color, pointstyle, linestyle does not matter here
```

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Problem IV: Integrals and Derivatives [30 pts.]

A train's velocity as a function of time are given in the table below. Using the method specified at the top of the table, calculate vales for the position (integral of velocity), acceleration (derivative of velocity), and jerk (second derivative of velocity) of the train at the given times. The train passes the station (0 m) at 10 s.

$h=10$

t (s)	v(t) (m/s)	r(t) Trapezoidal	r(t) Composite	a(t) 2PB	a(t) 2PF	a(t) 3P	j(t) 3P
10	5	0	0	must use 2PF 1	$\frac{v_2-v_1}{h}=1$	$\frac{-v_3+4v_2-3v_1}{2h}=$ $\frac{-45+60-15}{2h}=0$	use $j(t(2))$ 0.2
20	15	$0+\frac{h}{2}(v_2+v_1)=$ $0+5(20)=100$	T/ 100	$\frac{v_2-v_1}{h}=\frac{15-5}{10}=1$	$\frac{v_3-v_2}{h}=3$	$\frac{v_3-v_1}{2h}=\frac{45-5}{20}=2$	$\frac{v_3-2v_2+v_1}{h^2}=$ $\frac{45-30+5}{100}=0.2$
30	45	$100+\frac{h}{2}(v_3+v_2)=$ $100+5(60)=400$	S 1/3: $\frac{h}{3}(v_1+4v_2+v_3)$ $\frac{10}{3}(5+60+45)=366.7$	$\frac{v_3-v_2}{h}=\frac{45-15}{10}=3$	$\frac{v_4-v_3}{h}=-1.5$	$\frac{v_4-v_2}{2h}=\frac{30-15}{20}=.75$	$\frac{v_4-2v_3+v_2}{h^2}=$ $\frac{30-90+15}{100}=-0.45$
40	30	$400+\frac{h}{2}(v_4+v_3)=$ $400+5(75)=775$	S 3/8: $\frac{3h}{8}(v_1+3v_2+3v_3+v_4)$ $\frac{30}{8}(5+45+135+30)=$ 806.25	$\frac{v_4-v_3}{h}=\frac{30-45}{10}=-1.5$	must use 2PB -1.5	$\frac{3v_4-4v_3+v_2}{2h}=$ $\frac{90-180+15}{20}=-3.75$	use $j(t(3))$ -0.45

Assuming some dependent set of measurements $y(k)$ for k=1 : N and a constant spacing between times h, fill in the table with the equation you would use to find either the best approximation for the integral given the boundaries or the approximation required for the 1st or 2nd derivative.

INT	$\int_{t(2)}^{t(3)} y\ dt$	T/ $\frac{h}{2}(y_2+y_3)$	INT	$\int_{t(3)}^{t(6)} y\ dt$	S 3/8/ $\frac{3h}{8}(y_3+3y_4+3y_5+y_6)$
INT	$\int_{t(1)}^{t(5)} y\ dt$	S 1/3 twice/ $\frac{h}{3}(y_1+4y_2+2y_3+4y_4+y_5)$ or $\frac{h}{3}(y_1+4y_2+y_3)+\frac{h}{3}(y_3+4y_4+y_5)$	INT	$\int_{t(4)}^{t(9)} y\ dt$	S 1/3 & 3/8/ $\frac{h}{3}(y_4+4y_5+y_6)+\frac{3h}{8}(y_6+3y_7+3y_8+y_9)$ or $\frac{3h}{8}(y_4+3y_5+3y_6+y_7)+\frac{h}{3}(y_7+4y_8+y_9)$
2PB	$\left.\frac{dy}{dt}\right\rvert_{k=1,N}$	$\frac{y(k)-y(k-1)}{h}$	3P	$\left.\frac{dy}{dt}\right\rvert_{k=1,N}$	$\frac{y(k+1)-y(k-1)}{2h}$
2PB	$\left.\frac{dy}{dt}\right\rvert_{1}$	2PF: $\frac{y(2)-y(1)}{h}$	3P	$\left.\frac{dy}{dt}\right\rvert_{1}$	$\frac{-y(3)+4y(2)-3y(1)}{2h}$
2PB	$\left.\frac{dy}{dt}\right\rvert_{N}$	2PB: $\frac{y(N)-y(N-1)}{h}$	3P	$\left.\frac{dy}{dt}\right\rvert_{N}$	$\frac{3y(N)-4y(N-1)+y(N-2)}{2h}$
2PF	$\left.\frac{dy}{dt}\right\rvert_{k=1,N}$	$\frac{y(k+1)-y(k)}{h}$	3P	$\left.\frac{d\ y}{dt}\right\rvert_{k=1,N}$	$\frac{y(k+1)-2y(k)+y(k-1)}{h^2}$
2PF	$\left.\frac{dy}{dt}\right\rvert_{1}$	2PF: $\frac{y(2)-y(1)}{h}$	3P	$\left.\frac{d\ y}{dt}\right\rvert_{1}$	$\left.\frac{d^2y}{dt^2}\right\rvert_{2}=\frac{y(3)-2y(2)+y(1)}{h^2}$
2PF	$\left.\frac{dy}{dt}\right\rvert_{N}$	2PB: $\frac{y(N)-y(N-1)}{h}$	3P	$\left.\frac{d\ y}{dt}\right\rvert_{N}$	$\left.\frac{d^2y}{dt^2}\right\rvert_{N-1}=\frac{y(N)-2y(N-1)+y(N-2)}{h^2}$