Duke University
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EGR 53L Fall 2005

Test II

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Name (please print) Draft 1.0 (11/12/2005)

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

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Problem I: [15 pts.] Finding Roots Part 1

(1) Show the Matlab command you can use to find all the solutions to the equation $3+2x^2 = x^5$

Rewrite as x^5-2x^2-3=0 to turn into root finding problem, then use

roots([1 0 0 -2 0 -3])

(2) It can be shown that there is a value of x bracketed by 0 and 1 that solves the equation $fun(x,a) = x^{1/3} + \cos(x) - ax = 0$ for any value of a such that $2 < a < 10$. Write a .m function for fun, then write a script that gets a guaranteed-valid value of a from a user and uses it to find the value of x for which $fun(x,a) = 0$. Be sure to keep asking for values of a until the user enters a valid number. Put the .m function in the box and write the script that uses it below.

```
%function fun.m

function out = fun(x, a)
out = x.^(1/3) + cos(x) - a*x
```

```
a = 0;  % OR a = input('a: ');
while (a<=2) | (a>=10) % OR while ~((2<a) & (a<10))
    a = input('a: ');
end

fzero('fun', [0 1], optimset, a)
```

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Problem II: [15 pts.] Finding Roots Part 2

(1) Given some function $f = \cos(x) + .5$, which has a derivative of $f' = -\sin(x)$, and an initial guess for a root at 1 rad, determine the next two predictions of the root as produced by the Newton-Raphson method. What are the final values for the x tolerance and the f tolerance?

k	x(k)	f(x(k))	f'(x(k))	- f/f'	x(k+1)
1	1	1.040302	-0.84147	1.23629	2.23629
2	2.23629	-0.11745	-0.78661	-0.14931	2.086982
3	2.086982	0.006434			

final ftol = |f(x(3))| = 0.006434
final xtol = |x(3)-x(2)| = -0.14931

(2) Given some function $f = \cos(x) + .5$, which has a derivative of $f' = -\sin(x)$, and an initial bracket for a root between $x = 0$ and $x = 4$ radians, determine the first three values for predictions of the root as produced by the bisection method. What are the final values for the x tolerance and the f tolerance?

f(x_l)	x_l	f(x_m)	x_m	f(x_r)	x_r
1.5	0	0.083853	2	-0.15364	4
0.083853	2	-0.48999	3	-0.15364	4
0.083853	2	-0.30114	2.5	-0.48999	3

x(1) = 2, x(2) = 3, x(3) = 2.5

final ftol = |f(x(3))| = -0.3011
final xtol = 0.5*(x_r(3)-x_l(3)) = 0.5

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Problem III: [15 pts.] Matlab Interpretation

(1) Show the output for the following Matlab code:

```
 for i = 1:6
     if i == 3
         break;
     end
         disp(i)
 end
fprintf('End of loop and i is %0.0f', i);
```

1
2
End of loop and i is 3

%note - break happens when i is 3, so the 3 does not get displayed, but i remains 3 when the fprintf happens

(2) Given:

$$A = \begin{bmatrix} 1 & 4 & 7 & -3 \\ 2 & -5 & 1 & 2 \end{bmatrix}$$

Show the output of the following Matlab code:

```
for i = 1:size(A,1)
    for j = 1:size(A,2)
        if A(i,j) > 0
            x(j,i) = 1.0;
        else
            x(j,i) = 0.0;
        end
    end
end
disp(x)
```

1 0
1 0
1 1
0 1

% Note - size(A,1) gives number of rows, size(A,2) gives number of columns; also note x(j,i) in the program

(3) Calculate and display the final values of `radius` given:

```
r = inline('sqrt(x.^2 +y.^2)', 'x', 'y');
radius = r([2 2 3], [1 2 3])
```

radius = [sqrt(5) sqrt(8) sqrt(18)] %or
radius = [2.236 2.828 4.243]

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Problem IV: [15 pts.] Norms, Conditions, and Statistics

Given:

```
x = [3 -4 4]
A = [5 2 3; 4 6 6; 3 8 9]
B = [1 2; -2 5]
```

determine the following quantities by hand and then show the Matlab code you would use to calculate them.

(1) a=$\|x\|_1$

(2) b=$\|x\|_2$

(3) c=$\|x\|_e$

(4) d=$\|x\|_\infty$

(5) e=$\bar{x}$

(6) f=S_x

(7) g=$\|A\|_1$

(8) h=$\|A\|_e$

(9) i=$\|A\|_\infty$

(1)
a = norm(x, 1)
a = |3| + |-4| + |4| = 11
(2)
b = norm(x) %OR norm(x,2) %OR norm(x, 'fro')
b = sqrt(3^2+4^2+4^2) = sqrt(41) = 6.40
(3)
c = norm(x, 'fro') %OR norm(x,2) %OR norm(x)
c = sqrt(s^2+4^2+4^2) = sqrt(41) = 6.40
(4)
d = norm(x, inf)
d = max(|3|, |-4|, |4|) = 4
(5)
e = mean(x)
e = (1/3)(3-4+4) = 1
(6) (standard deviation)
f = std(f)
f = sqrt(sum((3-1)^2+(-4-1)^2+(4-1)^2)/2)=sqrt(19)
(6) (sum of squares of data residuals)
f = sum((x-mean(x)).^2)
f = (3-1)^2+(-4-1)^2+(4-1)^2 = 38
(7)
g = norm(A,1)
g =max 1 norm of columns, so max(12, 16, 18)=18
(8)
h = norm(A, 'fro')
h = sqrt of sum of squares of element vals, so
sqrt(25+16+9+4+36+64+9+36+81) = sqrt(280) = 16.7
(9)
i = norm(A, inf)
i = max 1 norm of rows, so max(10, 16, 20) = 20

(10) j=condition number of B, based on 1-norm

j = cond(A, 1)
j = norm(B, 1) * norm(inv(B), 1)
norm(B, 1) = max 1 norm of cols of B, so max(3, 7) = 7
inv(B) = 1/9 [5 -2; 2 1]
norm(inv(B), 1) = max 12 norm of cols of inv(B), so max(7/9, 3/9) = 7/9
j = 49/9 = 5.444

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Problem V: [20 pts.] Linear Algebra

(1) Assuming you have determined the following equations to be true:

$$1p + 4r - 7s = 10$$
$$2p - 5r + 8s = -11$$
$$3p + 6r - 9s = 12$$

show the Matlab code you would use to solve for p, r, and s. In other words, at the end of your code, the variables `p`, `r`, and `s` should exist as 1x1 matrices containing the appropriate values. Note that you should **not** try to solve this by hand.

```
M = [1 4 -7; 2 -5 8; 3 6 -9];
y = [10; -11; 12];
a = M\y; % OR a = inv(M)*y
p = a(1);
r = a(2);
s = a(3);
```

(2) Assuming the three equations above, show the Matlab code you would use to determine the condition number for the linear system based on the most commonly used norm.

```
cond(M) %OR cond(M, 2)
```

(3) Assuming you have determined the following equations to be true:

$$2.00t - 1.00u = -1.00$$
$$-6.50t + 3.00u = 2.00$$

(a) Show by hand how you would set up the matrix equation to solve for t and u and then solve for t and u by hand using that matrix equation.
(b) Show by hand how you would get the condition number for the system using the Frobenius norm, then find the condition number.
(c) Assuming you know your parameters of your linear system to three significant figures, what does the condition number mean for the accuracy of your values of t and u above?

(a)

$$\begin{bmatrix} 2 & -1 \\ -6.5 & 3 \end{bmatrix} \begin{bmatrix} t \\ u \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} t \\ u \end{bmatrix} = inv\left(\begin{bmatrix} 2 & -1 \\ -6.5 & 3 \end{bmatrix}\right) \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} t \\ u \end{bmatrix} = \frac{\begin{bmatrix} 3 & 1 \\ 6.5 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}}{6 - 6.5} = \frac{\begin{bmatrix} -1 \\ -2.5 \end{bmatrix}}{-.5}$$

$$\begin{bmatrix} t \\ u \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

(b) $\|M\|_e * \|M^{-1}\|_e$

$$\left\| \begin{bmatrix} 2 & -1 \\ -6.5 & 3 \end{bmatrix} \right\|_e = \sqrt{2^2 + 1^2 + 6.5^2 + 3^2} = 7.5$$

$$\left\| \begin{bmatrix} -6 & -2 \\ -13 & -4 \end{bmatrix} \right\|_e = \sqrt{6^2 + 2^2 + 13^2 + 4^2} = 15$$

cond(M, 'Fro') = 112.5

(c) Since $\log_{10}(112.5) = 2.051$, you may lose 2 significant figures, so t + u are only accurate to ~1 digit.

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Problem VI: [20 pts.] Fitting

Assuming you have the following measurements, where the x values are independent and the y values are dependent (note - they are columns):

```
x = [-1 0 1 2]';
y = [2 4 -2 -8]';
```

(1) Assume that you are trying to fit the data to the equation:

$$\hat{y}_a = a_1(x) + a_2(e^x)$$

Show the Matlab commands you would use to solve for a_1 and a_2. At the end of your script, you should have two 1x1 matrices called `a1` and `a2`.

(2) Assume that you are trying to fit the data to the equation:

$$\hat{y}_b = b_1(x) + b_2$$

Determine by hand, clearly showing your work, the values of b_1 and b_2

(3) The best-fit using a quadratic yields the equation:

$$\hat{y}_c = -2x^2 - 1.6x + 2.8$$

Is this a good fit? Why or why not? Provide the necessary **quantitative** proof (that is, you must calculate r^2, S_r, and S_t by hand and use them to make your case).

(1)

```
M = [x exp(x)]
a = M\y;
a1 = a(1);
a2 = a(2);
```

(2) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \\ -8 \end{bmatrix}$

$M'M = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$

$inv(M'M) = \dfrac{\begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}}{20}$

$M'y = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -2 \\ -8 \end{bmatrix} = \begin{bmatrix} -20 \\ -4 \end{bmatrix}$

→ $inv(M'M)M'y = \dfrac{\begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}}{20} \begin{bmatrix} -20 \\ -4 \end{bmatrix} = \dfrac{\begin{bmatrix} -72 \\ 16 \end{bmatrix}}{20}$

$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -3.6 \\ 0.8 \end{bmatrix}$

(3) $\hat{y} = -2x^2 - 1.6x + 2.8$

$\hat{y} = [2.4 \;\; 2.8 \;\; -.8 \;\; -8.4]$

$\bar{y} = \frac{1}{4}(2+4-2-8) = -1$

$S_t = ((2+1)^2 + (4+1)^2 + (-2+1)^2 + (-8+1)^2)$

$S_t = 84$

$S_r = ((2-2.4)^2 + (4-2.8)^2 + (-2+.8)^2 + (-8+8.4)^2)$

$S_r = 3.2$

$r^2 = \dfrac{S_t - S_r}{S_t} = .9619;$ OK fit...