Lab 8:
Root-finding Problems

8.1 Introduction

For this lab, you will be solving problems that rely on root-finding for their answers. Upon completion of this lab, you should be able to use `fzero` to find a single solution to a function given a set of inputs as well as generate a parametric array of solutions. You should also be able to use the `roots` command to find all of the roots of a polynomial.

8.2 Resources

The additional resources required for this assignment include:

- Books: Chapra - especially Chapra 6.4
- Pratt Pundit Pages: `MATLAB:fzero`

8.3 Getting Started

1. Log into one of the PCs in the lab using your NET ID. Be sure it is set to log on to acpub.
2. Start a browser and point it to `http://pundit.pratt.duke.edu/wiki/Lab:B209`. Check out the `Using the PCs to Run MATLAB Remotely` section for how to get connected and make sure the connection is working.
3. Once connected to a machine you believe will also display graphics, switch into your `EGR53` directory and create a `lab8` directory inside it:

   ```
   cd EGR53
   mkdir lab8
   ```

4. Switch to your ~`/EGR53/lab8` directory:

   ```
   cd lab8
   ```

5. Copy all relevant files from Dr. G's public `lab8` directory:

   ```
   cp -i ~mrg/public/EGR53/lab8/* .
   ```

   Do not forget the space and the "." at the end.
6. Open MATLAB by typing `matlab &` at the prompt that appears in your terminal window. It will take MATLAB a few seconds to start up.

8.4 Problems

8.4.1 Basic Root-Finding Problems

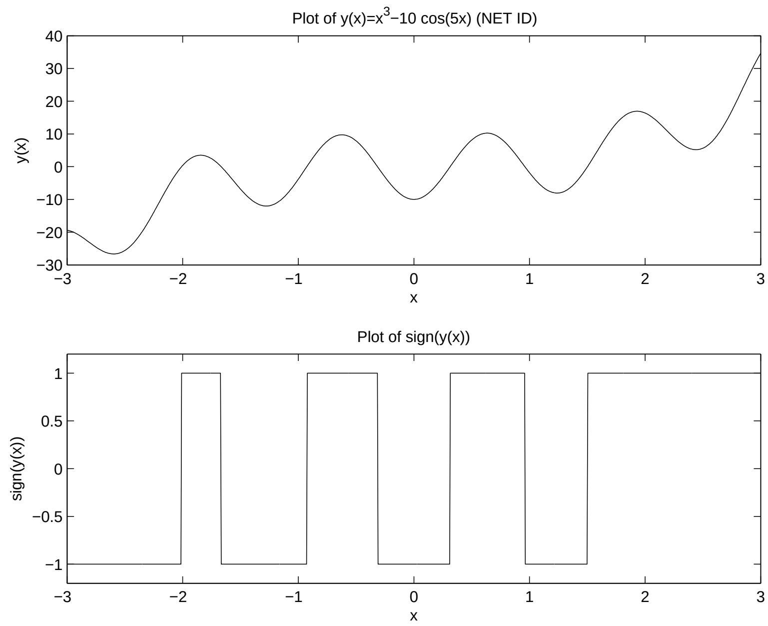
For this problem, you will be finding the real roots of four different equations - one that is a polynomial and three others. In each case, you will create a figure with a plot of the function in the top half and a plot of the *sign* of the function in the bottom half. The latter will help locate approximately where the roots will be found. However, the axis limits MATLAB sets for the sign plots will make the curve sit directly on the figure boundary - you should use the `axis` command to expand the y axis limits to go between -1.2 and 1.2 while keeping the same x axis limits as the top graph.

You will then use whatever combination of anonymous functions, the `roots` command, and the `fzero` command to find *all* the *real* roots of the functions. The number of real roots for each equation is given. Note that `fzero` can be used with named or unnamed anonymous functions as well as .m files. See the Pundit page on `MATLAB:fzero` for examples.

Your first step should be to determine a reasonable overall range for plotting your function - this should show *all* of the sign changes at the roots but not extend too far out of that domain. For example, having x go between -4 and 4 works well for the first problem. Then, use whatever commands you need to get the real roots. You should include everything (plotting and root finding commands) for all four problems *in one script*. Include your four figures in the lab report, and report your roots (use `format short e` in a table such as the one below:

Function	Real Roots	Roots
$y = x^4 + 2x^3 + 3x^2 + 4x - 5$	2	
$y = 2x + 9\cos(x)$	3	
$y = (x+1)^{(7/2)} + e^{-x} - e^{x} - 5, x > 0$	2	
$y = \frac{1}{2} + \cos(6x) + (x/2)^2 - (x/5)^4, x > 0$	3	

To give an example of the figures you will create, here is a sample set assuming you want to graphically locate the roots of $y = x^3 - 10\cos(5x)$:



8.4.2 Based on Chapra Chapter 6, Problem 13

This problem, which comes from chemistry and may be seen in biomedical engineering, involves the dissociation of water into its component parts and back again. You will be using `fzero` to determine the roots of the given equation, only instead of finding a single value of the mole fraction for a single parameter set of p_t and K, you will be finding an array of values for varying p_t settings.

To start, write an anonymous function that calculates the following:

$$f(x, K, p_t) = \frac{x}{1-x}\sqrt{\frac{2p_t}{2+x}} - K$$

As noted on the Pundit page for `MATLAB:fzero`, one issue when using functions of multiple variables is that you must tell MATLAB which variable it controls and which will be constant: `fzero` can only manipulate one variable at a time. Assume you want to find the value of x for which the function f above is 0, given $K = 0.05$ and p_t=3 atm. In this case, the first argument is the one that MATLAB is allowed to control while K and p_t are constant. The command in MATLAB to perform this task (further assuming an initial guess x=0.5) would be:

```
[xValue, fValue] = fzero(@(xDummy) f(xDummy, 0.05, 3), 0.5)
```

MATLAB will report:

```
xValue =
   2.8249e-02

fValue =
     0
```

This means that x is 2.8249% when K is 0.05 and p_t is 3 atm.

Next, determine how to put this process in a loop that will allow you to obtain values of x for twenty different values of p_t between 1 and 5 atm, keeping K=0.05. You need to think about how to best index the x values so that the entries in the vector x correspond to the entries in the vector p_t you created. There is help available on Pundit for this process. Once you have a vectors of x values that contain the mole fractions for given pressures that solves the equation, make a plot of mole fraction versus total pressure. In the lab report, state what you believe is the relationship between x and p_t for a given K.

8.4.3 Chapra Chapter 6, Problem 16

Do **(a)**. And **(b)**. Report your results. Make sure your graph is complete, including units. You should include a grid. In the lab report, give at least two numerically good reasons you believe your graph is correct. Note that you will most likely want to give a reasonable initial bracket, rather than an initial guess, to `fzero`.

8.4.4 Based on Chapra Chapter 6, Problem 19

Do what the book asks. Then create a graph of d values for 50 h values between 0 m and 1 m. Report your results. Make sure your graph is complete, including units. You should include a grid. In the lab report, give at least two numerically good reasons you believe your graph is correct. Note that you will most likely want to give a reasonable initial bracket, rather than an initial guess, to `fzero`. Also - there is a small typographical error in the book; the units for k_2 should be g/(s^2 m$^{0.5}$).

WARNING: d when h is 0 should *not* be zero. Although *mathematically* that is a solution to the equation, *physically* it makes no sense - the ball, if placed on the spring, will cause *some* deflection. Choose your initial guess or bracket wisely to make sure your graph represents mathematically *and* physically correct answers. This is a case where you also have to be careful that `fzero`'s search algorithm does *not* expand to the point of making $d < 0$ since the $d^{5/2}$ would become imaginary at that point.