Laboratory 9: Laplace Transforms Using Maple

9.1 Introduction

This lab focuses on using Laplace transforms to model circuits and on using Maple to solve Laplace transforms and inverse Laplace Transforms.

9.2 Resources

The additional resources required for this assignment include:

- Books: Alexander & Sadiku
- Pratt Pundit Pages: `Maple`, `Maple/Laplace Transforms`,

9.3 Getting Started

There are example files in the Resources section of the Sakai page, specifically under the Chapter 16 folder. Any file having the word "mathematica" in it is from Mathematica in case you want to see how things work with that.

9.4 Laplace Transforms and Inverse Laplace Transforms in Maple

Take a look at the Pratt Pundit page on `Maple/Laplace Transforms` for examples of how Maple can be made to determine Laplace transforms and inverse Laplace transforms. Next, go through the `NewStarter` worksheet. It will show you how to set up your worksheet and then takes you through some basic and intermediate techniques with Laplace transforms. The examples in that sheet are specifically targeted at the things you will need to know for this assignment.

For your worksheets with this lab, you will want to copy in the "Typical Starting Commands" at the top. You can also open the `NewStarter2022` worksheet, save it under a new name, and go from there.

9.5 Using evalf Wisely

As wonderful as the `evalf` command is, be sure to *only* use it in the last stage when you are making an answer presentable. Using it before then will cause Maple to actually roundoff the values it is using, which could result in substantially different answers. On the other had - sometimes, the Laplace or inverse Laplace result from Maple will contain an expression involving complex roots. You can use `evalf` to get Maple to work on solving for those roots and presenting a more easily interpreted answer. The upshot is - be sure to use `evalf` with numerical problems, but only at the end.

9.6 Assignment

For this lab, you will be submitting Maple worksheets to perform some forward and inverse Laplace transforms. You *can* put everything for this week in one worksheet or you can split it up into as many worksheets as seem reasonable. Be sure to use the `Insert-Paragraph-` command to annotate your worksheet with respect to which problem and which part of which problem you are solving. For any graph, include a reasonable title and set of axis labels. You will be turning in both the worksheets and PDF versions of the worksheets.

9.6.1 Basic Laplace Transforms

Use Maple to calculate the Laplace transform for each of the following four signals:

$$w_1(t) = 2e^{-17t}\,u(t) \qquad w_2(t) = t\,u(t)$$
$$w_3(t) = \cos(40t)\,u(t) \qquad w_4(t) = u(t) - u(t - 25\cdot 10^{-3}) + u(t - 100\cdot 10^{-3}) - u(t - 300\cdot 10^{-3})$$

This is as simple as defining the signal as a variable and then using the `laplace` function on that. See the "Basics" section of the starter file for details.

9.6.2 Laplace Transforms and Circuits

(1) Use Laplace transforms to find and plot the impulse response $h_{\text{a}}(t)$ and the step response $sr_{\text{a}}(t)$ of the voltage across the capacitor for the circuit given in Figure 10.70 of Problem 10.21(a) on p.445 of the book assuming:

$$L = 20\text{ mH} \qquad R = 1.5\text{ k}\Omega \qquad C = 2.2\text{ nF}$$

You will first need to solve for the transfer function

$$\mathcal{H}_{\text{a}}(s) = \frac{\mathcal{V}_{\text{o}}(s)}{\mathcal{V}_{\text{i}}(s)}$$

then determine the impulse response by simply taking the inverse Laplace transform of the transfer function. To get the *step* response, recall that for linear systems with impulse response $h(t)$ and input $x(t)$, the Laplace transform of the output $y(t)$ can be obtained with the convolution property,

$$\mathcal{Y}(s) = \mathcal{X}(s)\,\mathcal{H}(s)$$

All you will therefore need to do is find the Laplace transform of a step function and multiply it by the transfer function, then take the inverse Laplace transform of the result. You need to present the transfer function symbolically but only need to have the impulse and step responses numerically. In each case, plot the output for 100 μs. You should give the plots sensible titles and well-directed axis labels. The circuit schematic you need is on the following page of this assignment. The "System Responses" section of the starter document should be helpful here.

(2) Use Laplace transforms to find (but not plot) the impulse response $h_{\text{b}}(t)$ and find and plot the step response $sr_{\text{b}}(t)$ of the voltage across the inductor for the circuit given in Figure 10.71 of Problem 10.21(b) on p.443 of the book assuming:

$$L = 20\text{ mH} \qquad R = 1.5\text{ k}\Omega \qquad C = 2.2\text{ nF}$$

You will first need to solve for the transfer function

$$\mathcal{H}_{\text{b}}(s) = \frac{\mathcal{V}_{\text{o}}(s)}{\mathcal{V}_{\text{i}}(s)}$$

then determine the impulse response by simply taking the inverse Laplace transform of the transfer function. You need to present the transfer function symbolically but only need to have the impulse and step responses numerically. For this circuit, plot the *step* response for 150 μs. You should give the plot a sensible title and well-directed axis labels. You should provide a brief explanation of why think you were not asked to plot the impulse response. You can provide this as a paragraph in the Maple worksheet. The circuit schematic you need is on the following page of this assignment.

(3) Use Laplace transforms to find the transfer function $\mathcal{H}_3(s)$ between the input voltage $v_S(t)$ and the output voltage $v_o(t)$ in the circuit given in Figure 16.56 of Problem 16.33 on p. 748 of the book assuming:

$L = 10$ H $\quad R_1 = 22$ kΩ $\quad R_2 = 47$ kΩ $\quad C = 2$ μF

That is, find:

$$\mathcal{H}_3(s) = \frac{\mathcal{V}_o(s)}{\mathcal{V}_S(s)}$$

Next, find an expression for the impulse response $h_3(t)$.

Finally, use Laplace transforms to determine expressions in time for the voltages that would result if each of the four signals ($w_1(t)$, $w_2(t)$, $w_3(t)$, and $w_4(t)$) from Assignment 9.6.1 were used as the input voltages. For each of these, make a plot of both the input signal and the output signal on the same graph. Plot these for 500 ms, and be sure to clearly title and label your plots and include a legend. You should plot the input as a solid black line and the output as a solid red line. Use Maple's built in help for plot options to determine how to do this.

Note - for the last plot, Maple sometimes has issues plotting expressions with multiple Heaviside functions within them. See the Pundit page on `Maple/Plotting` - and especially the section on `Plotting Piecewise Functions` - for information on how to get that plot to work; if you try to plot it without turning it into a piecewise function, you will most likely get some spots where the output simply does not print. The circuit schematic you need is below.

Mathematica note: Mathematica is occasionally very picky about integers and floats. For this problem, the first three systems work best if everything is given in integer form; for the last signal, it only really plots correctly using floats. Your substitution would thus look like:

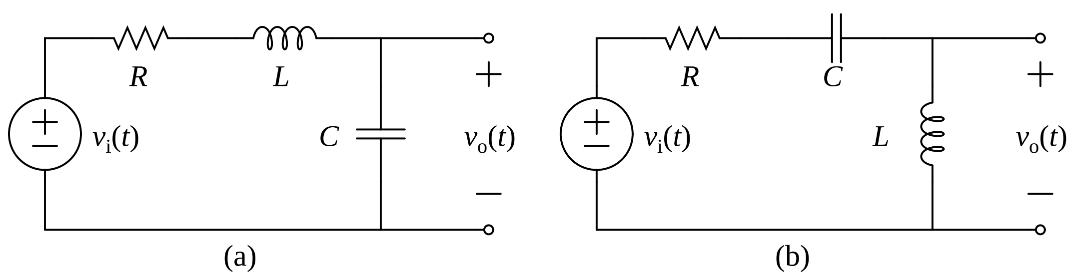
```
H3*W4  /. N[MyVals]
```

where the `N[]` command converts integers to floats.

9.7 Figures

The following figures are adapted from Alexander & Sadiku, 7th edition.

9.7.1 Figure 10.70, Redrawn



9.7.2 Figure 16.56, Redrawn