Duke Unibersity Edmund T. Pratt, Jr. School of Engineering

$\stackrel{\rm EGR 224 \ Summer \ 2020}{{\rm Test \ II}}$

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Name (please print):			
NetID (please print):	Start Time:	End Time:	

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

The test is open book, open class notes (yours and mine), open Sakai page for this class, and open Pundit. No other resources are allowed. If you have a question about whether something is allowed, ask the instructor. Be sure that you are in a place where you can work undisturbed for the duration of the test. If a situation arises that disrupts your work, let the instructor know - you will be allowed to pause the clock and resume work later. The Start Time above should be when you first started working on the test (after saving it, printing it out, etc.) and the End Time should be when you stopped doing work on the test and started working on scanning / photographing and uploading it.

Please be sure to put each answer in the correct box and make sure that your name *and* NetID are *clearly* written at the top of *every* page. If you need more space for a particular problem or want to show more work, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the top center of that page and submit those extra pages in problem-order after all preprinted pages of the test. Also, in the box for the problem, write a note that says "see extra page."

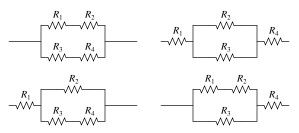
You will be turning your test into Gradescope. Carefully scan or photograph the test pages in order (with any additional pages properly labeled and after all the original test pages) and make a PDF of the scans / photographs. When you upload the PDF, you will also need to indicate where the answers for each problems are. If you used extra sheets, you only need to indicate the original test page for the problem.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

You may use the \parallel symbol for impedances in parallel and do not need to expand that construction unless you are required to determine a numerical answer. Be clear with your use of parentheses, however; simply writing something like

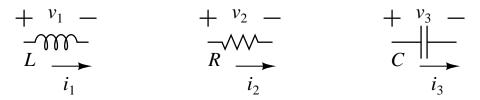
$$R_{\rm eq} = R_1 + R_2 \parallel R_3 + R_4$$

is too vague since it could refer to any of the four combinations below:



Problem I: [16 pts.] The Basics

(1) Write the equations in the time domain that relate the current to the voltage for the following three elements:



(2) For the reactive elements above, write the equation for the energy stored in each element:

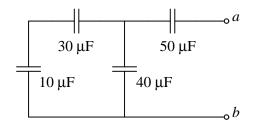
(3) Circle the appropriate entries in the sentence below:

The (current through / voltage drop across) an inductor and the (current through / voltage drop across) a capacitor must be continuous.

(4) Fill in the following table:

Quantity	Symbol	Name of real part	Symbol	Name of imaginary part	Symbol
Impedance					
Admittance					

(5) What is the equivalent capacitance as seen by the terminals a and b?





Problem II: [16 pts.] Phasors and Impedance

(1) Clearly using phasors, simplify the following signal into a single cosine:

$$v_{\rm a}(t) = 2\sin(20t) - 3\cos(20t)$$
 V

(2) Clearly using phasors, simplify the following signal into a single cosine:

 $i_{\rm b}(t) = 10\cos(100t + 18^{\circ}) - 12\cos(100t - 41^{\circ}) \,\mathrm{mA}$

(3) A resistor is connected to a reactive element inside a box, but you do not know what the element is or if they are connected in series or parallel. Two terminals are sticking out of the box. You conduct two experiments by applying sinusoidal voltages of different amplitudes, frequencies, and phases across the terminals and then measuring the steady state current going into the box. The inputs and outputs you measured are below:

Source Voltage (V)	Source Current (mA)
$5\cos(10t)$	$2.25\cos(10t+50.9^{\circ})$
$8\cos(200t+45^{\circ})$	$5.70\cos(200t+48.5^{\circ})$

(Note - there may be some small round-off in the calculations). What is the reactive element? How are the resistor and reactive element connected? What are their values? *Hint:* look at the formulas for and values of the impedance and admittance.

(4) Design a series-connected combination of a resistor and a reactive element that has an impedance of $\mathbb{Z} = 3 - j4 \ k\Omega$ at a frequency of 2 krad/s (2000 rad/s). Draw and label the circuit.

(5) Design a parallel-connected combination of a resistor and a reactive element that has an impedance of $\mathbb{Z} = 5 + j12 \ k\Omega$ at a frequency of 5 Mrad/s (5000000 rad/s). Draw and label the circuit.

Problem III: [18 pts.] Filters, ACSS, and Transfer Functions¹

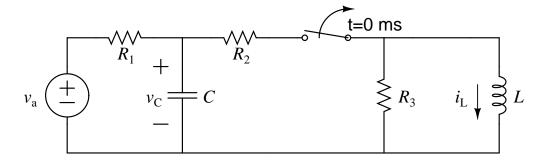
A voltage-to-voltage circuit has a transfer function and input voltage, respectively, of:

$$\mathbb{H}(j\omega) = \frac{\mathbb{V}_{\text{out}}}{\mathbb{V}_{\text{in}}} = \frac{4j\omega}{j\omega + 10} \qquad v_{\text{in}}(t) = 4 + 6\cos(5t + 10^{\circ}) + 7\cos(100t - 60^{\circ})$$

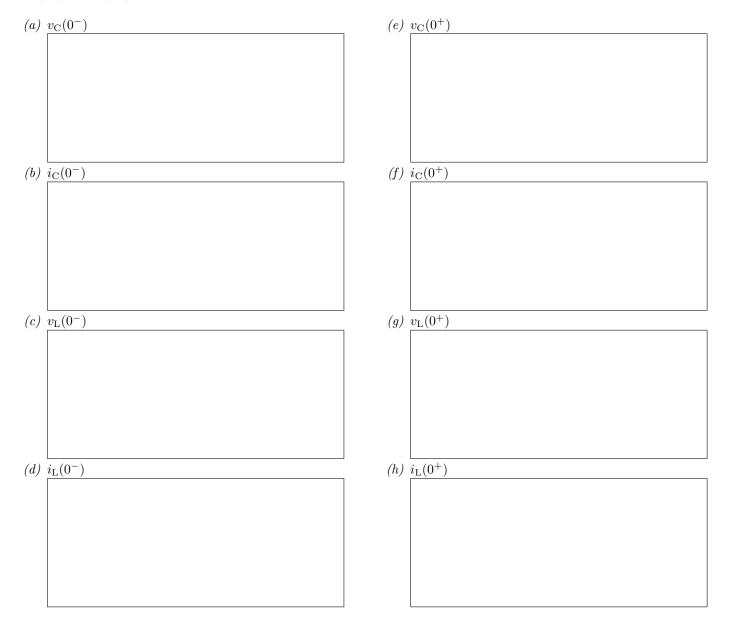
- (1) Sketch the straight line approximations to the Bode magnitude and phase plots. Be sure to label your axes and provide indications of slopes and meaningful values on the graphs.
- (2) What kind of filter does the transfer function represent? Why do you think that?
- (3) What is the maximum gain of the filter? What is the cutoff frequency?
- (4) Determine the steady-state function for the output voltage.
- (5) Find an equation in the time domain generally relating $v_{out}(t)$ and its derivatives to $v_{in}(t)$ and its derivatives.

Problem IV: [22 pts.] Transient Response

For the circuit below, assume that the switch has been closed for a very long time before t = 0 s. At t=0 s the switch opens.



(1) Assuming that v_a is constant for all times before t = 0, determine the following in terms of the symbolic element and source values (based on the passive sign convention). Put your answers in the box below. Also, you may use $v_{\rm C}(0^-)$, $i_{\rm C}(0^-)$, $v_{\rm L}(0^-)$ and $i_{\rm L}(0^-)$ in your solutions for the variables at 0^+ without further substitution.



(2) Assuming the circuit has the following element and source values:

$$\begin{array}{ll} R_1 = 1 \ \mathrm{k}\Omega & R_2 = 3 \ \mathrm{k}\Omega & R_3 = 10 \ \mathrm{k}\Omega \\ C = 100 \ \mu\mathrm{F} & L = 20 \ \mathrm{mH} & v_\mathrm{a}(t) = 12 \ \mathrm{V} \end{array}$$

determine the current through the inductor, $i_{\rm L}(t)$, for t > 0 s. Also indicate the time constant of the response and then make an accurate graph of $i_{\rm L}(t)$ for three time constants.

(3) Assuming the circuit has the same values as above, determine the voltage across the capacitor, $v_{\rm C}(t)$, for t > 0 s. Also indicate the time constant of the response and then make an accurate graph of $v_{\rm C}(t)$ for three time constants.

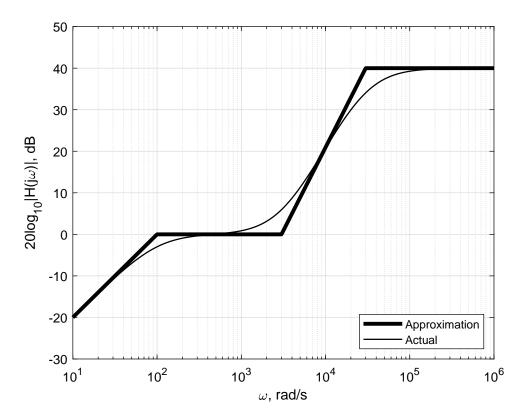
Problem V: [14 pts.] Bode Plots

Sketch the straight-line approximation for the Bode plots of **both** the magnitude and the phase of the transfer functions below. Be sure to properly label the axes, slopes, magnitudes, and angles.

(1)
$$\mathbb{H}_{1}(j\omega) = \frac{20}{j\omega+4}$$
(2)
$$\mathbb{H}_{2}(j\omega) = \frac{40000j\omega}{(j\omega+20)(j\omega+5000)}$$

Problem VI: [14 pts.] Bode Plots⁻¹

Given the following Bode magnitude plot of some transfer function $\mathbb{H}(j\omega)$ (along with its straight line approximation):



(a) Assuming all poles in the system are overdamped or critically damped, determine the formula for a transfer function $\mathbb{H}(j\omega)$ which is represented in the figure.

(b) What kind of filter is this? Also state why you believe that, *approximately* what the cutoff frequency/frequencies is/are, and what the passband gain is/is.