Problem I

(1) $v_1 = L\frac{di_1}{dt}$ $v_2 = R i_2$ $i_3 = C\frac{dv_3}{dt}$

(2) $\frac{1}{2}Li_1^2$ $\frac{1}{2}Cv_3^2$

(3) jwL R $\frac{1}{jwC}$

(4) current ... voltage

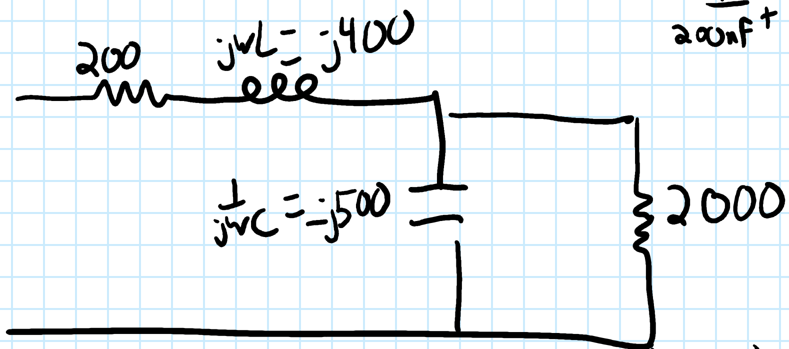
Problem II

(1) $40\,mH \parallel 20\,mH = \frac{40 \cdot 20}{40+20} = \frac{800}{60} = 13.\overline{3}\,mH + 90\,mH = 103.\overline{3}\,mH$

(2) C in parallel add, so $C_{right} = 600\,nF$

C in series add in inverse, so $C_{eff} = \dfrac{1}{\frac{1}{200nF} + \frac{1}{800nF} + \frac{1}{600nF}} = 126.3\,nF$

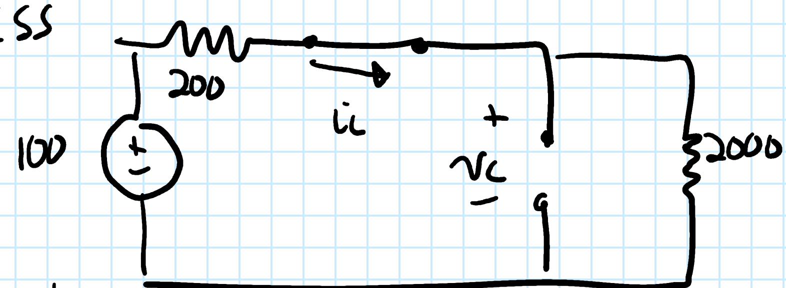
(3)



$Z_{R,L} = 200 + j400$ $\quad Z_{CR_2} = \dfrac{(-j500)(2000)}{-j500+2000} = 485.1\angle -75.96°$

$Z_{R,L} + Z_{CR_2} = 325\angle -12.53° = 317.3 - 70.51\,\Omega$

(4) DCSS

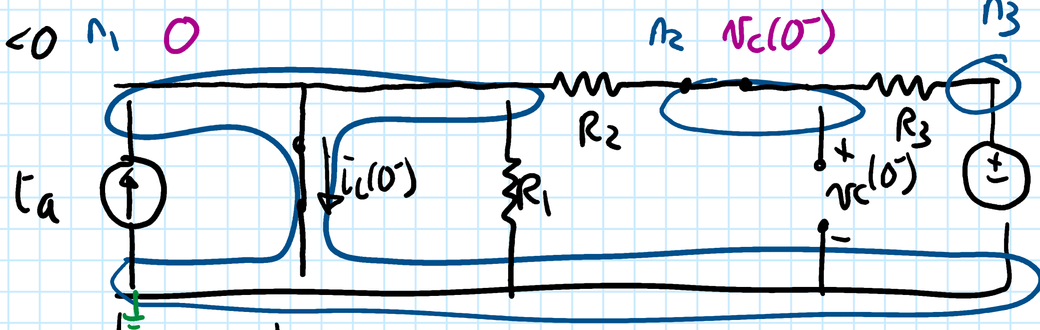


$i_L = \dfrac{100}{2200} = 0.04546\ A$ $\qquad v_C = \dfrac{100 \cdot 2000}{2200} = 90.91\ V$

$E = \frac{1}{2}Li^2 = 82.65\,\mu J$ $\qquad E = \frac{1}{2}Cv^2 = 1.653\,mJ$

Problem III

$t < 0$



Several ways to solve. NVM?

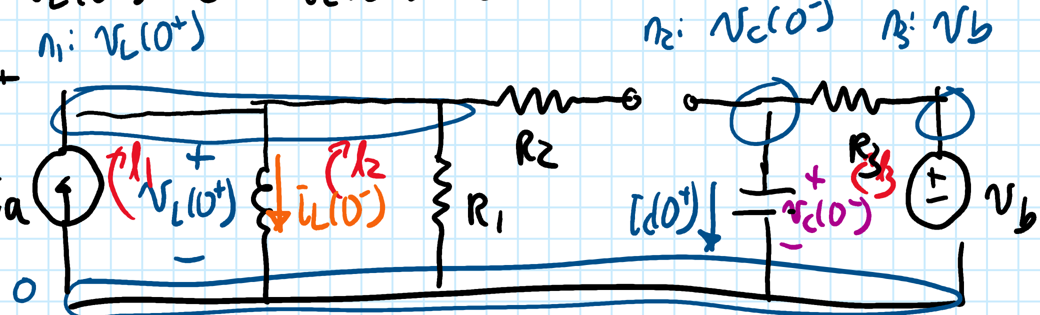
KCL, n_2: $\frac{v_c(0^-)-0}{R_2} + \frac{v_c(0^-)-v_b}{R_3} = 0$ $\quad v_c(0^-) = \boxed{\frac{R_2 v_b}{R_2+R_3}}$

KCL, n_1: $-i_a + i_L(0^-) + \frac{0 - v_c(0^-)}{R_2} = 0$

$$i_L(0^-) = i_a + \frac{R_2 v_b}{R_2(R_2+R_3)} = \boxed{i_a + \frac{v_b}{R_2+R_3}}$$

$v_L(0^-) = 0 \quad i_c(0^-) = 0$

$t = 0^+$



$\boxed{i_L(0^+) = i_L(0^-)} \quad \boxed{v_c(0^+) = v_c(0^-)}$

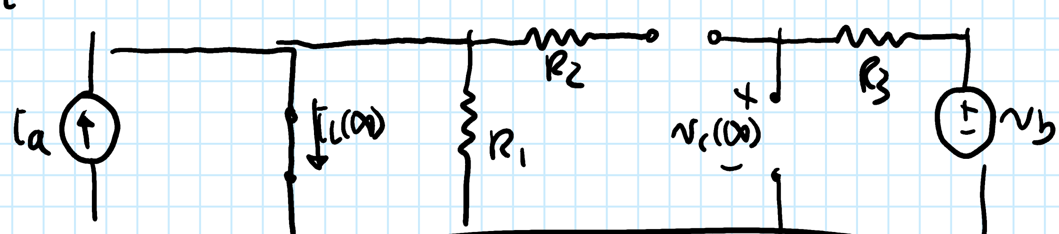
KCL, n_1: $-i_a + i_L(0^-) + \frac{v_L(0^+)}{R_1} = 0$

$$v_L(0^+) = R_1(i_a - i_L(0^-)) = R_1\left(i_a - i_a - \frac{v_b}{R_2+R_3}\right) = -\frac{R_1 v_b}{R_2+R_3}$$

KVL, l_3: $-v_c(0^-) - R_3 i_c(0^+) + v_b = 0$

$$i_c(0^+) = \frac{v_b - v_c(0^-)}{R_3} = \frac{v_b - \frac{R_2 v_b}{R_2+R_3}}{R_3} = \boxed{\frac{v_b}{R_2+R_3}}$$

$t \to \infty$



$$\boxed{v_L(\infty) = 0 \quad i_L(\infty) = i_a \quad v_c(\infty) = v_b \quad i_c(\infty) = 0}$$

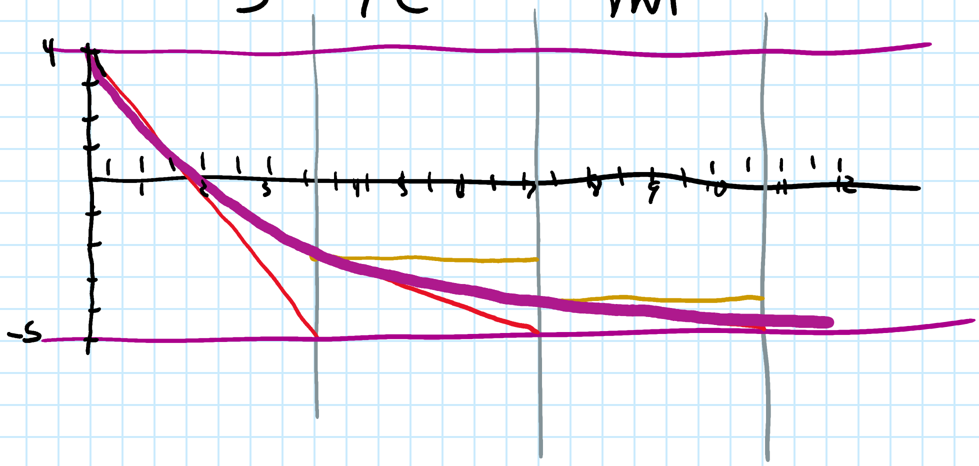
Problem IV

$$7\frac{di_o}{dt} + 2i_o = -10\text{ mA}$$

$$3.5\frac{di_o}{dt} + i_o = -5\text{ mA} \quad i_o(0) = 4\text{mA}$$

$\tau = 3.5 \quad i_o(0) = 4\text{ mA} \quad i_o(\infty) = -5\text{ mA}$

$$i_o(t) = -5 + (4-(-5))e^{-t/3.5}\text{ mA}$$
$$= -5 + 9e^{-t/3.5}\text{ mA}$$



Problem V

$$\mathbb{H} = \frac{10}{jw+5}$$

$$\frac{\mathbb{X}_{out}}{\mathbb{X}_{in}} = \frac{10}{jw+5} \qquad (jw+5)\mathbb{X}_{out} = 10\mathbb{X}_{in}$$

$$\boxed{\frac{dx_{out}(t)}{dt} + 5x_{out}(t) = 10x_{in}(t)}$$

$$x_{in}(t) = 1 + 3\cos(5t) + 7\cos(9t + 20°)$$

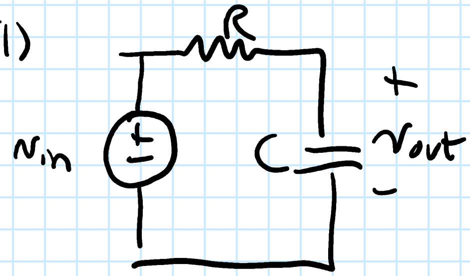
w	$\mathbb{X}_{in}$	$\mathbb{H}(jw)$	$\mathbb{X}_{in} \cdot \mathbb{H}(jw)$
0	1	$2\angle 0°$	$2\angle 0°$
5	$3\angle 0°$	$\frac{10}{j5+5} = 1.414\angle -45°$	$4.243\angle -45°$
9	$7\angle 20°$	$\frac{10}{j9+5} = 0.9713\angle -60.95°$	$6.8\angle -40.95°$

$$\boxed{x_{out}(t) = 2 + 4.243\cos(5t - 45°) + 6.8\cos(9t - 40.95°)}$$

or $+6.8\sin(9t + 49.05°)$

Problem VI

(1)

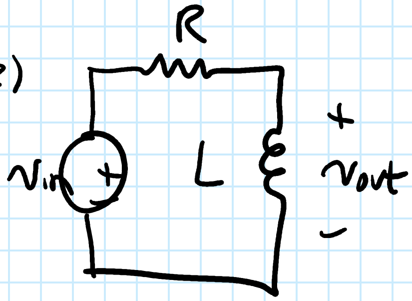


$$H = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC} = \frac{1}{1+j\frac{\omega}{2000}}$$

$$\omega_{cutoff} = \frac{1}{RC} = 2000$$

$$C = \frac{1}{(2000)(1000)} = 500nF$$

(2)



$$H = \frac{j\omega L}{R+j\omega L} = \frac{j\omega L/R}{1+j\omega L/R} = \frac{j\omega/5000}{1+j\omega/5000}$$

$$\omega_{cutoff} = \frac{R}{L} = 5000$$

$$L = \frac{R}{5000} = \frac{\cancel{2000}}{5000} = 0.4H$$

Problem VII

(1) $x(t) = 2r(t) - 4u(t-1) - 2r(t-2)$

(2) $h(t) = 2u(t) - 2u(t-3)$

(3) $x(t) * h(t) =$

$\rightarrow \quad 4q(t) - 8r(t-1) - 4q(t-2)$

$\rightarrow \quad -4q(t-3) + 8r(t-4) + 4q(t-5)$

(4) $t<0: \quad 0$

$0<t<1: \quad 4q(t) = 4\left(\frac{1}{2}t^2\right) = 2t^2$

$1<t<2 \quad 2t^2 - 8r(t-1) = 2t^2 - 8(t-1) = 2t^2 - 8t + 8$

$2<t<3 \quad 2t^2 - 8t + 8 - 4\left(\frac{1}{2}(t^2 - 4t + 4)\right) = 2t^2 - 8t + 8 - 2t^2 + 8t - 8$
$= 0$

$3<t<4 \quad 0 - 4q(t-3) = -4\left(\frac{1}{2}(t^2 - 6t + 9)\right) = -2t^2 + 12t - 18$

$4<t<5 \quad -2t^2 + 12t - 18 + 8r(t-4) = -2t^2 + 12t - 18 + 8t - 32 =$
$-2t^2 + 20t - 50$

$t>5 \quad -2t^2 + 20t - 50 + 4\left(\frac{1}{2}(t^2 - 10t + 25)\right)$
$-2t^2 + 20t - 50 + 2t^2 - 20t + 50 = 0$

Oh, Canada!

```
> restart
> u := t→Heaviside(t) :
> r := t→t·u(t) :
> x := t→2·r(t) − 4·u(t − 1) − 2·r(t − 2) :
> h := t→2·u(t) − 2·u(t − 3) :
> y := t→int(x(tau)·h(t − tau), tau = −infinity ..infinity) :
> expand(y(t))
```

$$\begin{cases} 0 & t \le 0 \\ 2t^2 & t \le 1 \\ 2t^2 - 8t + 8 & t \le 2 \\ 0 & t \le 3 \\ -2t^2 + 12t - 18 & t \le 4 \\ -2t^2 + 20t - 50 & t \le 5 \\ 0 & 5 < t \end{cases}$$

```
> plot([x(t), h(t), y(t)], t = −1 ..5)
```