Problem I

Monday, April 11, 2022

(1) $v_1 = L\frac{di_1}{dt}$ $v_2 = Ri_2$ $i_3 = C\frac{dv_3}{dt}$

(2) $E_L = \frac{1}{2}Li^2$ $E_C = \frac{1}{2}Cv^2$

(3) jwL R $\frac{1}{jwC}$

(4) current through inductor

voltage across capacitor

Problem II

(1) $-2\angle 90^\circ + 5\angle 0^\circ = 5.385\angle +21.80^\circ$

$\rightarrow 5.385\cos(22t + 21.80^\circ)$

(2) $5\angle 30^\circ + 6\angle 60^\circ = 10.63\angle 46.40^\circ$

$\rightarrow 10.63\cos(7t + 46.40^\circ)$

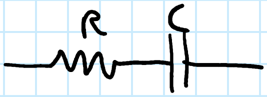
(3) $\mathbb{V} = 5\angle 0^\circ \quad \mathbb{I} = 553.3\cdot 10^{-6}\angle 51.71^\circ$

$\mathbb{Z} = \frac{\mathbb{V}}{\mathbb{I}} = 9037\angle -51.71^\circ = 5600 - j7093\ \Omega$

$\mathbb{Y} = \frac{1}{\mathbb{Z}} = 6.857\cdot 10^{-5} + j8.685\cdot 10^{-5}\ \mho$

$X<0$: CAPACITOR:

series:



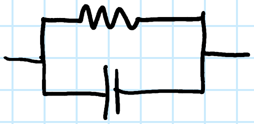
$\mathbb{Z} = R + \frac{1}{j\omega C}$

$R = 5600$

$\frac{1}{wC} = 7093$

$C = \frac{1}{7093\cdot w}$, $w=3$, $C = 47\mu F$

parallel:



$\mathbb{Y} = \frac{1}{R} + j\omega C$

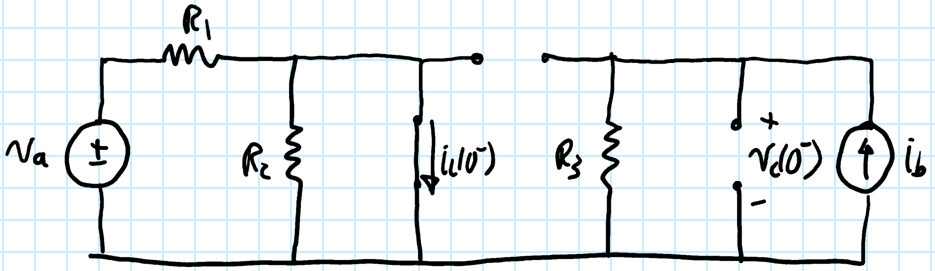
$R = \frac{1}{6.873\cdot 10^{-5}} = 14550\Omega$

$wC = 8.685\cdot 10^{5}$

$C = \frac{8.685\cdot 10^{5}}{w}$, $w=3$, $C = 28.95\mu F$

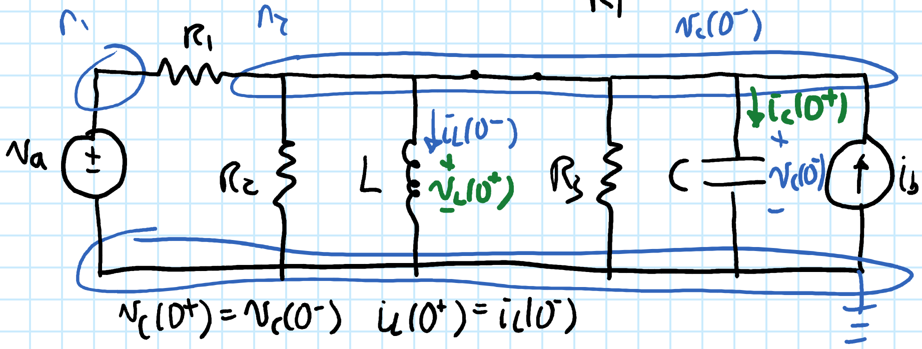
Problem III

$t=0^-$:



$v_L(0^-)=0$ $i_c(0^-)=0$ $i_L(0^-)=\frac{v_a}{R_1}$ $v_c(0^-)=R_3 i_b$

$t=0^+$



$v_c(0^+)=v_c(0^-)$ $i_L(0^+)=i_L(0^-)$

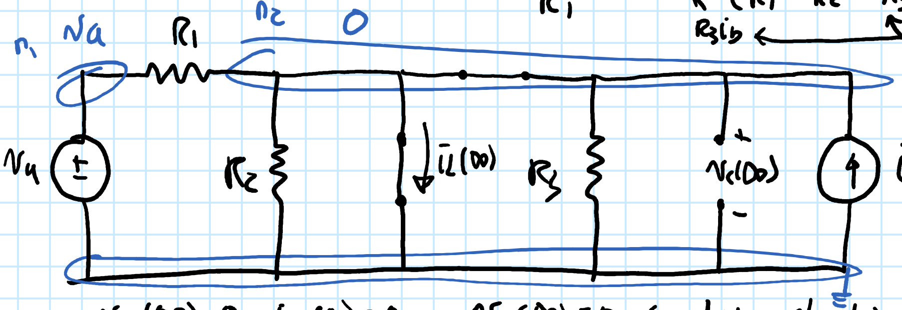
$v_L(0^+)=v_c(0^-)$

$$\text{KCL}, n_2: \frac{v_c(0^-)-v_a}{R_1}+\frac{v_c(0^-)}{R_2}+i_L(0^+)+\frac{v_c(0^-)}{R_3}+i_c(0^+)-i_b=0$$

$$i_c(0^+)=\frac{v_a}{R_1}-\underset{R_3 i_b}{v_c(0^-)}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)+i_b=\frac{v_a}{R_1}-R_3 i_b\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

cancels

$t\to\infty$



$v_L(\infty)=0$ $i_c(\infty)=0$ $v_c(\infty)=0$ (inductor shorts)

$\text{KCL}, n_2: \frac{0-v_a}{R_1}+i_L(\infty)-i_b=0$ $i_L(\infty)=\frac{v_a}{R_1}+i_b$

$v_c(0^-)=R_3 i_b \longrightarrow$	$v_c(0^+)=R_3 i_b$	$v_c(\infty)=0$
$\underline{i_c(0^-)=0}$	$i_c(0^+)=\frac{v_a}{R_1}-R_3 i_b\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$	$\underline{i_c(\infty)=0}$
$\underline{v_L(0^-)=0}$	$v_L(0^+)=R_3 i_b$	$\underline{v_L(\infty)=0}$
$i_L(0^-)=\frac{v_a}{R_1} \longrightarrow$	$i_L(0^+)=\frac{v_a}{R_1}$	$i_L(\infty)=\frac{v_a}{R_1}+i_b$

Problem IV

$$4\frac{dv_o}{dt}+3v_o=6$$

normalize to get v_o alone

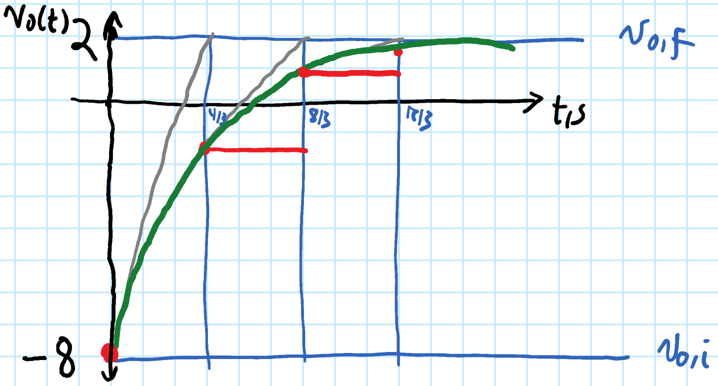
$$\underbrace{\frac{4}{3}}_{\tau}\frac{dv_o}{dt}+v_o=\underbrace{2}_{v_o(\infty)}$$

$v_o(0) = -8$

$$v_o(t)=v_{o,f}+(v_{o,i}-v_{o,f})e^{-(t-t_0)/\tau}$$

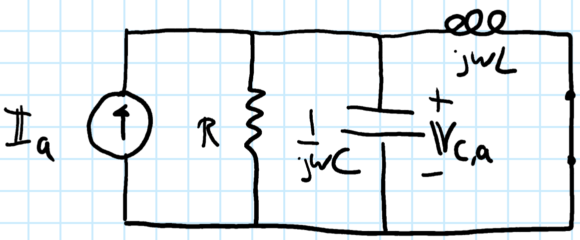
$$=2+(-8-2)\,e^{-t/(4/3)}$$

$$=2-10e^{-t/(4/3)}$$



Problem V

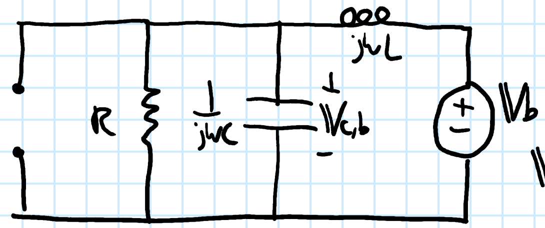
(1)



$$\mathbb{V}_{C,a} = \mathbb{Z}_{eq}\,\mathbb{I}_a \qquad \mathbb{H}_a = \frac{\mathbb{V}_{C,a}}{\mathbb{I}_a} = \mathbb{Z}_{eq}$$

$$\mathbb{H}_a = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{j\omega LR}{(j\omega)^2 LRC + (j\omega)L + R}$$

(2)



$$\mathbb{Z}_{RC} = \frac{\frac{R}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{j\omega RC + 1}$$

$$\mathbb{V}_{C,b} = \frac{\mathbb{V}_b \mathbb{Z}_{RC}}{j\omega L + \mathbb{Z}_{RC}} = \frac{\mathbb{V}_b \frac{R}{j\omega RC + 1}}{j\omega L + \frac{R}{j\omega RC + 1}} = \frac{\mathbb{V}_b R}{(j\omega)^2 LRC + j\omega L + R}$$

$$\mathbb{H}_b = \frac{\mathbb{V}_{C,b}}{\mathbb{V}_b} = \frac{R}{(j\omega)^2 LRC + j\omega L + R}$$

(3)

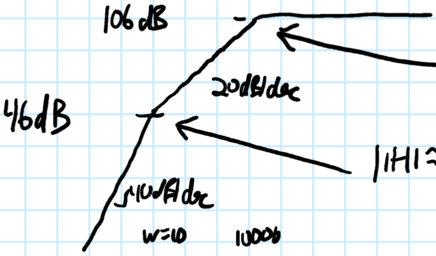
ω	$\mathbb{I}_a$	$\mathbb{H}_a$	$\mathbb{V}_b$	$\mathbb{H}_b$	$\mathbb{I}_a\mathbb{H}_a + \mathbb{V}_b\mathbb{H}_b$
0	3	0	4	1	4
8000	0	N/R	$8\angle -40°$	$1.184\angle -28.26°$	$9.471\angle -68.26°$
20000	$7 \cdot 10^{-3}\angle 20°$	$1715\angle -30.96°$	0	N/R	$12.00\angle -10.96°$

$$v_c(t) = 4 + 9.471\cos(8000t - 68.26°) + 12.00\cos(20000t - 10.96°)\ \text{V}$$

Problem VI

$$H = \frac{200000\,(jw)^2}{(jw+10)(jw+10000)} = \frac{2\,(jw)^2}{\left(1+j\frac{w}{10}\right)\left(1+j\frac{w}{10000}\right)}$$

(1)



$$|H| = \frac{(2)(10000)^2}{(1000/10)(1)} = 2\cdot10^5 \rightarrow 106\text{dB}$$

$$|H| \approx \frac{(2)(10)^2}{(1)(1)} = 200 \rightarrow 46\text{dB}$$

(2)

(3) HPF MAX GAIN = 200000 or 106 dB

$w_{cutoff} \approx 10000$ rad/s

(4)

$$\frac{V_{out}}{V_{in}} = \frac{200000\,(jw)^2}{(jw)^2+(10010)\,jw+100000}$$

$$\left((jw)^2+10010\,jw+100000\right)V_{out} = 200000\,(jw)^2\,V_{in}$$

$$\frac{d^2 v_{out}(t)}{dt^2} + 10010\frac{d v_{out}(t)}{dt} + 100000\, v_{out}(t) = 200000\, \frac{d^2 v_{in}(t)}{dt^2}$$

Problem VII

- COMES IN FLAT: NO LONE $(j\omega)$
- UP AT 100: $(1+j\omega/100)$ on top
- down AT 2000: $(1+j\omega/2000)$ on bottom
- double down at $1\cdot10^5$ $(1+j\omega/10^5)^2$ on bottom

$$H = \frac{K(1+j\omega/100)}{(1+j\omega/2\cdot10^3)(1+j\omega/10^5)^2} \qquad H(100) \approx 1000 \quad (60\,dB \to 1000)$$

$$H(100) \approx \frac{K\cdot1}{1\cdot1^2} = K \qquad K=1000$$

$$H = \frac{1000(1+j\omega/100)}{(1+j\omega/2\cdot10^3)(1+j\omega/10^5)^2} = \frac{2\cdot10^{14}(j\omega+100)}{(j\omega+2000)(j\omega+10^5)^2}$$

$$H(2000) \approx \frac{1000\cdot2000/100}{1\cdot1^2} = 20000 \to 86\,dB$$