Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 224 Spring 2021 Final Exam

Name (please print):

NetID (please print): Start Time: End Time:

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

The test is open book, open class notes (yours and mine), open Sakai page for this class, and open Pundit. No other resources are allowed. If you have a question about whether something is allowed, ask the instructor. Be sure that you are in a place where you can work undisturbed for the duration of the test. If a situation arises that disrupts your work, let the instructor know - you will be allowed to pause the clock and resume work later. The Start Time above should be when you first started working on the test (after saving it, printing it out, etc.) and the End Time should be when you stopped doing work on the test and started working on scanning / photographing and uploading it.

Please be sure that your name *and* **NetID are** *clearly* **written at the top of** *every* **page**. If you need more space for a particular problem or want to show more work, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page and submit those extra pages in problem-order.

You will be turning your test into Gradescope. Carefully scan or photograph the test pages in problem-order and make a single PDF of the scans / photographs. When you upload the PDF, you will also need to indicate where the answer for each problem is. **Be sure to indicate all pages with answers to a given problem**.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

If we discover that you put all or part of this exam on Chegg or some other disallowed site during the test window, or are found to have otherwise engaged in academic dishonesty, you will fail the class and potentially face other sanctions.

Name (please print):
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Problem I: [15 *pts.*] Non-Laplace Basics

(1) State the ideal op-amp assumption(s). Be sure to clearly define the variable(s) you are talking about, perhaps by drawing and labeling the inside of an op-amp with the simple model we discussed in class or at least giving any variable you describe a name.

(2) A first order system, driven by a constant source and with a known value of the output $y(t)$ at time $t = 0$ sec of 10 is modeled with the following equation:

$$2\frac{dy(t)}{dt} + 5y(t) = -15$$
$$y(0) = 10;$$

(a) State the value of the time constant τ for the system,

(b) Determine an expression for $y(t)$ for $t \geq 0$ sec, and

(c) Make an *accurate* and *labeled* sketch of $y(t)$ for at least three time constants past 0 sec.

(3) Clearly using phasors, simplify the following signal into a single cosine:

$$v_{\text{a}}(t) = 4\cos(160t - 35°) + 5\sin(160t + 82°)$$

(4) You are given a box with two terminals sticking out of it and told the box contains one resistive element and one reactive element. After finding the magnitude ratio and phase differences between the applied voltage and the delivered current, you determine that the impedance between the terminals when the frequency is 1000 rad/s is $\mathbb{Z}(j1000) = 2000 - j4000\ \Omega$.

(a) If the elements are in series, what kinds of elements are in the box and what are their values?

(b) If the elements are in parallel, what kinds of elements are in the box and what are their values?

(c) If you apply a voltage source $v_{\text{in}}(t) = 5\cos(1000t)$ to the box's terminals for a very long time, find a formula for the steady-state instantaneous power delivered by the source to the impedance in the box as a function of time, $p(t)$.

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Problem II: [15 *pts.*] Laplace Basics

For these problems, you must show how you would find the answers by hand.

(1) Determine the Laplace transform $\mathbb{A}(s)$ of:

$$a(t) = \left(t + 2e^{-4t}\right)\ u(t-1)$$

(2) Determine the Laplace transform $\mathbb{B}(s)$ of:

$$b(t) = \left(9e^{-8t}\cos(7t) + 6e^{-5t}\sin(4t)\right)\ u(t)$$

(3) Determine the inverse Laplace transform $f(t)$ of:

$$\mathbb{F}(s) = \frac{s+20}{s^2+10s+21}$$

(4) Determine the inverse Laplace transform $g(t)$ of:

$$\mathbb{G}(s) = \frac{s+20}{s^2+10s+34}$$

(5) Determine the inverse Laplace transform $h(t)$ of:

$$\mathbb{H}(s) = \frac{s+20}{s^2+10s+25}$$

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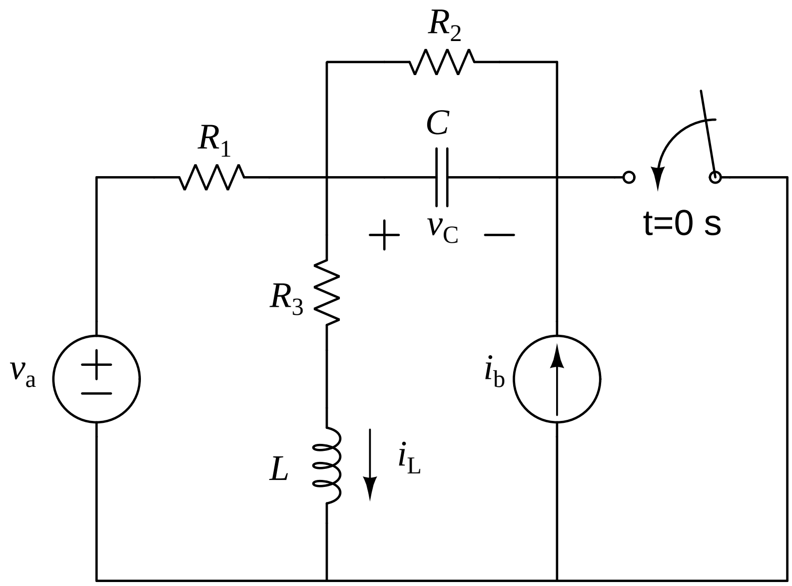
Problem III: [15 pts.] The Switch Also Closes

For the circuit below, assume that the switch has been open for a very long time before $t = 0$ s. At t=0 s the switch closes.



Assuming that v_a and i_b are constant for all times before (and after) $t = 0$, determine the following in terms of the *symbolic* element and source values (based on the passive sign convention). Also, you may use $v_C(0^-)$, $i_C(0^-)$, $v_L(0^-)$ and $i_L(0^-)$ in your solutions for the variables at 0^+ and ∞ without further substitution.

(a) $v_C(0^-)$

(b) $i_C(0^-)$

(c) $v_L(0^-)$

(d) $i_L(0^-)$

(e) $v_C(0^+)$

(f) $i_C(0^+)$

(g) $v_L(0^+)$

(h) $i_L(0^+)$

(i) $v_C(\infty)$

(j) $i_C(\infty)$

(k) $v_L(\infty)$

(l) $i_L(\infty)$

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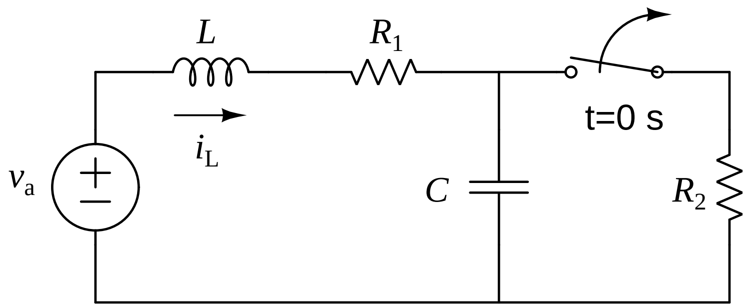
Problem IV: [20 *pts.*] Complete Response

For the circuit below, assume that the voltage source is a constant for all time and that the switch has been closed for a very long time before $t = 0$ s. At t=0 s the switch opens.



Important: For the first five parts below, answer symbolically; you may substitute in numbers only for part **(6)**.

(1) How much energy is stored in the circuit at $t = 0^-$ s?

(2) How much energy is stored in the circuit at $t = 0^+$ s?

(3) How much energy is stored in the circuit as $t \rightarrow \infty$ s?

(4) **Clearly use Laplace-based techniques** to find an expression for the unilateral Laplace transform of the current through the inductor, $\mathcal{I}_{\mathrm{L}}(s)$. Your result needs to be reduced to a ratio of polynomials in s.

(5) Clearly use the Initial and Final Value Theorems to determine $i_{\mathrm{L}}(0)$ and $i_{\mathrm{L}}(\infty)$; do these agree with what you found above? (you *did* find the currents through the inductor at 0 and ∞ above, right?)

(6) Assuming:

$$v_{\mathrm{a}}(t) = 20 \text{ V} \qquad R_1 = 1 \text{ k}\Omega \qquad R_2 = 1 \text{ k}\Omega \qquad L = 10 \text{ mH} \qquad C = 50 \text{ nF} = 50 \times 10^{-9} \text{ F}$$

Find an expression for the current through the inductor, $i_{\mathrm{L}}(t)$, for all $t > 0$ s. *Note:* In the real world, numbers aren't always pretty. You can round any numerical values you come up with here to four significant figures. **Hint:** the Laplace transform of a constant is *not* just that constant - be careful when representing v_{a} in the frequency domain. There is an s involved.* **Note:** you should be able to do a quick test of your answer using the solution to parts **(5)**.

*Well, a $1/s$, really**
**I've said too much...

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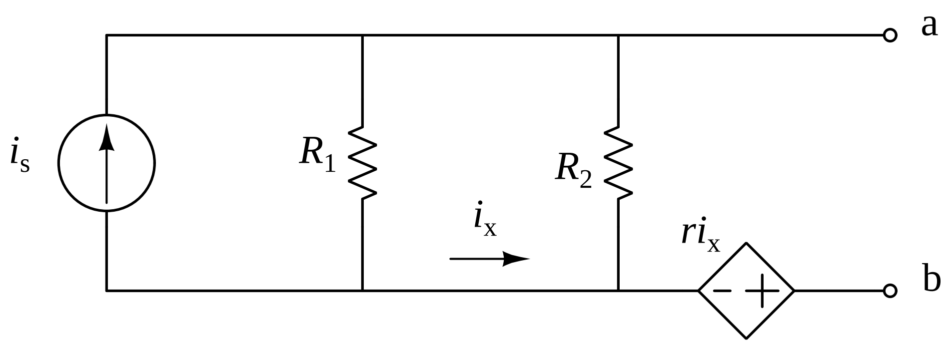
Problem V: [10 *pts.*] Blast from the Past

For the circuit:



(1) Clearly determine the values of and draw the Thévenin equivalent for the circuit as seen from terminals a and b. Be sure to include terminal labels in your circuit drawing. You may not leave your answer as a system of equations - any variable you have in your drawing must clearly be defined as a function of known values. Once you have solved for a variable in terms of known values, that variable may also be considered known.

(2) Clearly determine the values of and draw the Norton equivalent for the circuit as seen from terminals a and b. Be sure to include terminal labels in your circuit drawing. You may not leave your answer as a system of equations - any variable you have in your drawing must clearly be defined as a function of known values. Once you have solved for a variable in terms of known values, that variable may also be considered known.

(3) If a load resistor R_L were placed between the terminals a and b, what value of this resistor would maximize the power transferred to that load? What is the value of the power transferred to that load?

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Problem VI: [10 *pts.*] Filter Design and Analysis

(1) Filter 1: Design an active voltage-to-voltage low-pass filter with a maximum gain of 4 and a cutoff frequency of 2000 rad/s using a 100 nF capacitor, resistors of your choosing, ideal operational amplifiers, and wires. Indicate the transfer function $\mathbb{G}(s)$ for this filter, draw the circuit, then draw the straight-line approximation for the Bode magnitude plot and the Bode phase plot (be sure to label slopes and important values correctly). Your transfer function is allowed to have a passband phase of either 0 or 180° (which is to say, there can be either a + or - in front of the whole thing).

(2) Filter 2: Determine the transfer function $\mathbb{H}(s)$ for a second-order band-pass filter with a passband gain of 10, a logarithmic center frequency of 4000 rad/s, and a quality of 0.2. Write the transfer function for the band-pass filter using one of the two "standard" forms we discussed in class for band-pass filters. Determine and report the bandwidth, cutoff frequencies, linear center frequency, and damping ratio for the filter. You do *not* need to design a circuit for this filter, nor do you need Bode plots.

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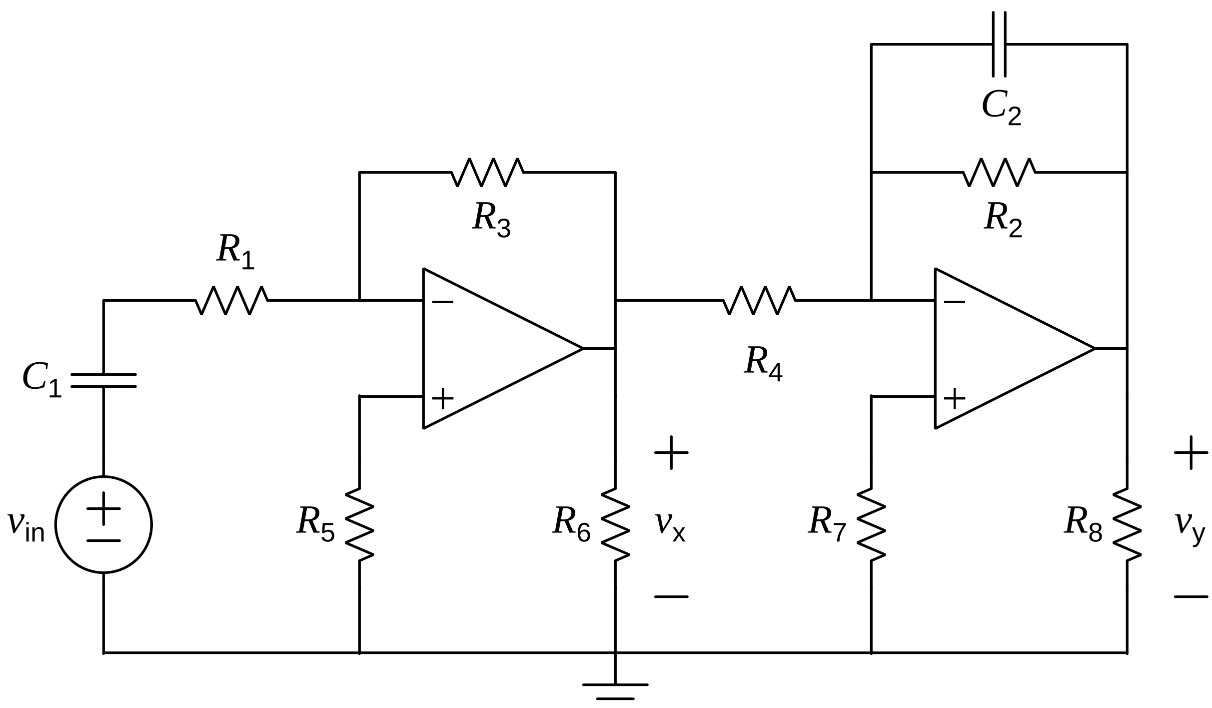
Problem VII: [15 *pts.*] Op-Amps

All problems below assume ideal op-amps.

(1) Using op-amps, design a circuit where the relationship between the output voltage v_x and an input voltage source v_a is $v_\text{x} = 5v_\text{a}$ and where there is no power delivered by the input voltage source. Any resistor values should be between 1 kΩ and 20 kΩ. Assume that you only have access to the positive terminal of v_a and that its negative terminal is attached to ground.

(2) Using op-amps, design a circuit where the relationship between the output voltage v_y and two different input voltage sourcess v_b and v_c is $v_\text{y} = 4(v_\text{b} + v_\text{c})$ and where there is no power delivered by the input voltage sources. Any resistor values should be between 1 kΩ and 20 kΩ. Assume that you only have access to the positive terminals of v_b and v_c and that their negative terminals are attached to ground.

(3) Given the following circuit:



and assuming that the values for the passive elements are known, first relabel the circuit in the frequency domain, specifically using Laplace transforms. Then, complete the following:

(a) Assuming the op amps above are ideal, determine expressions for the following transfer functions:

$$\mathbb{H}_1(s) = \frac{\mathbb{V}_\text{x}(s)}{\mathbb{V}_\text{in}(s)} \qquad \mathbb{H}_2(s) = \frac{\mathbb{V}_\text{y}(s)}{\mathbb{V}_\text{x}(s)} \qquad \mathbb{H}_3(s) = \mathbb{H}_1(s)\ \mathbb{H}_2(s) = \frac{\mathbb{V}_\text{y}(s)}{\mathbb{V}_\text{in}(s)}$$

(b) What kind of filter does each of the three transfer functions represent? There should be three answers here - be sure to clearly indicate which transfer function you are talking about for each.