### Duke Unibersity Edmund T. Pratt, Jr. School of Engineering

# EGR 224 Spring 2019 Test II

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Name (please print)

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:

### Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each answer in the correct box and make sure that your name and NET ID are clearly written at the top of every page. If you absolutely need more space for a particular problem, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the top center of that page and submit those extra pages in problem-order after all the original pages of the test. Also, in the box for the problem, write a note that says "see extra page."

To turn in your test, carefully stack the test pages in order (with any additional pages properly labeled and after all the original test pages), staple them back together if need be, and place the test in the box at the front of the room with the top left corner of the test going in the back left of the box.

Your calculator may only be used as a calculation device, not a memory storage unit. Using a calculator for any purpose other than performing "just-in-time" numerical calculations is a violation of the community standard.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

#### Problem I: [12 pts.] Phasors and Impedance

(1) Clearly using phasors, simplify the following signal into a single cosine:

$$v_{\rm a}(t) = 6\cos(12t) - 4\sin(12t)$$
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(2) Clearly using phasors, simplify the following signal into a single cosine:

 $i_{\rm b}(t) = 4\sin(100t + 12^{\circ}) + 8\cos(100t - 25^{\circ}) \,\mathrm{mA}$ 

(3) A resistor is connected to a reactive element inside a box, but you do not know what the element is or if they are connected in series or parallel. Two terminals are sticking out of the box. You conduct two experiments by applying a sinusoidal voltage across the terminals and then measuring the steady state current going into the box. The inputs and outputs you measured are below:

Source Voltage (V)Source Current (mA) $10\cos(10^5t)$  $8.17\cos(10^5t - 56.2^o)$  $10\cos(10^6t)$  $4.60\cos(10^6t - 8.50^o)$ 

(Note - there may be some small round-off in the calculations). What is the reactive element? How are the resistor and reactive element connected? What are their values? *Hint:* look at the formulas for and values of the impedance and admittance.

### Problem II: [12 pts.] ACSS

A voltage-to-voltage circuit has a transfer function and input voltage, respectively, of:

$$\mathbb{H}(j\omega) = \frac{5}{j\omega + 5} \qquad v_{\rm in}(t) = 3 + 4\cos(2t) + 6\cos(50t)$$

- (1) What kind of filter does the transfer function represent?
- (2) Design a series-connected combination of a 10 k $\Omega$  resistor and a reactive element that has the transfer function of this filter. Draw and label the circuit in the time domain, clearly showing where the output voltage is measured.
- (3) Determine the steady-state function for the output voltage.

## Problem III: [12 pts.] Unilateral Laplace Transforms

(1) Determine the Unilateral Laplace transform  $\mathcal{A}(s)$  of:

 $a(t) = 5t + \cos(3t)$ 

(2) Determine the Unilateral Laplace transform  $\mathcal{B}(s)$  of:

$$b(t) = e^{-2t} + e^{-4t}\sin(8t)$$

(3) Determine the inverse Unilateral Laplace transform c(t) of:

$$\mathcal{C}(s) = \frac{5s+3}{s^2 + 12s + 40}$$

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## Problem IV: [12 pts.] LTI Systems 1

A system can be modeled with the differential equation:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2\frac{dx(t)}{dt} + 5x(t)$$

Determine:

(1) The transfer function for the system,  $\mathbb{H}(s) = \frac{\mathbb{Y}(s)}{\mathbb{X}(s)}$ 

(2) The impulse response for the system, h(t)

(3) The step response for the system,  $s_{\rm r}(t)$ 

### Problem V: [12 pts.] LTI Systems 2

A system with an input voltage  $v_{\rm s}(t)$  and an output voltage  $v_{\rm out}(t)$  has an impulse response of:

$$h(t) = e^{-3t}u(t)$$

- (1) Determine a differential equation relating  $v_{out}(t)$  and its derivatives to  $v_s(t)$  and its derivatives.
- (2) Assuming  $v_{\text{out}}(0^-) = 2$  V and  $v_s(t) = 6e^{-2t}u(t)$  for t > 0, clearly use the Unilateral Laplace transform to determine  $v_{\text{out}}(t)$  for t > 0.

#### Problem VI: [16 pts.] Transient Response

For the circuit below, assume that the switch has been closed for a very long time before t = 0 s. At t=0 s the switch opens.



(1) Assuming that  $v_a$  is constant for all times before t = 0, determine the following in terms of the symbolic element and source values (based on the passive sign convention). Put your answers in the box below. Also, you may use  $v_{\rm C}(0^-)$  and  $i_{\rm C}(0^-)$ in your solutions for the variables at  $0^+$  without further substitution.



(2) Assuming the circuit has the following element and source values:

$$R_1 = 1 \ \mathrm{k}\Omega$$
  $R_2 = 2 \ \mathrm{k}\Omega$   $C = 5 \ \mu\mathrm{F}$   $v_{\mathrm{a}}(t) = 20 \ \mathrm{V}$ 

determine the voltage across the capacitor,  $v_{\rm C}(t)$ , for t > 0 s. Also indicate the time constant of the response and then make an accurate graph of  $v_{\rm C}(t)$  for three time constants.

## Problem VII: [12 pts.] Band-Pass Filter Design

Determine the transfer function  $\mathbb{G}(s)$  for a second-order band-pass filter with a passband gain of 10 and cutoff frequencies of 100 and 2000 rad/s. Write the transfer function for the band-pass filter using one of the two "standard" forms we discussed in class for band-pass filters. Also state the logarithmic center frequency, linear center frequency, bandwidth, and quality of the filter. You do *not* need to find the corner frequencies nor do you need to sketch Bode plots or draw a circuit.

## Problem VIII: [12 pts.] Bode Plots

Sketch the straight-line approximation for the Bode plots of both the magnitude and the phase of the transfer functions below. Be sure to properly label the axes, slopes, magnitudes, and angles.

(1) 
$$\mathbb{H}_{1}(s) = \frac{s}{s+1}$$
  
(2)  $\mathbb{H}_{2}(s) = \frac{10}{(s+100)(s+1000)}$