

Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 224 Spring 2018

Test II

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Name (please print) _____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: _____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Your calculator may only be used as a calculation device, not a memory storage unit. Using a calculator for any purpose other than performing "just-in-time" numerical calculations is a violation of the community standard.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):

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Problem I: [25 pts.] Phasors and Impedance

- (1) Clearly using phasors, simplify the following signal into a single cosine:

$$v_a(t) = 2 \sin(10t) + 3 \cos(10t) \text{ V}$$

- (2) Clearly using phasors, simplify the following signal into a single cosine:

$$i_b(t) = -20 \cos(200t + 9^\circ) + 18 \sin(200t - 54^\circ) \text{ mA}$$

- (3) A resistor is connected to a capacitor inside a box, but you do not know if they are connected in series or parallel. Two terminals are sticking out of the box. You conduct two experiments by applying a sinusoidal voltage across the terminals and then measuring the steady state current going into the box. The inputs and outputs you measured are below:

Source Voltage (V)	Source Current (mA)
$10 \cos(10t)$	$3.84 \cos(10t + 54.8^\circ)$
$10 \cos(20t)$	$5.44 \cos(20t + 35.4^\circ)$

(Note - there may be some small roundoff in the calculations). How are the resistor and capacitor connected? What are their values? *Hint:* look at the formulas for and values of the impedance and admittance.

- (4) Design a series-connected combination of a resistor and a reactive element that has an impedance of $\mathbb{Z} = 2000 - j1000 \Omega$ at a frequency of 5,000 rad/s. Draw and label the circuit.
- (5) Design a parallel-connected combination of a resistor and a reactive element that has an impedance of $\mathbb{Z} = 2000 - j1000 \Omega$ at a frequency of 5,000 rad/s. Draw and label the circuit.
- (6) A voltage-to-voltage circuit has a transfer function of $\mathbb{H}(j\omega) = \frac{j\omega}{20 + j\omega}$ and in input voltage of $v_{in}(t) = 2 \cos(t) + 3 \cos(20t)$ V. Determine the steady-state function for the output voltage.

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Problem II: [20 pts.] Unilateral Laplace Transforms

(1) Determine the Unilateral Laplace transform $\mathcal{A}(s)$ of:

$$a(t) = 5 + \sin(4t)$$

(2) Determine the Unilateral Laplace transform $\mathcal{B}(s)$ of:

$$b(t) = e^{-t} + e^{-3t} \cos(6t)$$

(3) Determine the inverse Unilateral Laplace transform $c(t)$ of:

$$\mathcal{C}(s) = \frac{s + 4}{(s + 1)(s + 2)(s + 3)}$$

(4) Determine the inverse Unilateral Laplace transform $d(t)$ of:

$$\mathcal{D}(s) = \frac{2s - 6}{s^2 + 10s + 34}$$

(5) Determine the inverse Unilateral Laplace transform $f(t)$ of:

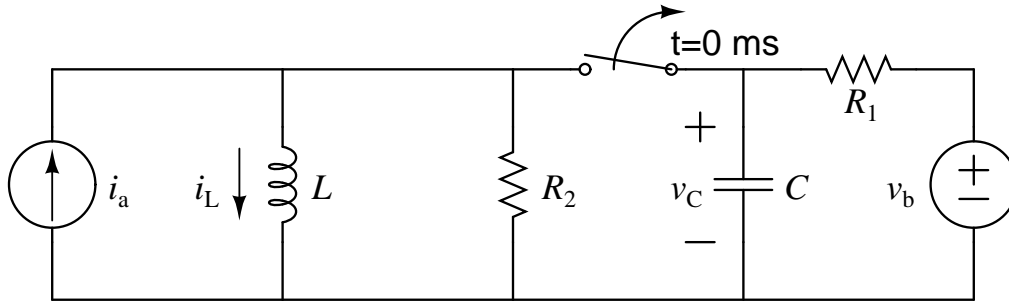
$$\mathcal{F}(s) = \frac{s + 20}{s^2 + 9s + 8}$$

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Problem III: [30 pts.] Complete Response

For the circuit below, assume that the switch has been closed for a very long time before $t = 0$ s. At $t = 0$ s the switch opens.



- (1) Assuming that i_a and v_b are constant for all times before $t = 0$, determine the following in terms of the symbolic element and source values (based on the passive sign convention); **write your final answer next to the item** - your work can be on extra paper. Also, you may use $v_C(0^-)$, $i_C(0^-)$, $v_L(0^-)$, and $i_L(0^-)$ in your solutions for the variables at 0^+ without further substitution.

(a) $v_C(0^-)$

(e) $v_L(0^-)$

(b) $i_C(0^-)$

(f) $i_L(0^-)$

(c) $v_C(0^+)$

(g) $v_L(0^+)$

(d) $i_C(0^+)$

(h) $i_L(0^+)$

- (2) Assuming the circuit has the following element and source values:

$$R_1 = 2 \text{ k}\Omega$$

$$R_2 = 1 \text{ k}\Omega$$

$$C = 2 \text{ }\mu\text{F}$$

$$L = 500 \text{ mH}$$

$$i_a(t) = 10e^{-1000t}u(t) \text{ mA}$$

$$v_b(t) = 20 \text{ V}$$

where $u(t)$ is the unit step function.

- (a) Determine the voltage across the capacitor, $v_C(t)$, for $t > 0$ s. **Write your final answer below.** On a separate sheet of paper, indicate the time constant of the response and then make an accurate graph of $v_C(t)$ for three time constants.

- (b) Determine the current through the inductor, $i_L(t)$, for $t > 0$ s. **Write your final answer below.** You do not need to make a graph of this result.

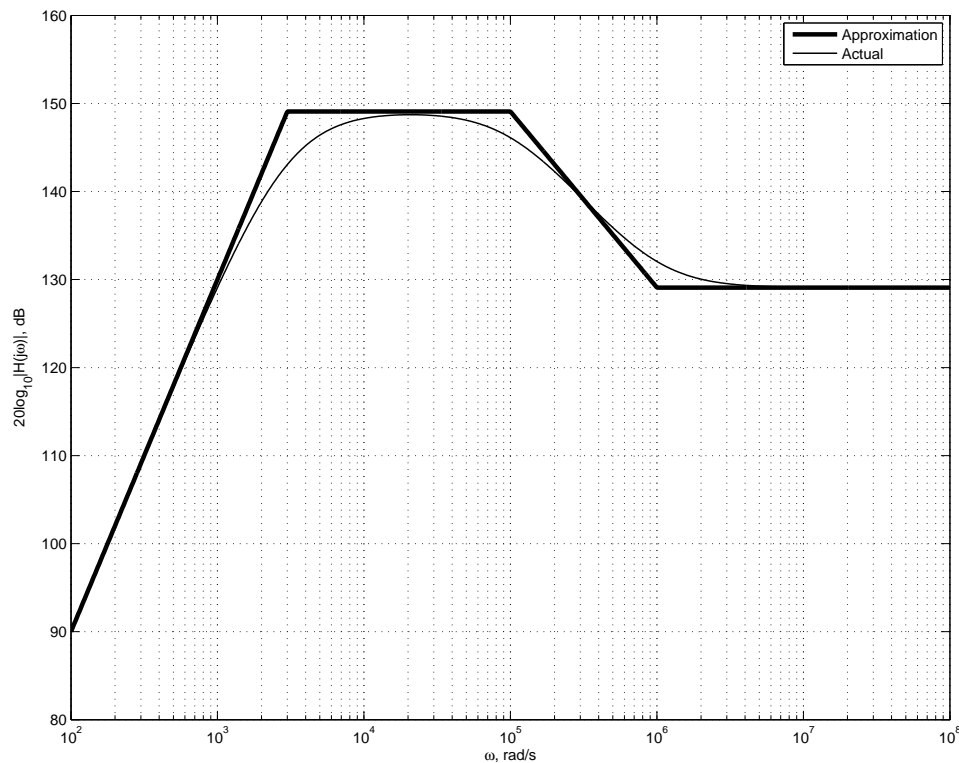
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Problem IV: [25 pts.] Design and Analysis

(1) First order:

- (a) Determine the transfer function $\mathbb{G}_1(s) = \frac{V_{out}(s)}{V_{in}(s)}$ for a first-order low-pass filter with a maximum gain of 1 and a cutoff frequency of 5000 rad/s.
- (b) For the system with a transfer function of $\mathbb{G}_1(s)$, Write the differential equation that relates $v_{out}(t)$ and its derivatives to $v_{in}(t)$ and its derivatives.
- (c) Sketch an accurate straight-line approximation to the Bode magnitude plot of $\mathbb{G}_1(s)$. Be sure to label your axes, all slopes, and the frequencies and magnitudes at all corners.
- (d) Sketch an accurate straight-line approximation to the Bode phase plot of $\mathbb{G}_1(s)$. Be sure to label your axes, all slopes, and the frequencies and angles at all corners.
- (e) Assuming you only have a single 1 k Ω resistor, along with a supply of any inductors, capacitors, or wires you might need, design a voltage-to-voltage filter with a transfer function of $\mathbb{G}_1(s)$. Be sure to clearly input where $v_{out}(t)$ is measured.

- (2) Determine the transfer function $\mathbb{G}_2(s)$ for a second-order band-pass filter with a passband gain of 25 and cutoff frequencies of 60 and 1500 rad/s. Write the transfer function for the band-pass filter using one of the two “standard” forms we discussed in class for band-pass filters. Also state the logarithmic center frequency, bandwidth, and quality of the filter.
- (3) Given the following Bode magnitude plot of some transfer function $\mathbb{H}(s)$ (along with its straight line approximation):



- (a) Assuming all poles in the system are purely real, determine the formula for a transfer function $\mathbb{H}(s)$ or $\mathbb{H}(j\omega)$ which is represented in the figure.
- (b) What kind of filter is this? Also state why you believe that, *approximately* what the cutoff frequency/frequencies is/are, and what the passband gain is/is.