Duke University
Edmund T. Pratt, Jr. School of Engineering

EGR 224 Spring 2016

Test II

Michael R. Gustafson II

Name (please print) Solution

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

For 2 points: Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Your calculator may only be used as a calculation device, not a memory storage unit. Using a calculator for any purpose other than performing "just-in-time" numerical calculations is a violation of the community standard.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):
Community Standard (print NetID):

Problem I: [32 pts.] Basics

The following items get at the "core" of some of the concepts you have learned in the course. Note however that they are *not* related to each other. **Put your final answers in the appropriate spaces below**; you may put work on additional pages.

(1) Clearly using phasors, simplify the following signal into a single cosine:

$$i_a(t) = 18\sin(500t - 19^o) - 41\cos(500t + 89^o)$$

$$18\angle -19^\circ - 90^\circ - 41\angle 89^\circ = 18\angle -109^\circ - 41\angle 89^\circ = 58.38\angle -96.5^\circ$$

$$58.38\cos(500t - 96.5^\circ)$$

(2) A resistor R is in **parallel** with a reactive element. At a frequency of 20000 rad/s, the **impedance** of the combination is found to be $4000+j5000\ \Omega$. Draw the combination below, including the actual values of the resistor and the reactive element.

$Z = 4000 + j5000$

$Y = \frac{1}{Z} = 9.756\cdot 10^{-5} - j1.220\cdot 10^{-4}\ \mho$

$Y = \frac{1}{R} + \frac{1}{j\omega L}$ so

$R = \frac{1}{9.756\cdot 10^{-5}} = 10250\ \Omega$

$L = \frac{1}{\omega \cdot 1.22\cdot 10^{-4}} = 0.4\ \mathrm{H}$

(3) Determine the inverse Laplace transform $v(t)$ of:

$$\mathbb{V}(s) = \frac{2s+17}{s^2+12s+45} = \frac{2s+17}{(s+6)^2+(3)^2} = \frac{2(s+6)+\frac{5}{3}(3)}{(s+6)^2+(3)^2}$$

$$v(t) = e^{-6t}\left(2\cos 3t + \frac{5}{3}\sin 3t\right)u(t)$$

(4) A system can be modeled with the differential equation:

$$2\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 12y(t) = 3\frac{dx(t)}{dt} + 5x(t)$$

Determine:

$2s^2 Y + 10sY + 12Y = 3sX + 5X$ so

(a) The transfer function for the system, $\mathbb{H}(s) = \frac{\mathbb{Y}(s)}{\mathbb{X}(s)}$

(b) The impulse response for the system, $h(t)$

(c) The step response for the system, $s_r(t)$

$$H = \frac{Y}{X} = \frac{3s+5}{2s^2+10s+12}$$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{2s^2+10s+12}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{3}{2}s+\frac{5}{2}}{(s+2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s+2} + \frac{B}{s+3}\right\}$$

$$A = \lim_{s\to -2} \frac{\frac{3}{2}s+\frac{5}{2}}{s+3} = -\frac{1}{2}$$

$$B = \lim_{s\to -3} \frac{\frac{3}{2}s+\frac{5}{2}}{s+2} = 2$$

$$h(t) = \left(-\frac{1}{2}e^{-2t} + 2e^{-3t}\right)u(t)$$

$$s_r(t) = \mathcal{L}^{-1}\left\{\frac{H}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{3}{2}s+\frac{5}{2}}{s(s+2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{C}{s} + \frac{D}{s+2} + \frac{E}{s+3}\right\}$$

$$C = \lim_{s\to 0} \frac{\frac{3}{2}s+\frac{5}{2}}{(s+2)(s+3)} = \frac{5}{12}$$

$$D = \lim_{s\to -2} \frac{\frac{3}{2}s+\frac{5}{2}}{(s)(s+3)} = \frac{1}{4}$$

$$E = \lim_{s\to -3} \frac{\frac{3}{2}s+\frac{5}{2}}{s(s+2)} = -\frac{2}{3}$$

$$s_r(t) = \left(\frac{5}{12} + \frac{1}{4}e^{-2t} - \frac{2}{3}e^{-3t}\right)u(t)$$

(5) A system can be modeled with the differential equation:

$$\frac{dz(t)}{dt} + 4z(t) = 10\,e^{-t}u(t)$$

If $z(0^-)=6$, clearly use the Unilateral Laplace Transform to determine $z(t)$ for $t > 0$.

$$sZ - z(0^-) + 4Z = \frac{10}{s+1}$$

$$(s+4)Z = \frac{10}{s+1} + 6 = \frac{6s+16}{s+1}$$

$$Z = \frac{6s+16}{(s+1)(s+4)} = \frac{F}{s+1} + \frac{G}{s+4}$$

$$F = \lim_{s\to -1} \frac{6s+16}{s+4} = \frac{10}{3}$$

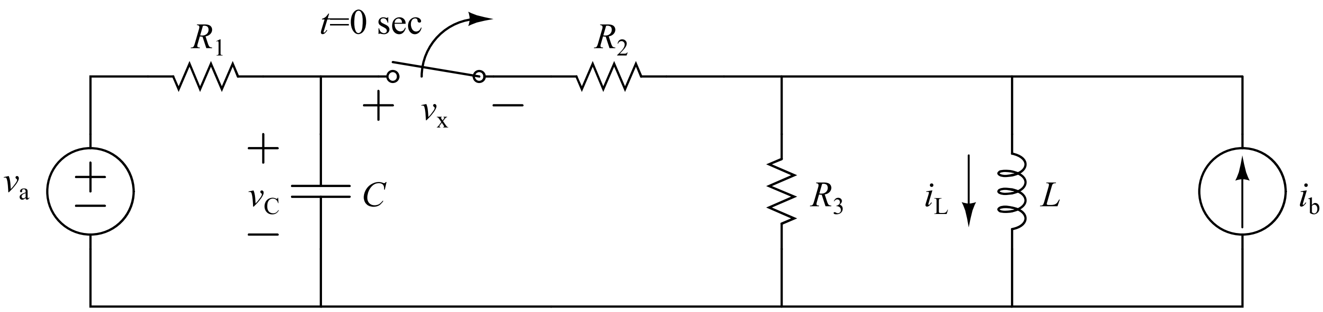
$$G = \lim_{s\to -4} \frac{6s+16}{s+1} = \frac{8}{3}$$

$$z(t) = \left(\frac{10}{3}e^{-t} + \frac{8}{3}e^{-4t}\right)u(t)$$

Name (please print):
Community Standard (print NetID):

Problem II: [24 pts.] Complete Response

For the circuit below, assume that the switch has been closed for a very long time before $t = 0$ s. At t=0 s the switch opens.



(1) Assuming that v_a and i_b are constant for all times before $t = 0$, determine the following in terms of the symbolic element and source values; **write your final answer next to the item** - your work can be on extra paper. Also, you may use $v_C(0^-)$, $i_C(0^-)$, $v_L(0^-)$, and $i_L(0^-)$ in your solutions for the variables at 0^+ without further substitution. C is open, L is short

(a) $v_C(0^-)$ $\quad \dfrac{V_a R_2}{R_1+R_2}$

(e) $v_L(0^-)$ $\quad 0$

(b) $i_C(0^-)$ $\quad 0$

(f) $i_L(0^-)$ $\quad \dfrac{V_a}{R_1+R_2} + i_b$

(c) $v_C(0^+)$ $\quad V_c(0^-) = \dfrac{V_a R_2}{R_1+R_2}$

(g) $v_L(0^+)$ $\quad R_3\left(i_b - i_L(0^-)\right) = \dfrac{-V_a R_3}{R_1+R_2}$

(d) $i_C(0^+)$ $\quad \dfrac{V_a - V_c(0^-)}{R_1} = \dfrac{V_a}{R_1+R_2}$

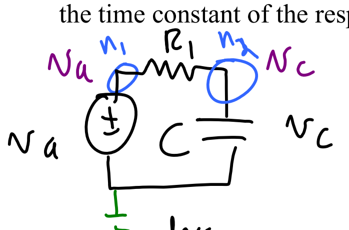
(h) $i_L(0^+)$ $\quad i_L(0^-) = \dfrac{V_a}{R_1+R_2} + i_b$

(2) Assuming the circuit has the following element and source values:

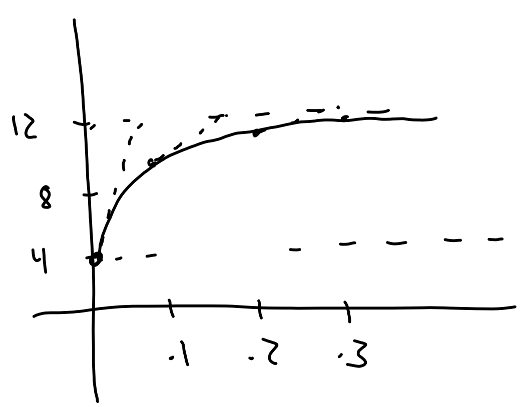
$R_1 = 2\ \text{k}\Omega$ $\quad R_2 = 1\ \text{k}\Omega$ $\quad R_3 = 4\ \text{k}\Omega$ $\quad C = 50\ \mu\text{F}$ $\quad L = 100$ mH

$v_a(t) = 12$ V $\quad i_b(t) = 5$ mA

Determine the voltage across the capacitor, $v_C(t)$, for $t > 0$ s. **Write your final answer below**. On a separate sheet of paper, indicate the time constant of the response and then make an accurate graph of $v_C(t)$ for three time constants.



KCL n_2: $\dfrac{V_c - V_a}{R_1} + C\dfrac{dV_c}{dt} = 0$



$R_1 C \dfrac{dV_c}{dt} + V_c = V_a$

$0.1 \dfrac{dV_c}{dt} + V_c = 12$

$V_c(0^-) = \dfrac{V_a R_2}{R_1+R_2} = 4$, $V_{c,f} = 12$, $\tau = 0.1$

$V_c(t) = V_{c,f} + (V_{c,i} - V_{c,f})\, e^{-(t-t_0)/\tau} = \boxed{12 - 8e^{-t/0.1}}$

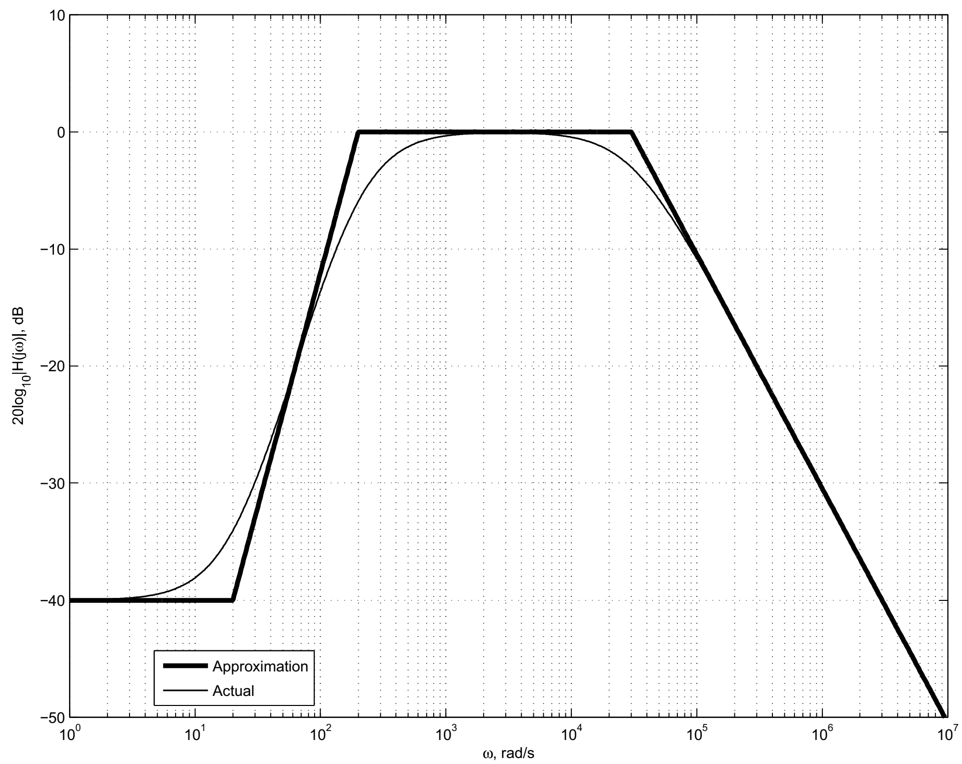
Name (please print):
Community Standard (print NetID):

Problem III: [24 `pts.`] Design and Analysis

(1) Assuming you have a single 1 kΩ resistor and any inductors, capacitors, and wires you need, design a voltage-to-voltage circuit that represents a first-order low pass filter with a maximum gain of 1 and a cutoff frequency of 6000 rad/s. Be sure to clearly indicate where the input and output voltages would be on the circuit.

(2) Determine the transfer function $\mathbb{G}(s)$ for a second-order band-pass filter with a passband gain of 1, a logarithmic center frequency of 10000 rad/s, and a quality of 0.4. Write the transfer function for the band-pass filter using one of the two "standard" forms we discussed in class for band-pass filters.

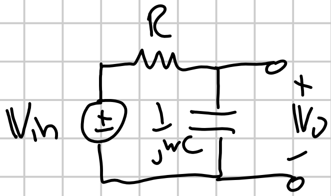
(3) Given the following Bode magnitude plot of some transfer function $\mathbb{H}(s)$ (along with its straight line approximation):



(a) Assuming all poles in the system are purely real, determine the formula for a transfer function $\mathbb{H}(s)$ or $\mathbb{H}(j\omega)$ which is represented in the figure.

(b) What kind of filter is this? Also state why you believe that, *approximately* what the cutoff frequency/frequencies is/are, and what the passband gain is/is.

(1) LPF:



or

$H = \dfrac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \dfrac{1}{1 + j\omega RC}$

$\omega_{co} = 1/RC$ so $C = 1/\omega_{co}R = 167\,nF$

$H = \dfrac{R}{j\omega L + R} = \dfrac{1}{1 + j\omega L/R}$

$\omega_{co} = R/L$ so $L = R/\omega_{co} = 167\,mH$

(2) Gain = 1, $\omega_n = 10000$, $Q = 0.4$, $K = 1$ so $\zeta = \dfrac{1}{2Q} = 1.25$

Either: $\dfrac{K\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2}$ or $\dfrac{K}{1 + jQ\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)}$

(3) $\dfrac{K\left(1 + \frac{j\omega}{20}\right)^2}{\left(1 + \frac{j\omega}{200}\right)^2 \left(1 + \frac{j\omega}{30000}\right)}$

for K: $\lim_{\omega \to 0} |H| = 10^{-2} = .01 = K$

$\dfrac{.01\,(1 + j\omega/20)^2}{(1 + j\omega/200)^2\,(1 + j\omega/30000)} = \dfrac{30000\,(j\omega + 20)^2}{(j\omega + 200)^2\,(j\omega + 30000)}$

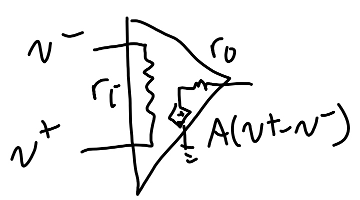
(4) BPF; two cutoffs, $\omega_{cutoff} \approx 200,\ 30000$

Max gain $= 10^0 = 1$ or 0dB

Name (please print):
Community Standard (print NetID):

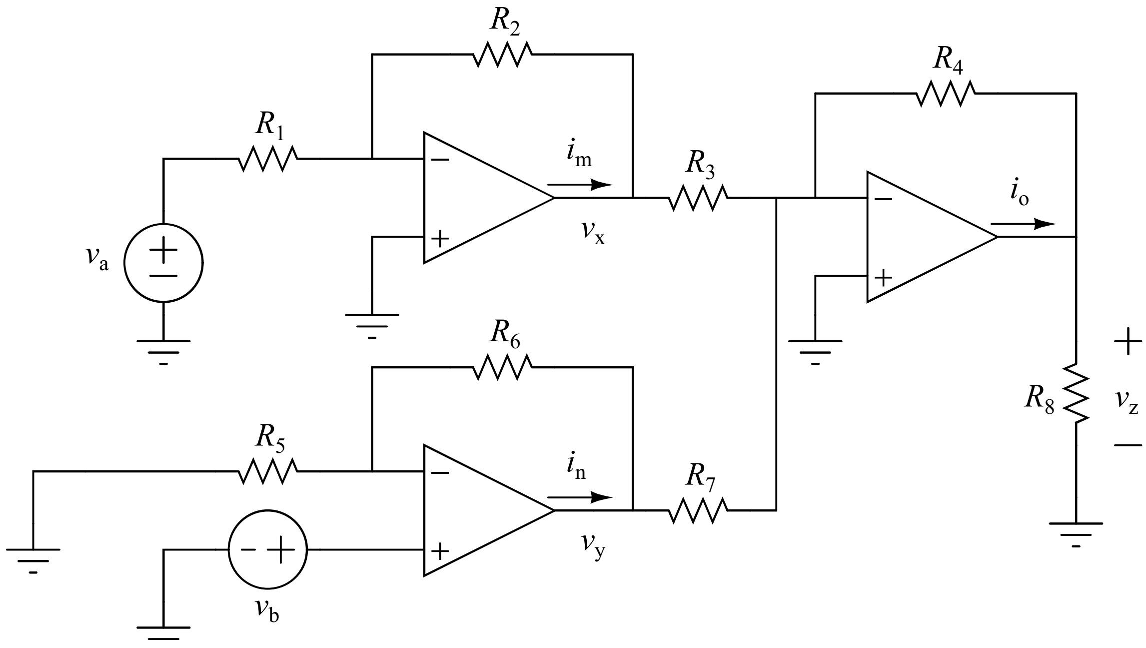
Problem IV: [18 `pts.`] Operational Amplifiers

(1) State the ideal op-assumptions; be sure to describe the meaning of any variables you name:



$A \to \infty$
$r_o \to 0$
$r_i \to \infty$

(2) Assuming the op-amps in the following circuit are ideal:



and that v_a and v_b are known, clearly label the circuit and then symbolically solve for the items below. Please put your final answers here though your work may be on a different page:

(1) v_x $\quad -\frac{R_2}{R_1} v_a$

(2) v_y $\quad \left(1+\frac{R_6}{R_5}\right) v_b$

(3) v_z $\quad -R_4\left(\frac{v_x}{R_3} + \frac{v_y}{R_7}\right)$

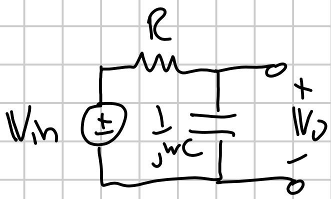
(4) i_m $\quad \frac{v_x}{R_2} + \frac{v_x}{R_3}$

(5) i_n $\quad \frac{v_y - v_b}{R_6} + \frac{v_y}{R_7}$

(6) i_o $\quad \frac{v_z}{R_8} + \frac{v_z}{R_4}$

Note: if you solve for an unknown variable in terms of known values, you may then use that variable in other answers without substitution. For instance, v_z should end up as merely a function of v_x and v_y and resistor values.

(1) LPF:



or

$$H = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$\omega_{co} = 1/RC$ so $C = 1/\omega_{co}R = 167\, nF$

$$H = \frac{R}{j\omega L + R} = \frac{1}{1 + j\omega L/R}$$

$\omega_{co} = R/L$ so $L = R/\omega_{co} = 167\, mH$

(2) Gain $= 1$, $\omega_n = 10000$, $Q = 0.4$, $K = 1$ so $\zeta = \frac{1}{2Q} = 1.25$

Either: $\dfrac{K\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2}$ or $\dfrac{K}{1 + jQ\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)}$

(3) $$\frac{K\left(1 + \frac{j\omega}{20}\right)^2}{\left(1 + \frac{j\omega}{200}\right)^2\left(1 + \frac{j\omega}{30000}\right)}$$

for K: $\lim_{\omega \to 0} |H| = 10^{-2} = .01 = K$

$$\frac{.01\,(1 + j\omega/20)^2}{(1 + j\omega/200)^2\,(1 + j\omega/30000)} = \frac{30000\,(j\omega + 20)^2}{(j\omega + 200)^2\,(j\omega + 30000)}$$

(4) BPF; two cutoffs, $\omega_{cutoff} \approx 200, 30000$

Max gain $= 10^0 = 1$ or 0dB