Duke University

Edmund T. Pratt, Jr. School of Engineering

EGR 224 Spring 2015

# Test I

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Name (please print) SOLUTION

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: ______________________

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## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

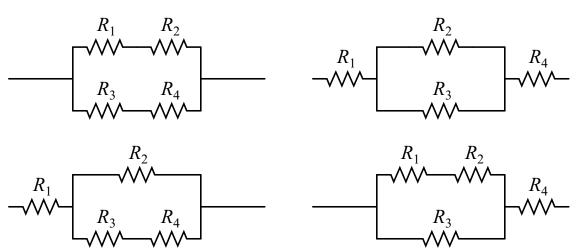
Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

You may use the $\|$ symbol for resistances in parallel and do not need to expand that construction. Be clear with your use of parentheses, however; simply writing something like

$$R_{\text{eq}} = R_1 + R_2 \parallel R_3 + R_4$$

is too vague since it could refer to any of the four combinations below:



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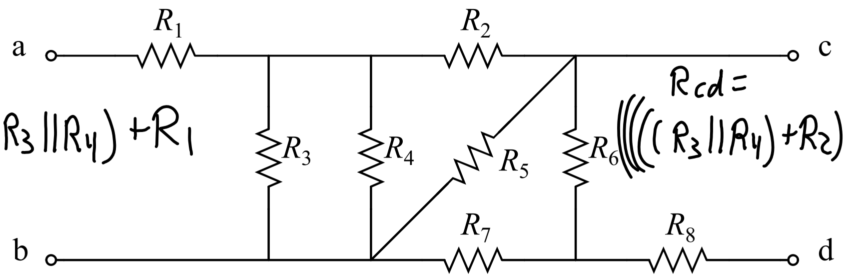
## Problem I: [22 pts.] Equivalents and Division

For all parts of this problem, you can *carefully* use the || symbol (and parentheses) as appropriate and do *not* need to simplify expressions using that symbol.

**(1)** For the following network:

$R_{ab} = ((((R_6+R_7)||R_5)+R_2)||R_3||R_4)+R_1$

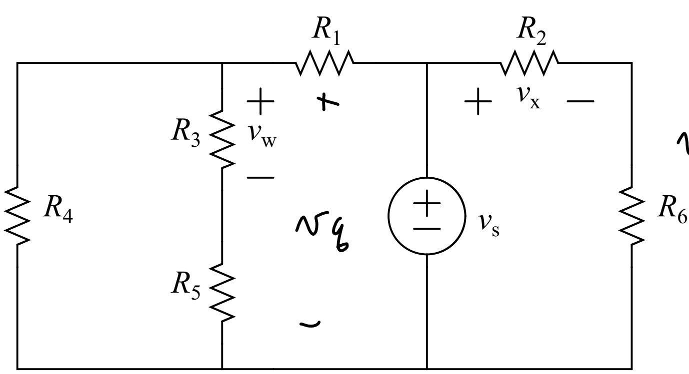



$R_{cd} = ((((R_3||R_4)+R_2)||R_5)+R_7)||R_6)+R_8$

*(a)* Find the equivalent resistance as seen from terminals **a** and **b**, $R_{ab}$
*(b)* Find the equivalent resistance as seen from terminals **c** and **d**, $R_{cd}$.

**(2)** For the following circuit:

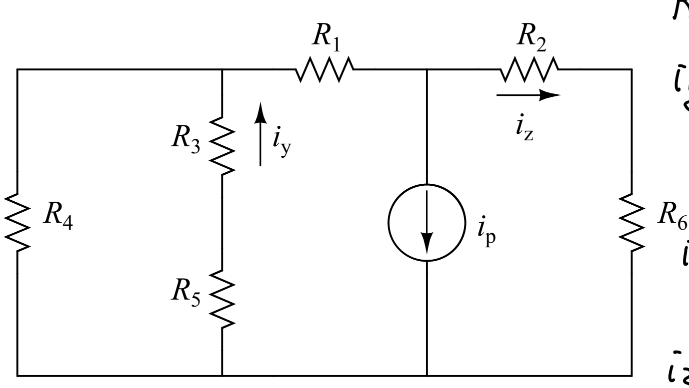
$$v_w = \underbrace{v_s \frac{(R_4||(R_3+R_5))}{R_1+(R_4||(R_3+R_5))}}_{v_q} \frac{R_3}{R_3+R_5}$$




$$v_x = \frac{v_s R_2}{R_2+R_6}$$

*clearly* show voltage division to obtain expressions for $v_w$ and $v_x$ in terms of the resistors and $v_s$.

**(3)** For the following circuit:




$R_{eq} = (R_2+R_6)||(R_1+(R_4||(R_3+R_5)))$

$$i_y = i_p \frac{R_{eq}}{R_1+(R_4||(R_3+R_5))} \frac{R_4||(R_3+R_5)}{R_3+R_5}$$

or

$$i_y = \frac{i_p(R_2+R_6)}{R_2+R_6+(R_1+(R_4||(R_3+R_5)))} \frac{R_4}{R_4+R_3+R_5}$$

$$i_z = \frac{-i_p R_{eq}}{R_2+R_6}$$

*clearly* show current division to obtain expressions for $i_y$ and $i_z$ in terms of the resistors and $i_p$.

or

$$\frac{-i_p(R_1+(R_4||(R_3+R_5)))}{(R_1+(R_4||(R_3+R_5)))+R_2+R_6}$$

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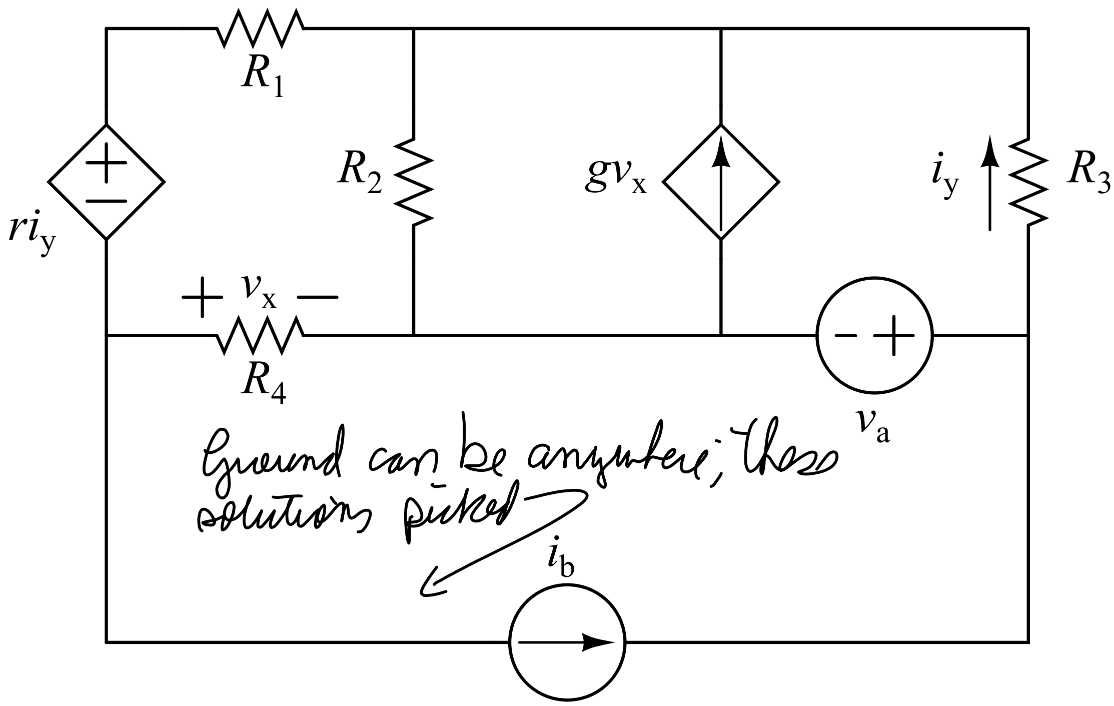
## Problem II: [15 pts.] Node Voltage Method

Many correct answers depending on labels and where ground is.
Regardless, 5 nodes - 1 - 2 v.s = 2 KCL

Given the following circuit:




and assuming that constants $g$ and $r$, the values for the passive elements ($R_1$ through $R_4$), and the values for the independent sources ($v_a$ and $i_b$) are known,

**(1)** *Clearly* demonstrate the use of the Node Voltage Method in labeling unknowns for the circuit and in determining a complete set of linearly independent equations that could be used to solve for these unknowns. List the set of unknowns you believe your equations will find. Please put the list of unknowns and the equations on a separate piece of paper and in a box; you can label the circuit above.

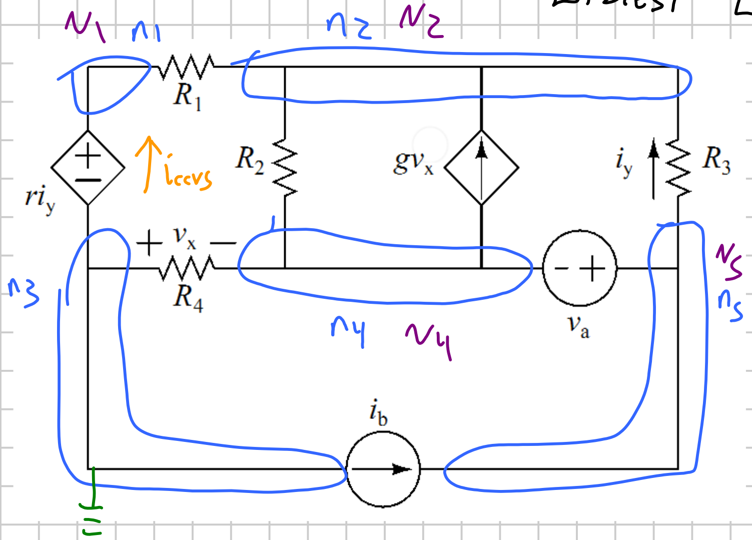
**(2)** Assuming you are able to solve for those unknowns, write expressions for the following. Put your expressions next to the appropriate bullet below even if your work is elsewhere:

- $p_{abs,R_2} =$

- $p_{del,VCCS} =$

- $p_{del,CCVS} =$

# "LAZIEST" LABELS




Unk: $v_1$ $v_2$ $v_4$ $v_5$ $v_x$ $i_y$

KCL, $n_2$: $\frac{v_2-v_1}{R_1} + \frac{v_2-v_4}{R_2} - gv_x + \frac{v_2-v_5}{R_3} = 0$

or $-i_y$

KCL, $sn_{45}$: $\frac{v_4-0}{R_4} + \frac{v_4-v_2}{R_2} + gv_x + \frac{v_5-v_2}{R_3} - i_b = 0$

or $-\frac{v_x}{R_4}$   or $+i_y$

SRC $ri_y$: $ri_y = v_1 - 0$

SRC $v_a$: $v_a = v_5 - v_4$

MEAS $v_x$: $v_x = v_3 - v_4$

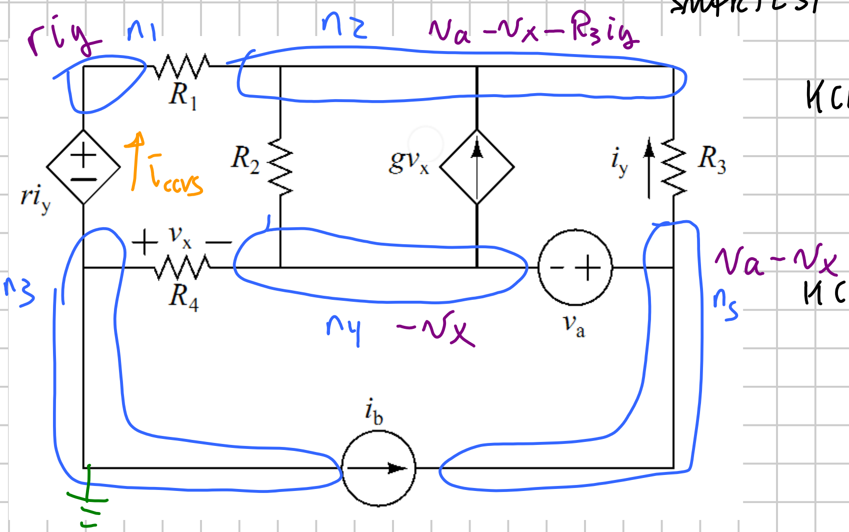
MEAS $i_y$: $i_y = \frac{v_5 - v_2}{R_3}$

$P_{abs,R_2} = (v_2-v_4)^2 / R_2$

$P_{del,vccs} = gv_x(v_2 - v_4)$

$i_{ccvs} = \frac{v_1-v_2}{R_1}$ or $\frac{-v_x}{R_4} - i_b$; $P_{del,vccs} = ri_y i_{ccvs}$

# "SMARTEST" LABELS




UNK: $v_x$, $i_y$

KCL, $n_2$: $\frac{v_a - v_x - R_3 i_y - ri_y}{R_1} + \ldots$

$\frac{v_a - R_3 i_y}{R_2} - gv_x - i_y = 0$

KCL, $sn_{45}$: $\boxed{\frac{-v_x}{R_4} + \frac{R_3 i_y - v_a}{R_2} + gv_x + i_y - i_b = 0}$

$P_{abs,R_2} = (v_a - R_3 i_y)^2 / R_2$

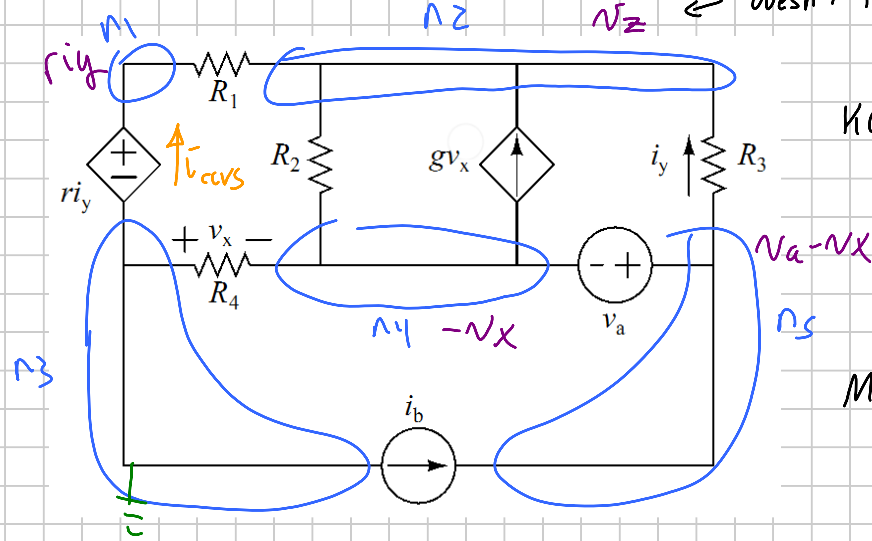
$P_{del,vccs} = gv_x (v_a - R_3 i_y)$

$i_{ccvs} = \frac{ri_y - (v_a - v_x - R_3 i_y)}{R_1}$ or $\frac{-v_x}{R_4} - i_b$

$P_{del,ccvs} = ri_y i_{ccvs}$

# "MOSTLY SMART" LABELS

← doesn't try to use $R_3 i_y$...




KCL, $n_2$: $\frac{v_z - r i_y}{R_1} + \frac{v_z + v_x}{R_2} - g v_x - i_y = 0$

KCL, $sn_{4,5}$: $\frac{-v_x}{R_4} + \frac{-v_x - v_z}{R_2} + g v_x + \cdots$

$i_y - i_b = 0$

MEAS, $i_y$: $i_y = \frac{v_a - v_x - v_z}{R_3}$

$P_{abs,R_2} = (v_z + v_x)^2 / R^2$

$P_{del,vccs} = g v_x (v_z + v_x)$

$i_{ccvs} = \frac{r i_y - v_z}{R_1}$ or $\frac{-v_x}{R_4} - i_b$

$P_{del,ccvs} = r i_y \, i_{ccvs}$

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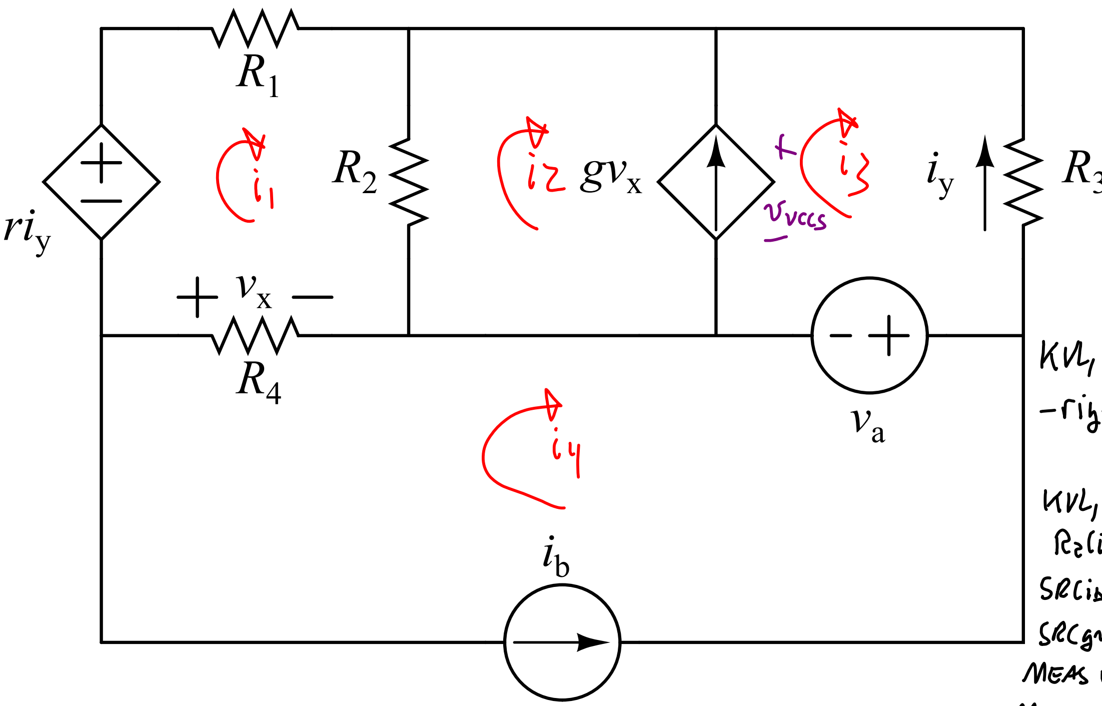
## Problem III: [15 pts.] Branch/Mesh Current Method

4mesh − 2 i.s = 2 KVL

Given the following circuit:




KVL, $l_1$:
$-ri_y + R_1 i_1 + R_2(i_1 - i_2) + R_3(i_1 - i_4) = 0$
or $-v_x$

KVL, $sl_{23}$:
$R_2(i_2 - i_1) + R_3 i_3 + v_a = 0$
or $-R_3 i_y$

SRC $i_b$: $i_b = -i_4$

SRC $gv_x$: $gv_x = i_3 - i_2$

MEAS $i_y$: $i_y = -i_3$

MEAS $v_x$: $v_x = R_4(i_4 - i_1)$

and assuming that constants $g$ and $r$, the values for the passive elements ($R_1$ through $R_4$), and the values for the independent sources ($v_a$ and $i_b$) are known,

**(1)** *Clearly* demonstrate the use of either the Branch Current Method or Mesh Current Method in labeling unknowns for the circuit and in determining a complete set of linearly independent equations that could be used to solve for these unknowns. Indicate which method (BCM or MCM) you are using. List the set of unknowns you believe your equations will find. Please put the list of unknowns and the equations on a separate piece of paper and in a box; you can label the circuit above.

**(2)** Assuming you are able to solve for those unknowns, write expressions for the following. Put your expressions next to the appropriate bullet below even if your work is elsewhere

- $p_{\text{abs},R_2} =$ $(i_1 - i_2)^2 R_2$

- $p_{\text{del,VCCS}} =$ $v_{VCCS} = R_2(i_1 - i_2)$ (KVL, $l_2$) or $v_a - R_3 i_y$ (KVL, $l_3$) or $v_a + R_3 i_3$

  $p_{del} = gv_x \, v_{VCCS}$

- $p_{\text{del,CCVS}} =$ $ri_y i_1$

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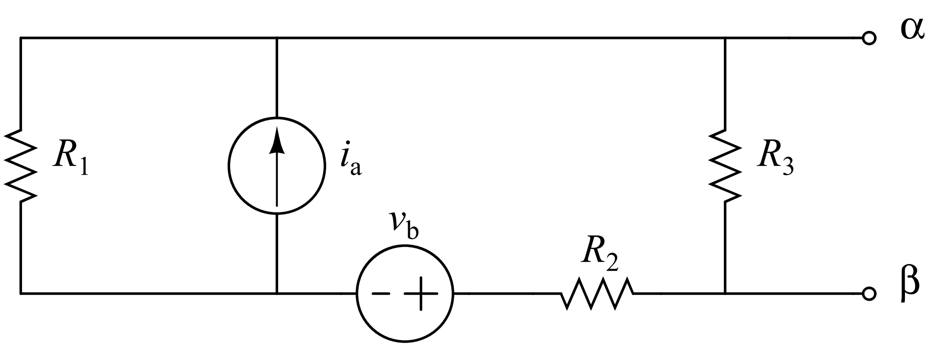
## Problem IV: [24 pts.] Thévenin/Norton

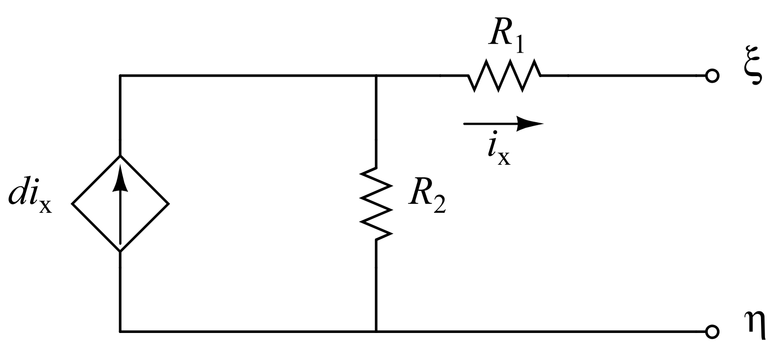
*Note:* for the parts below you must *fully solve* expressions for any variables that are unknown; you cannot simply leave unsolved systems of equations. You do *not*, however, need to simplify any compound fractions, nor do you need to expand any use of the parallel resistance symbol discussed on the cover page. Furthermore, once a variable is fully solved in terms of known values, that variable can also be considered "known" - you do not need to back-substitute.

**(1)** Given the following circuit:




and assuming that the values for the passive elements ($R_1$ through $R_3$) and the values for the independent sources ($i_a$ and $v_b$) are known, draw the Thévenin equivalent circuit with respect to terminals α and β in terms of the known values. Be sure to show your process clearly and indicate where α and β are in your equivalent circuit drawings.

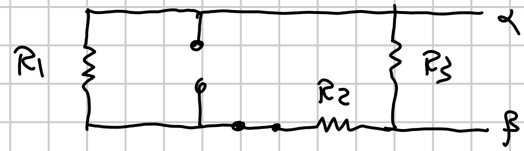
**(2)** Given the following circuit:




and assuming that the values for the passive elements ($R_1$ and $R_2$) and the value for the constant $d$ are known, draw the Norton equivalent circuit with respect to terminals ξ and η in terms of the known values. Be sure to show your process clearly and indicate where ξ and η are in your equivalent circuit drawings.

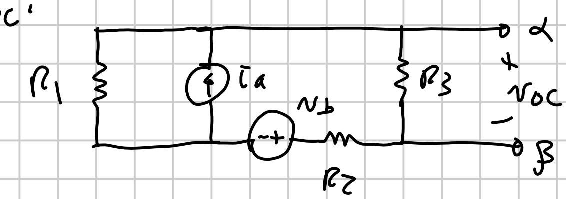
(1) $<0$ indep, $0$ dep $\Rightarrow$ find $R_{th}$ an $v_{oc}$

$R_{th}$: make sources $0$:




$$R_{TH} = R_3 \| (R_1 + R_2)$$

$v_{oc}$:

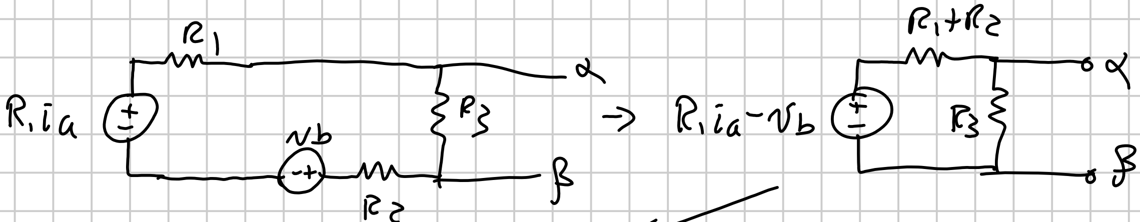



Superposition:

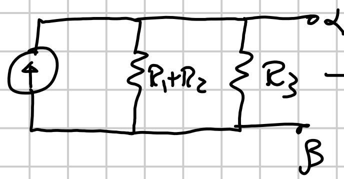
$$v_{oc} = \frac{i_a R_1}{R_1+R_2+R_3} R_3 - \frac{v_b R_3}{R_1+R_2+R_3}$$

or $\dfrac{i_a\,(R_1 \| (R_2+R_3))}{(R_2+R_3)} R_3$

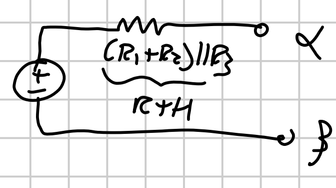
OR use Norton → Thévenin on current. ☆ see note next page




$$\frac{(R_1 i_a - v_b)}{R_1+R_2}$$




$$\rightarrow \underbrace{\frac{R_3(R_1 i_a - v_b)}{R_1+R_2+R_3}}_{v_{oc}}$$

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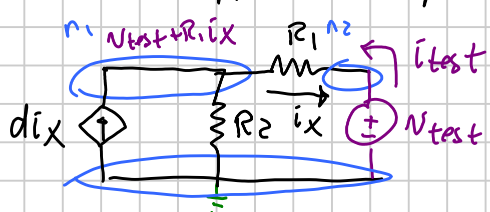
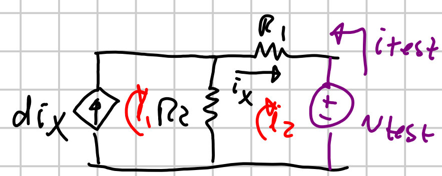
(2) $0$ indep, $<0$ dep — $v_{oc} = i_{sc} = 0$; use $R_T = v_{test}/i_{test}$




or




KCL, $n_1$: $-di_x + \dfrac{(v_{test} + R_1 i_x)}{R_2} + i_x = 0$

$$i_x = \frac{\frac{v_{test}}{R_2}}{d - \frac{R_1}{R_2} - 1} = \frac{v_{test}}{dR_2 - R_1 - R_2}$$

KVL, $\ell_2$: $R_2(i_2 - i_1) + R_1 i_2 + v_{test} = 0$

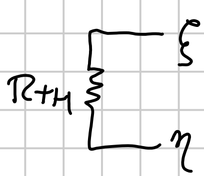
$i_1 = di_x \quad i_2 = i_x$

$$R_2(i_x - di_x) + R_1 i_x + v_{test} = 0$$

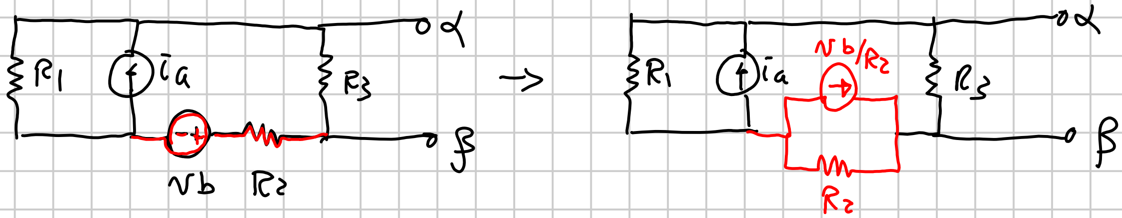
$$i_x = \frac{v_{test}}{dR_2 - R_2 - R_1}$$

$i_{test} = -i_x$, $\dfrac{v_{test}}{i_{test}} = -dR_2 + R_1 + R_2 = R_{TH}$

☆☆ see note next page

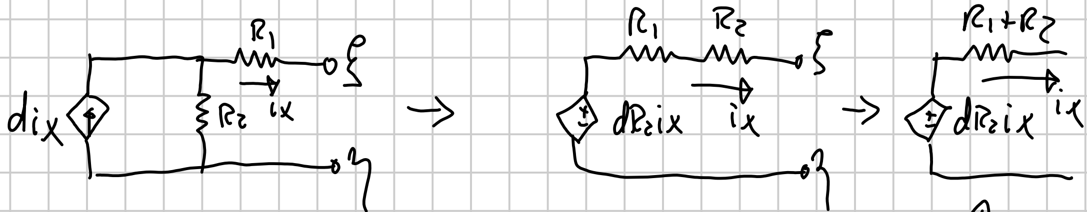


(1) ☆ Note – Trying to go Th->N on $v_b \circ R_2$ leads to



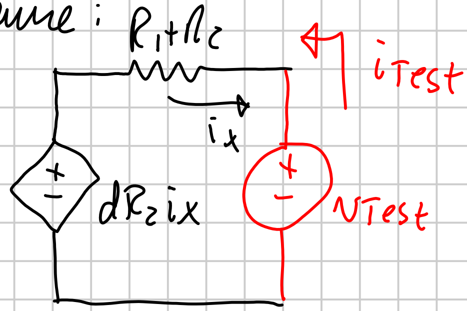

which doesn't really help...

(2) ☆☆ Could use N->Th here since $i_x$ measurement not lost;




Cannot leave this way; equivalent is always an indep. src (possibly 0) and a resistance (possibly negative)

Then use test source:




KVL: $-dR_2 i_x + (R_1+R_2) i_x + v_{Test} = 0$

$$i_x = \frac{v_{Test}}{dR_2 - R_1 - R_2}$$

$$R_{Th} = \frac{v_{Test}}{i_{Test}} = -\frac{v_{Test}}{i_x} = \underline{R_1 + R_2 - dR_2}$$

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## Problem V: [24 pts.] Transients and DC Steady State

**(1)** A first order circuit with a constant source is found to have the following modeling equation:

$$8\frac{dv(t)}{dt} + 2v(t) = 25$$

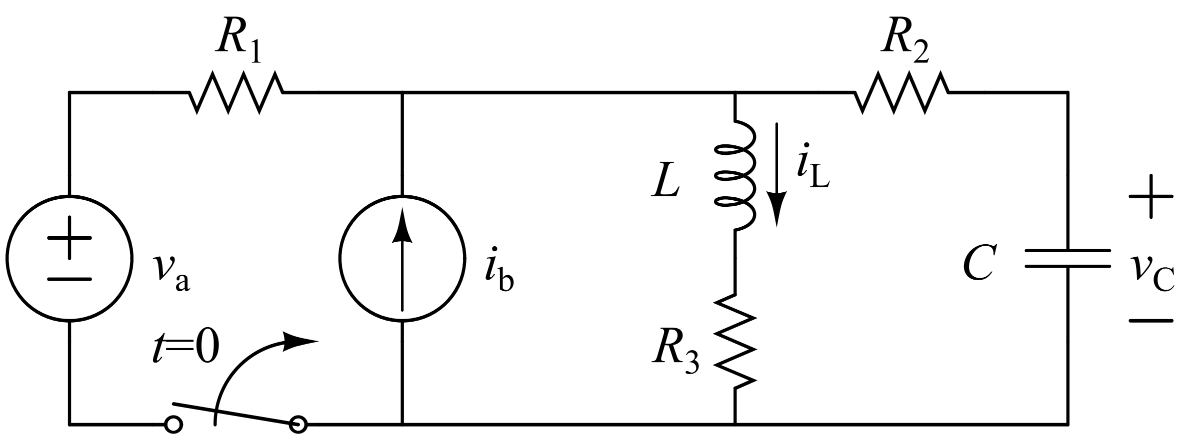
See next page

A measurement at time $t$=3 sec reveals that $v(3) = -10$ V. Given that information,

*(a)* Find an expression that could be used to calculate $v(t)$ for $t \geq 3$ and then

*(b)* Make a very accurate sketch of $v(t)$ starting at $t$=3 sec and continuing for at least three time constants. Be sure to clearly indicate how you made a very accurate sketch. Include axis labels.

**(2)** *A completely different circuit* is given below. The sources $v_a$ and $i_b$ are constants. The switch is initially closed for a really long time before it is opened at time $t$=0 sec:




Given that, find expressions for all the following values and put them in the boxes below. *Note* - once you have solved for an item in terms of the source and element values, you can use that value in subsequent boxes without having to substitute. $E$ refers to the energy stored in an element.

| | | | |
|---|---|---|---|
| $v_C(0^-) =$ $R_3 i_L(0^-)$ or $\frac{R_3 v_a}{R_1+R_3} + \frac{R_1 R_3 i_b}{R_1+R_3}$ | $v_C(0^+)=$ $v_c(0^-)$ | $v_C(\infty)=$ $R_3 i_b$ | $E_C(0^-)=$ $\frac{1}{2}(v(0^-))^2$ |
| $i_C(0^-) =$ $0$ | $i_C(0^+)=$ $i_b - i_L(0^-)$ | $i_C(\infty)=$ $0$ | $E_C(\infty)=$ $\frac{1}{2}(v(\infty))^2$ |
| $v_L(0^-) =$ $0$ | $v_L(0^+)=$ $R_2(i_b - i_L(0^-))$ or $R_2(i_c(0^+))$ | $v_L(\infty)=$ $0$ | $E_L(0^-)=$ $\frac{1}{2}L\, i(0^-)^2$ |
| $i_L(0^-) =$ $v_c(0^-)/R_3$ or $\frac{v_a}{R_1+R_3} + \frac{R_1 i_b}{R_1+R_3}$ | $i_L(0^+)=$ $i_L(0^-)$ | $i_L(\infty)=$ $i_b$ | $E_L(\infty)=$ $\frac{1}{2}L\, i(\infty)^2$ |

either $v_c(0^-)$ or $i_L(0^-)$ must be defined

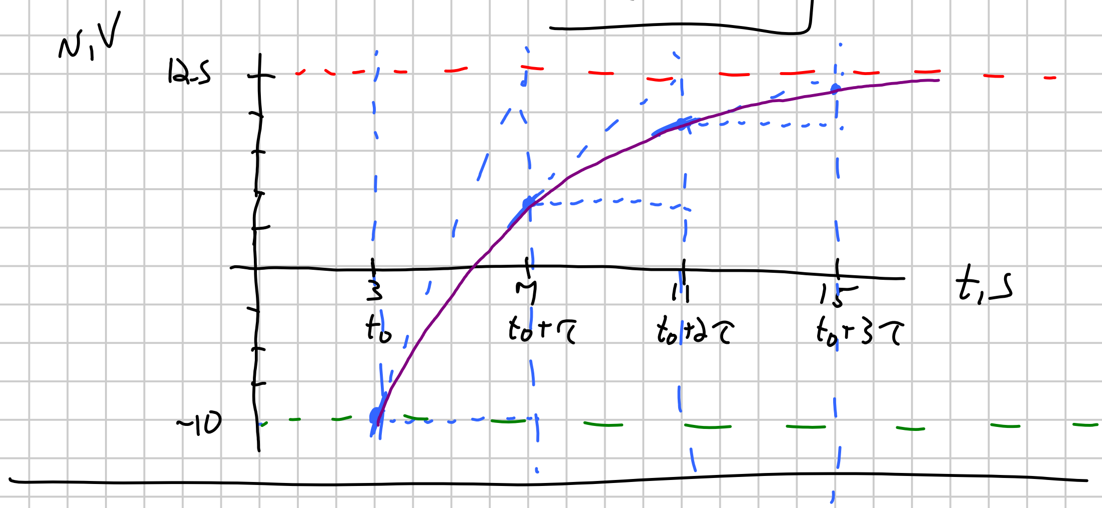
(1) $8\dot{v} + 2v = 25$ get $v$ alone:

$$4\dot{v} + v = 12.5$$

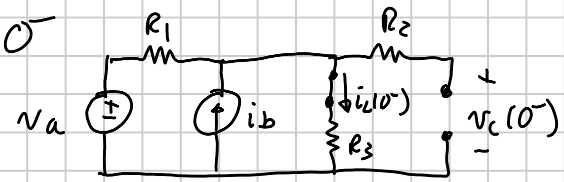
$\tau = 4$ $\quad v(\infty) = 12.5$ $\quad v(3) = -10$ $\quad t_0 = 3$

$$v = v(\infty) + (v(t_0) - v(\infty))\, e^{-(t-t_0)/\tau}$$

$$= \boxed{12.5 - 22.5\, e^{-(t-3)/4}}$$




$t = 0^-$

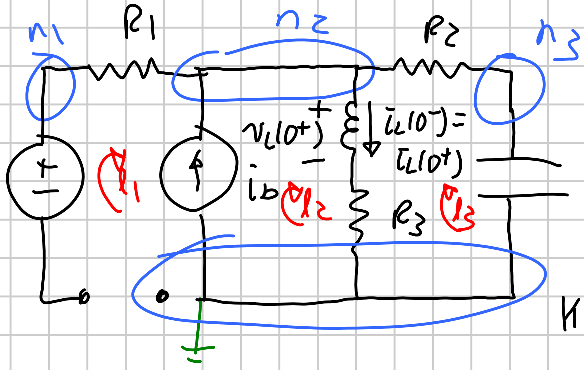



$i_c(0^-) = 0$ $\quad v_L(0^-) = 0$ $\quad$ DCSS

$i_L(0^-) = \dfrac{v_a}{R + R_3} + \dfrac{i_b R_1}{R_1 + R_3}$ superposition

$v_c(0^-) = i_L(0) R_3 = \dfrac{v_a R_3}{R_1 + R_3} + \dfrac{i_b R_1 R_3}{R_1 + R_3}$ Ohm's

$t = 0^+$




$+ \downarrow i_c(0^+)$

$v_c(0^-) = v_c(0^+)$

$-$

KCL, $sn_{23}$: $-i_b + i_L(0^-) + i_c(0^+) = 0$

$i_c(0^+) = i_b - i_L(0^-)$

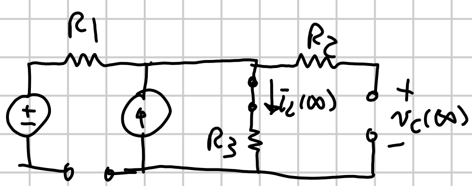
KVL, $l_3$: $-R_3 i_L(0^-) - v_L(0^+) + R_2 i_c(0^+) + v_c(0^-) = 0$

$v_L(0^+) = -R_3 i_L(0^-) + R_2 i_c(0^+) + v_c(0^-)$ but $v_c(0^-) = R_3 i_L(0^-)$

so

$\boxed{R_2 i_c(0^+)}$

$t \to \infty$




$i_c(\infty) = 0$ $\quad v_L(\infty) = 0$ $\quad$ DCSS

$i_L(\infty) = i_b$ $\quad v_c(\infty) = i_b R_3$