

EGR 119 SPRING 2012 Test II

Note Title

I

$$H(s) = \frac{10s(s+1000)}{(s+10)(s+10000)}$$



- 2) High pass; signals lower than $\approx 10^4$ rad/s are attenuated by more than half power.
- 3) $\approx 10^4$ that corner is a 3 dB drop for the high pass; other terms may make 3 dB point a little different but not too much

$$4) \quad \frac{Y}{X} = \frac{10s^2 + 10000s}{s^2 + 10010s + 100000}$$

$$(s^2 + 10010s + 100000)Y = (10s^2 + 10000s)X$$

$$\frac{d^2y}{dt^2} + 10010 \frac{dy}{dt} + 100000y = 10 \frac{d^2x}{dt^2} + 10000 \frac{dx}{dt}$$

$$5) \quad \mathcal{L}^{-1} \left\{ \frac{1}{s} \frac{10s(s+1000)}{(s+10)(s+10000)} \right\} = \mathcal{L}^{-1} \left\{ \frac{10(s+1000)}{(s+10)(s+10000)} \right\}$$

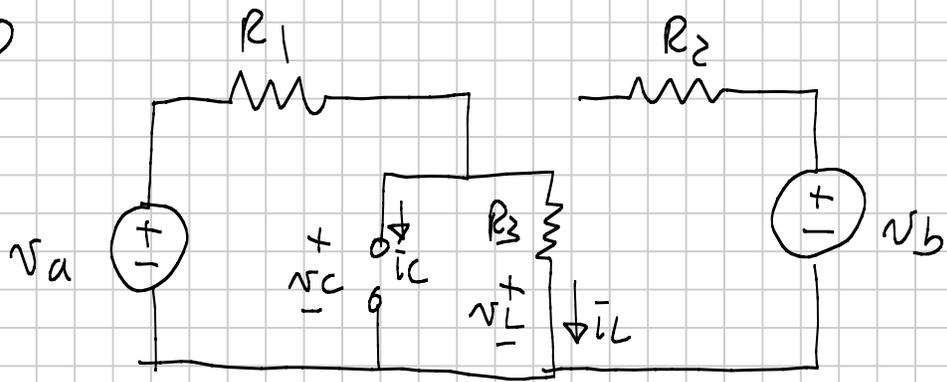
$$= \mathcal{L}^{-1} \left\{ \frac{A}{s+10} + \frac{B}{s+10000} \right\}$$

$$A = \frac{(10)(990)}{(9990)} \quad B = \frac{(10)(-9000)}{-9990}$$

$$A = 0.991 \quad B = 9.009$$

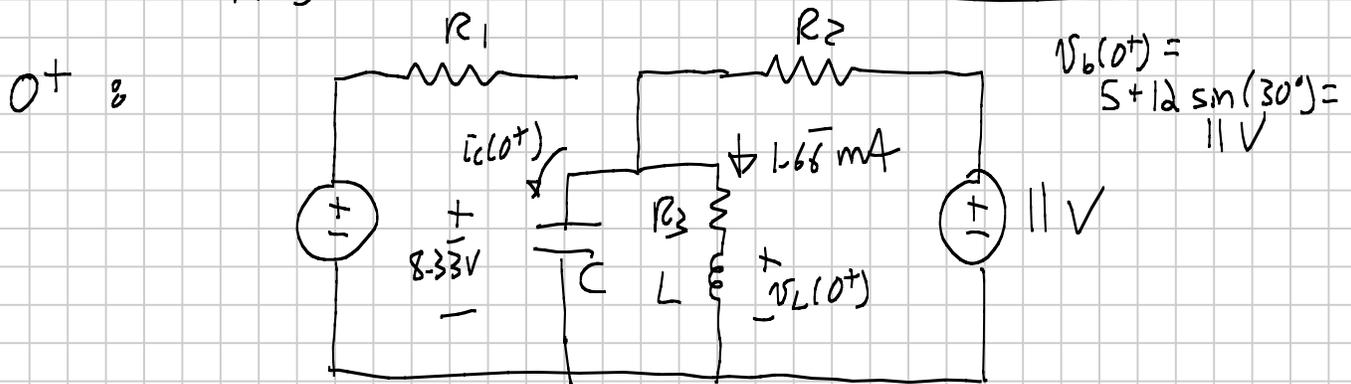
$$s_f(t) = (0.991 e^{-10t} + 9.009 e^{-10000t}) u(t)$$

II) $t < 0$



$$i_c(0^-) = 0 \quad v_c(0^-) = \frac{R_3}{R_1 + R_3} v_a = \frac{5}{6} \cdot 10 = 8.33 \text{ V}$$

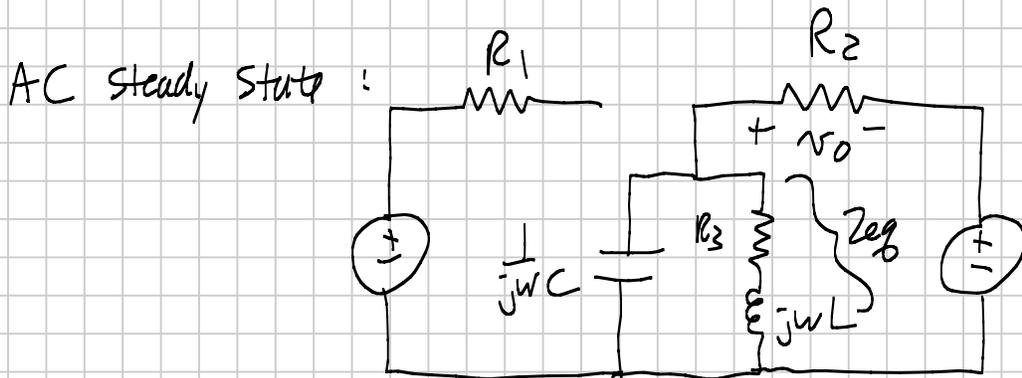
$$i_L(0^-) = \frac{v_a}{R_1 + R_3} = \frac{10}{6 \cdot 10^3} = 1.66 \text{ mA} \quad v_L(0^-) = 0 \text{ V}$$



$$v_c(0^+) = v_c(0^-) = 8.33 \text{ V} \quad i_L(0^+) = i_L(0^-) = 1.66 \text{ mA}$$

$$v_L(0^+) = v_c(0^+) - R_3 i_L(0^+) = 0 \text{ V}$$

$$i_c(0^+) = \frac{v_c(0^+) - 11}{R_2} \quad i_L(0^+) = -0.333 \text{ mA}$$



$$Z_{eq} = \frac{\frac{1}{j\omega C} (j\omega L + R_3)}{\frac{1}{j\omega C} + j\omega L + R_3} = \frac{j\omega L + R_3}{(j\omega)^2 LC + j\omega CR_3 + 1}$$

$$V_o = - \frac{R_2}{R_2 + Z_{eq}} V_b$$

$$V_b = 5 \quad \omega = 0 \quad Z_{eq} = R_3 \quad V_o = \frac{-R_2}{R_2 + R_3} \cdot 5V = 1.429 \angle 180^\circ$$

$$V_{o,s} = -1.429 V$$

$$V_b = 12 \sin(10t + 30^\circ) \\ = 12 \cos(10t - 60^\circ)$$

$$V_b = 12 \angle -60^\circ$$

$$\omega = 3000 \quad Z_{eq} = \frac{j(10)(8 \cdot 10^{-3}) + 5000}{- (10)^2 (8 \cdot 10^{-3})(100 \cdot 10^{-6}) + j(10)(100 \cdot 10^{-6})(5000) + 1}$$

$$= 980 \angle -78.7^\circ$$

$$V_o = \frac{-2000}{2000 + 980 \angle -78.7^\circ} \times 12 \angle -60^\circ = 10.03 \angle 143.7^\circ$$

$$V_{o,sin} = 10.03 \cos(10t + 143.7^\circ)$$

$$V_o = -1.429 + 10.03 \cos(10t + 143.7^\circ)$$

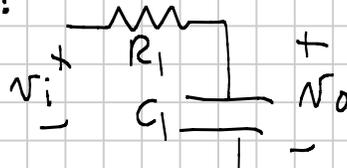
III

(1) $K=25$ $\omega = 1000 - 2\pi = 6283 \text{ rad/s}$

$$H = \frac{K a}{j\omega + a} = \frac{157080}{j\omega + 6283}$$

ω is always in rad/s

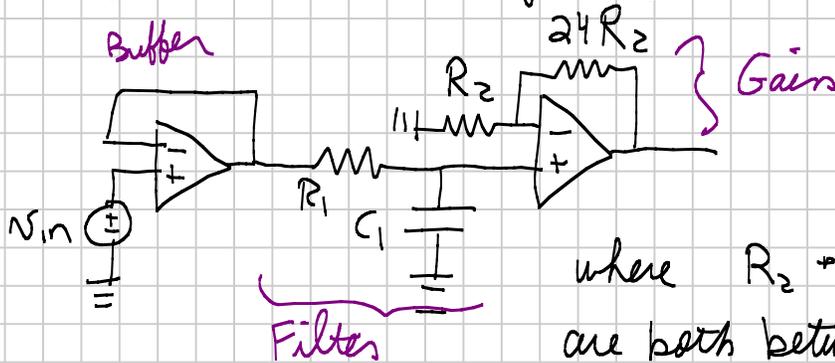
low pass in general:



$$\frac{V_o}{V_i} = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{1}{sR_1C_1 + 1} = \frac{1/CR_1}{s + 1/CR_1}$$

$$a = 6283 = \frac{1}{R_1 C_1} \quad R_1 = \frac{1}{6283 \cdot 100 \cdot 10^{-6}} = 1.5916 \Omega$$

Gain of 1 here, so need gain of 25; use op-amp:



where $R_2 = 24 R_1$
are both between
1k Ω and 100k Ω

Note: The 100 μ F in the problem was meant to be 100 nF... The 1.592 Ω resistor value is not really reasonable...

(2)

Given: $\omega_n = 10000\pi = 31416 \text{ rad/s}$

$Q = 0.25$

$K = 0.1$

can get:

(a) $\zeta = \frac{1}{2Q} = 2.0$

$\omega_{c,lm} = \frac{\omega_n \sqrt{1+4Q^2}}{2Q} = 70248 \text{ rad/s}$

$BW = \frac{\omega_n}{Q} = 125664 \text{ rad/s}$

$\omega_{hi,lo} = \omega_{c,lm} \pm \frac{BW}{2} = 7416, 133080 \text{ rad/s}$

(quick check: $\sqrt{\omega_{hi} \omega_{lo}} = 31416 \checkmark$)

b) $\frac{K 2\zeta \omega_n s}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ or $\frac{K}{|1 + jQ(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega})|}$

$\frac{12566.4 \text{ s}}{s^2 + 125664 \text{ s} + 9.870 \cdot 10^8}$ or $\frac{K}{|1 + j0.25(\frac{\omega}{31416} - \frac{31416}{\omega})|}$

c) Need corners:

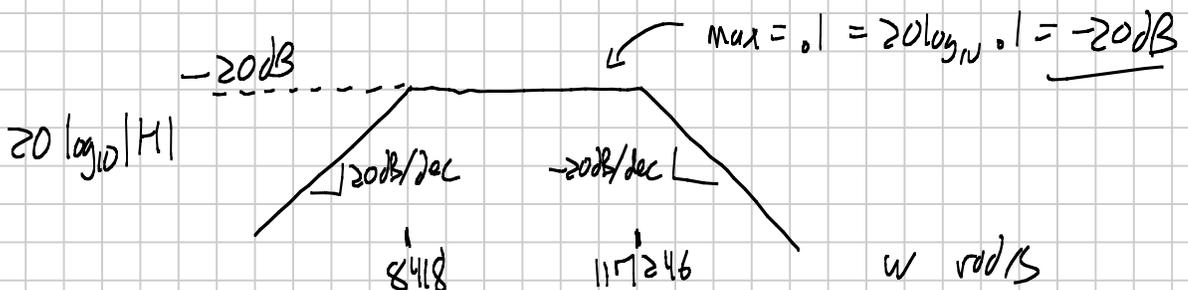
$\omega_1 \omega_2 = \omega_n^2 = 9.870 \cdot 10^8$

$\omega_1 + \omega_2 = BW = 125664$

$\omega_1 (125664 - \omega_1) = 9.870 \cdot 10^8$

$\omega_1^2 - 125664 \omega_1 + 9.870 \cdot 10^8 = 0$

$\omega_1 = 117246 \quad \omega_2 = 8418$



IV

$$Z_n = sL + R_1 + \frac{1}{sC_1} = \frac{s^2 L_1 C_1 + s C_1 R_1 + 1}{s C_1}$$

$$Z_f = R_2 + \frac{1}{sC_2} = \frac{s C_2 R_2 + 1}{s C_2}$$

$$H = -\frac{Z_f}{Z_n} = -\frac{s C_2 R_2 + 1}{s C_2} \cdot \frac{s C_1}{s^2 L_1 C_1 + s C_1 R_1 + 1}$$

$$\text{or } -\frac{C_1 C_2 R_2}{L_1 C_1 C_2} \frac{s + 1/C_2 R_2}{(s^2 + s R_1/L_1 + 1/L_1 C_1)} = -\frac{R_2}{L_1} \frac{s + 1/C_2 R_2}{s^2 + s R_1/L_1 + 1/L_1 C_1}$$

$$\text{numbers } 5 \cdot 10^6 \frac{s + 1.6 \cdot 10^6}{s^2 + 6 \cdot 10^6 s + 1 \cdot 10^{13}}$$

$$(b) \text{ discriminant} = (6 \cdot 10^6)^2 - 4(1 \cdot 10^{13}) = 3.6 \cdot 10^{13} - 4 \cdot 10^{13} = -4 \cdot 10^{12}$$

$$(s + 3 \cdot 10^6)^2 = s^2 + 6 \cdot 10^6 s + 9 \cdot 10^{12} \quad + 1 \cdot 10^{12} \text{ so}$$
$$(s + 3 \cdot 10^6)^2 + (1 \cdot 10^6)^2 \quad \underbrace{\hspace{10em}}_{1 \cdot 10^{13} \text{ to get}}$$

$$H = -\frac{5 \cdot 10^6 (s + 1.6 \cdot 10^6)}{(s + 3 \cdot 10^6)^2 + (1 \cdot 10^6)^2} = \frac{A(s + 3 \cdot 10^6) + B(1 \cdot 10^6)}{(s + 3 \cdot 10^6)^2 + (1 \cdot 10^6)^2}$$

$$A = -5 \cdot 10^6 \quad B = \frac{(-5 \cdot 10^6)(1.6 \cdot 10^6) - (A)(3 \cdot 10^6)}{1 \cdot 10^6} = +7 \cdot 10^6$$

$$h(t) = e^{-3 \cdot 10^6 t} \left(-5 \cdot 10^6 \cos(1 \cdot 10^6 t) + 7 \cdot 10^6 \sin(1 \cdot 10^6 t) \right) u(t) \text{ V}$$