

# EGR 119 Spring 2009 Test 2

Note Title

I)

1) 20 dB/dec means 1<sup>st</sup> order lowpass

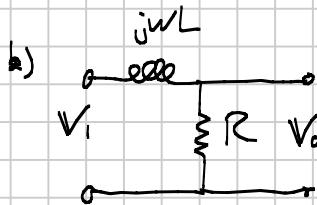
2 options : a) RC or b) L R

a) 

$$\frac{V_o}{V_i} = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} = \frac{1}{1 + jwCR} = \frac{1/RC}{jw + 1/RC}$$

cutoff at  $\frac{1}{RC} = 10000 \times 2\pi$

$$C = \frac{1}{20000\pi \times R} = 15.92 \text{ nF}$$

b) 

$$\frac{V_o}{V_i} = \frac{R}{R + jwL} = \frac{R/L}{jw + R/L}$$

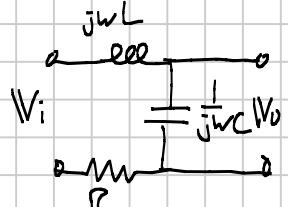
cutoff at  $\frac{R}{L} = 20000\pi \quad L = \frac{R}{20000\pi} = 15.92 \text{ mH}$

in either case :

$$\omega_{c0} = 20000\pi$$



2) 40 dB/dec means 2<sup>nd</sup> order; double corner.



$$\frac{V_o}{V_i} = \frac{\frac{1}{jwC}}{jwL + R + \frac{1}{jwC}} = \frac{1}{(jw)^2 LC + (jw)RC + 1}$$

$$= \frac{1/LC}{(jw)^2 + (jw)R/L + 1/LC}$$

double corner means  $(jw + 20000\pi)^2$  in denominator

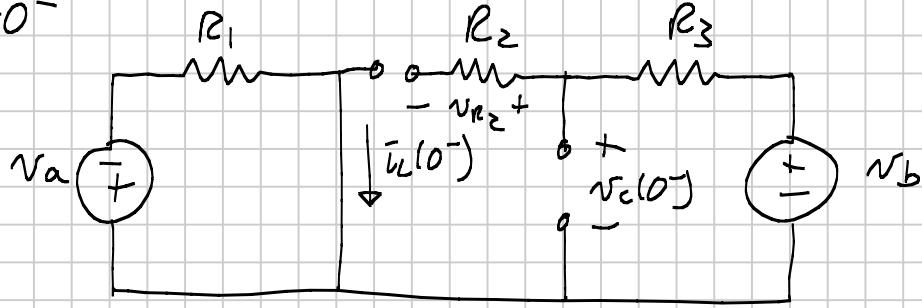
$$(jw)^2 + (40000\pi)(jw) + (20000\pi)^2$$

$$\frac{R}{L} = 40000\pi, \quad L = \frac{R}{40000\pi} = 7.958 \text{ mH}$$

$$\frac{1}{LC} = (20000\pi)^2, \quad C = \frac{1}{(20000\pi)^2 L} = 31.83 \text{ nF}$$



II)  $t=0^-$

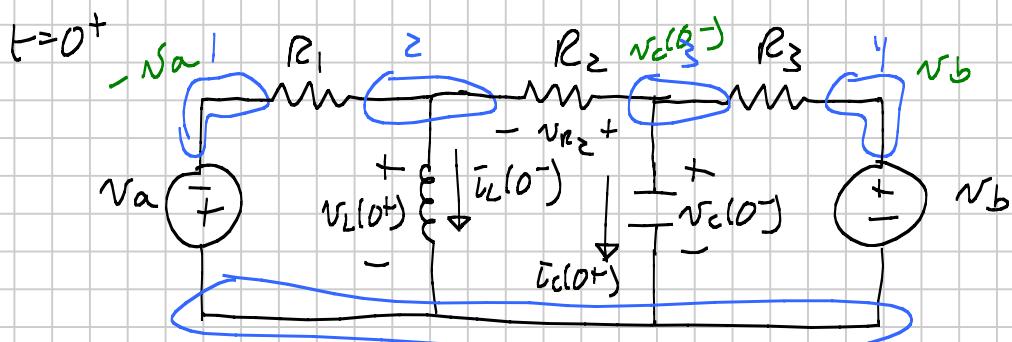


$$i_L(0^-) = \frac{v_a}{R_1}$$

$$v_c(0^-) = v_b$$

$$v_{R_2}(0^-) = 0 \text{ no curr}$$

$v_L(0^-) = 0$  (acts like short)  $i_c(0^-) = 0$  (acts like open)



$i_L(0^+) = i_L(0^-)$  and  $v_c(0^+) = v_c(0^-)$  by continuity

$$KCL, n_2: \frac{v_L(0^+) + v_a}{R_1} + i_L(0^-) + \frac{v_L(0^+) - v_c(0^-)}{R_2} = 0$$

$$\begin{aligned} v_L(0^+) &= -i_L(0^-) \frac{-v_a}{R_1} + \frac{v_c(0^-)}{R_2} = \frac{\frac{v_a}{R_1} - \frac{v_a}{R_1} + \frac{v_b}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \\ &= \underbrace{\frac{R_1}{R_1 + R_2} v_b} \end{aligned}$$

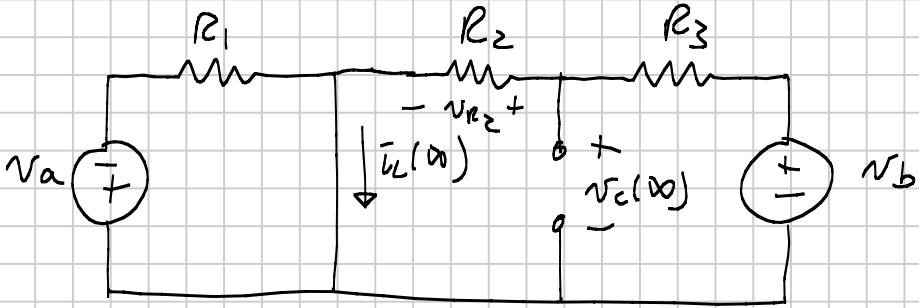
$$v_{R_2}(0^+) = v_c(0^-) - v_L(0^+) = v_b - \frac{R_1}{R_1 + R_2} v_b = \frac{R_2 v_b}{R_1 + R_2}$$

=

$$KCL, n_3: \frac{v_{R_2}}{R_2} + i_c(0^+) + \frac{v_c(0^-) - v_b}{R_3} \stackrel{\text{equal}}{=} 0$$

$$i_c(0^+) = -\frac{v_{R_2}}{R_2} = -\frac{v_b}{R_1 + R_2}$$

$t = 0^-$ :



Superposition:

$$V_L(\infty) = -\frac{V_a}{R_1} + \frac{V_b}{R_2 + R_3} \quad V_c(\infty) = 0 + \frac{R_2}{R_2 + R_3} V_b$$

$$V_L(0^-) = 0$$

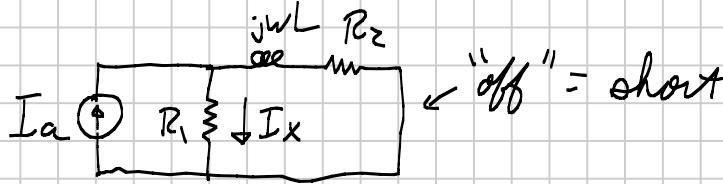
$$V_L(\infty) = 0$$

$$V_{R_2}(\infty) = V_c(\infty) = \frac{R_2}{R_2 + R_3} V_b$$

	$V_L$	$V_L$	$V_C$	$V_C$	$V_{R_2}$
$t = 0^-$	$-\frac{V_a}{R_1}$	0	0	$V_b$	0
$t = 0^+$	$-\frac{V_a}{R_1}$	$\frac{R_1 V_b}{R_1 + R_2}$	$-\frac{V_b}{R_1 + R_2}$	$V_b$	$\frac{R_2 V_b}{R_1 + R_2}$
$t = \infty$	$-\frac{V_a}{R_1} + \frac{V_b}{R_2 + R_3}$	0	0	$\frac{R_2}{R_2 + R_3} V_b$	$\frac{R_2}{R_2 + R_3} V_b$

III)

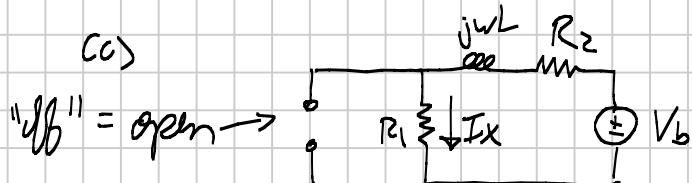
(a)



$$\frac{I_x}{I_a} = \frac{jwL + R_2}{jwL + R_2 + R_1}$$

(b) high pass:  $G_a(j\omega) = \frac{R_2}{R_1 + R_2}$   $G_a(j\infty) = 1$

$$\frac{R_2}{R_1 + R_2}$$



$$\frac{I_x}{V_b} = \frac{1}{jwL + R_1 + R_2}$$

(c) low pass:  $Y_b(j0) = \frac{1}{R_1 + R_2}$   $Y_b(j\infty) = 0$

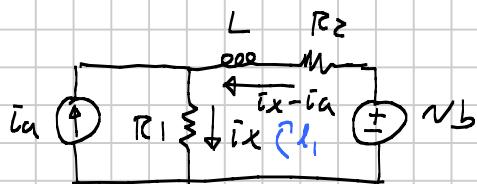
$$\frac{1}{R_1 + R_2}$$

(e)  $I_x = G_a I_a + Y_b V_b$

$$I_x = \frac{(jwL + R_2) I_a}{jwL + R_1 + R_2} + \frac{(1) V_b}{jwL + R_1 + R_2}$$

$$(jwL + R_1 + R_2) I_x = (jwL + R_2) I_a + V_b$$

$$L \frac{dI_x}{dt} + (R_1 + R_2) I_x = L \frac{dI_a}{dt} + R_2 I_a + V_b$$

on

$$KVL \lambda_1: -R_1 I_x - L \frac{d}{dt}(I_x - I_a) - R_2(I_x - I_a) + V_b = 0$$

$$L \frac{dI_x}{dt} + (R_1 + R_2) I_x = L \frac{dI_a}{dt} + R_2 I_a + V_b$$

(F)

from  $\bar{V}_a$

$$S: \quad I_x(j0) = G_a(j0) \cdot S = \frac{R_2}{R_1+R_2} \cdot S = \frac{3}{12} S = 1.25 \angle 0^\circ \text{ mA}$$

$$i_x = 1.25 \text{ mA}$$

$$2 \cos(10^4 t + 25^\circ) \quad I_x = G_a(j10^4) \cdot 2 \angle 25^\circ$$

$$I_a = 2 \angle 25^\circ \text{ mA} \quad = \frac{3000 + j40000}{12000 + j40000} \quad 2 \angle 25^\circ = 1.921 \angle 37.41^\circ \text{ mA}$$

$$i_{x2} = 1.921 \cos(10^4 t + 37.41^\circ) \text{ mA}$$

from  $V_b$

$$3 \sin(5000t - 15^\circ) = 3 \cos(5000t - 105^\circ)$$

$$\omega = 5000$$

$$V_b = 3 \angle -105^\circ \quad I_x = V_b(j5000) \cdot 3 \angle -105^\circ$$

$$= \frac{1}{12000 + j20000} 3 \angle -105^\circ = 0.129 \angle -164^\circ \text{ mA}$$

$$i_{x3} = 0.129 \cos(5000t - 164^\circ) \text{ mA}$$

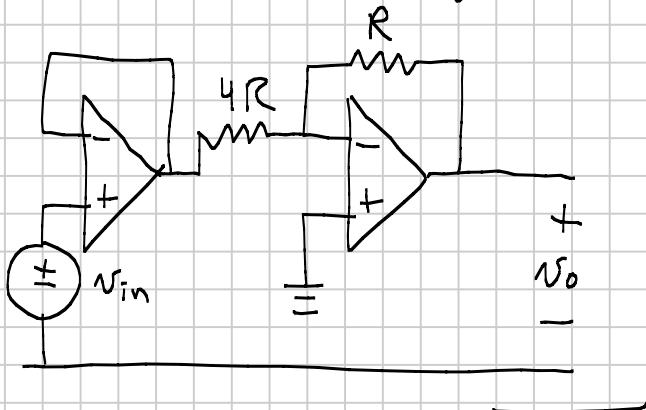
$$i_x = i_{x1} + i_{x2} + i_{x3}$$

$$= 1.25 + 1.921 \cos(10^4 t + 37.41^\circ) + 0.129 \cos(5000t - 164^\circ) \text{ mA}$$

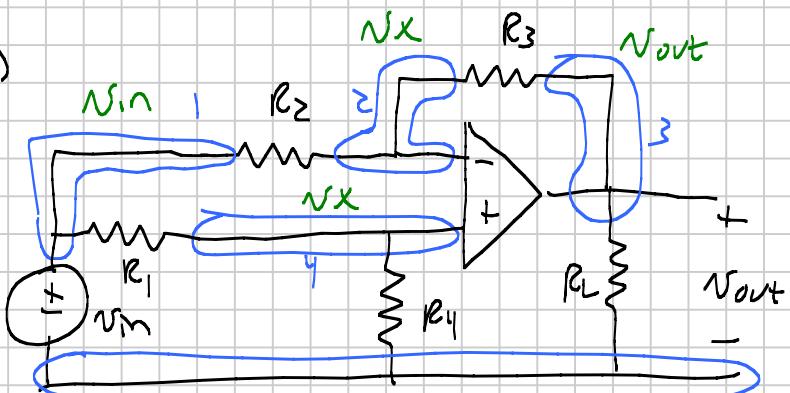
IV

(1) Gain =  $-V_4$  (inverting) no power loss (buffer)

Resistors in  $k\Omega$  range



2)



1, 3 not useful for KCL;

$$KCL_4: \frac{V_x - V_m}{R_1} + \frac{V_x}{R_4} = 0 \quad V_x = \frac{R_4}{R_1 + R_4} V_m$$

$$KCL_2: \frac{V_x - V_m}{R_2} + \frac{V_x - V_{out}}{R_3} = 0$$

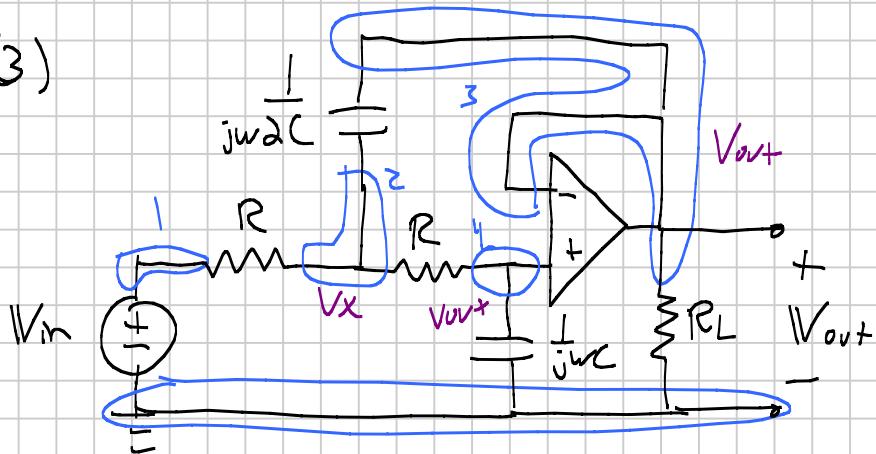
$$(R_2 + R_3)V_x - R_3V_m - R_2V_{out} = 0$$

$$R_2V_{out} = (R_2 + R_3)\left(\frac{R_4}{R_1 + R_4}\right)V_m - R_3V_m$$

$$R_2V_{out} = \frac{(R_2R_4 + R_3R_4 - R_1R_3 - R_3R_4)V_m}{R_1 + R_4}$$

$$V_{out} = \frac{(R_2R_4 - R_1R_3)V_m}{R_2(R_1 + R_4)}$$

(3)



l-3 not useful.

$$\text{KCL}_{\text{in}2}: \frac{V_x - V_{\text{in}}}{R} + \frac{V_x - V_o}{1/j\omega C} + \frac{V_x - V_o}{R} = 0$$

$$V_x - V_{\text{in}} + j\omega \alpha CR V_x - j\omega \alpha CR V_o + V_x - V_o = 0$$

$$V_x = \frac{V_{\text{in}} + j\omega \alpha CR V_o + V_o}{2 + j\omega \alpha CR}$$

$$\text{KCL}_{\text{in}4}: \frac{V_o - V_x}{R} + \frac{V_o}{1/j\omega C} = 0$$

$$V_o - V_x + j\omega CR V_o = 0$$

$$V_o - \frac{V_{\text{in}} + j\omega \alpha CR V_o + V_o}{2 + j\omega \alpha CR} + j\omega CR V_o = 0$$

$$(2 + j\omega \alpha CR) V_o - j\omega \alpha CR V_o - V_o + (2 + j\omega \alpha CR)(j\omega CR) V_o = V_{\text{in}}$$

$$(2 + j\omega \alpha CR - j\omega \alpha CR - 1 + j\omega \alpha CR + (\omega \alpha)^2 2C^2 R^2) V_o = V_{\text{in}}$$

$$V_o = \frac{V_{\text{in}}}{(j\omega \alpha)^2 2C^2 R^2 + (j\omega \alpha) 2CR + 1}$$