

**Duke University**  
**Edmund T. Pratt, Jr. School of Engineering**

**EGR 119 Spring 2008**  
**Test II**  
Michael R. Gustafson II

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Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Judicial Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

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### **Instructions for Problem Sections**

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently.

Make sure that your name and NET ID are *clearly* written at the top of every page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder. Put this signature page as the "cover page" for the first problem.

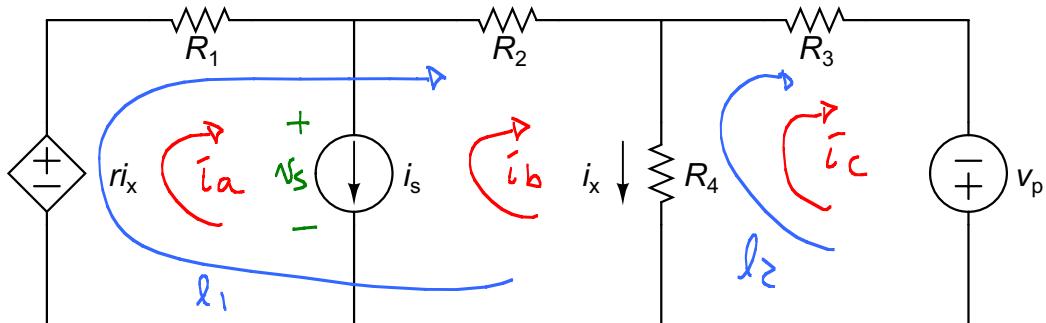
Note that there may be people taking the test after you, so you are not allowed to talk about the test until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):

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### Problem I: [20 pts.] Controlled Sources

Given the following circuit:



and assuming that the values for the passive elements, sources  $i_s$  and  $v_p$ , and constant  $r$  are known,

- Clearly demonstrate the use of the Mesh Current Method in labeling unknowns for the circuit and in determining coupled linear algebra equations that could be used to solve for any currents in the system.
- Using the unknowns defined above, give expressions for the power absorbed by current controlled voltage source and the power absorbed by the independent current source.

$$\text{NOTE} \rightarrow \bar{i}_x = \bar{i}_b - \bar{i}_c$$

$$\text{AUX : } \bar{i}_a - \bar{i}_b = \bar{i}_s$$

$$L1 : -r(\bar{i}_b - \bar{i}_c) + R_1(\bar{i}_a) + R_2(\bar{i}_b) + R_4(\bar{i}_b - \bar{i}_c) = 0$$

$$L2 : R_4(\bar{i}_c - \bar{i}_b) + R_3(\bar{i}_c) - v_p = 0$$

$$P_{\text{abs,ccvs}} = -r i_x \bar{i}_a = -r(\bar{i}_b - \bar{i}_c) \bar{i}_a$$

$P_{\text{abs}, i_s}$  → need voltage across  $\bar{i}_s$  called  $\bar{v}_s$  above

$$P_{\text{abs}, i_s} = +\bar{v}_s \bar{i}_s$$

$$\bar{v}_s \text{ using KVL is either } \bar{v}_s = r \bar{i}_x - R_1 \bar{i}_a$$

$$= r(\bar{i}_b - \bar{i}_c) - R_1 \bar{i}_a$$

$$\text{or } \bar{v}_s = R_2 \bar{i}_b + R_4(\bar{i}_b - \bar{i}_c)$$

$$\text{or } \bar{v}_s = R_2 \bar{i}_b + R_3 \bar{i}_c - v_p$$

$$\text{or, } (r(\bar{i}_b - \bar{i}_c) - R_1 \bar{i}_a) \bar{i}_s \text{ or}$$

$$(R_2 \bar{i}_b + R_4(\bar{i}_b - \bar{i}_c)) \bar{i}_s \text{ or}$$

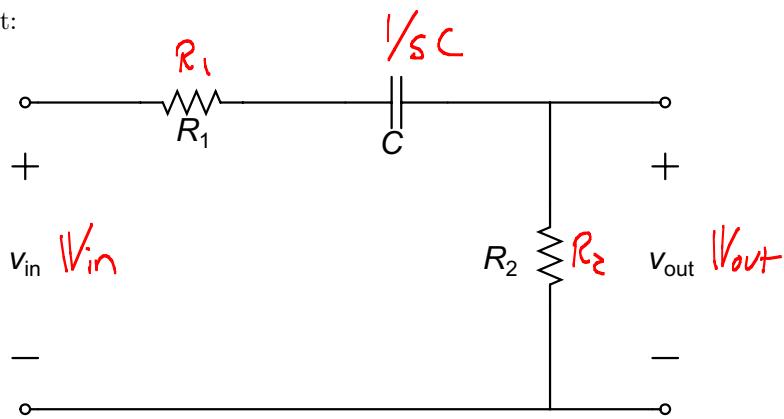
$$(R_2 \bar{i}_b + R_3 \bar{i}_c - v_p) \bar{i}_s$$

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### Problem II: [20 pts.] Passive Filter

Given the following circuit:



and assuming that the values for the passive elements are known, first redraw the circuit in the frequency domain, specifically using Laplace transforms. Then,

- (a) Determine an expression for the transfer function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

- (b) Re-cast the transfer function as a Fourier transform (i.e.,  $H(j\omega)$ ). Then determine the value for the maximum magnitude of the transfer function,  $H_{max}$ , and the frequency at which it occurs.  
 (c) What kind of filter is this?  
 (d) What is/are the half-power frequency/frequencies in terms of the passive element values?  
 (e) What is the step response  $s_r(t)$  of this filter (as a function of time)?

a)  $\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + \frac{1}{sC} + R_2} = \frac{sCR_2}{s(CR_1 + R_2) + 1}$

$2w^2C^2R_2^2(R_1 + R_2)^2 = w^2C^2R_2^2(R_1 + R_2)^2 + R_2^2$

$w^2C^2R_2^2(R_1 + R_2)^2 = R_2^2$

$\omega_{co} = \frac{1}{C(R_1 + R_2)}$

b)  $H(j\omega) = \frac{j\omega CR_2}{j\omega C(R_1 + R_2) + 1}$

$|H(j\omega)| = \frac{\omega CR_2}{\sqrt{\omega^2C^2(R_1 + R_2)^2 + 1}}$

$H_{max} = \frac{CR_2}{C(R_1 + R_2)} = \frac{R_2}{R_1 + R_2} \text{ as } \omega \rightarrow 0$

c) HIGHPASS

d)  $|H(\omega_{co})|^2 = \frac{H_{max}^2}{2}$

$\frac{w^2C^2R_2^2}{w^2C^2(R_1 + R_2)^2 + 1} = \frac{R_2^2}{(R_1 + R_2)^2 \cdot 2}$

e)  $H(s) = \frac{sCR_2}{s(CR_1 + R_2) + 1}$

$S(s) = \frac{H(s)}{s} = \frac{CR_2}{s(CR_1 + R_2) + 1}$

$S(s) = \frac{R_2}{s + \frac{1}{C(R_1 + R_2)}}$

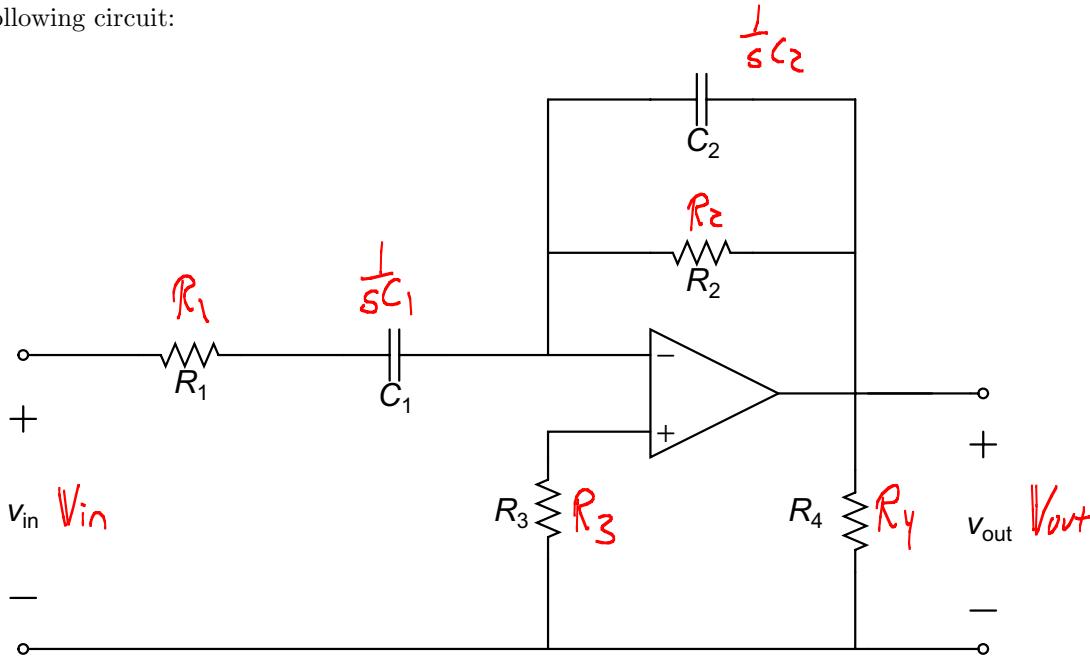
$s_r(t) = \frac{R_2}{R_1 + R_2} \exp\left(-\frac{t}{C(R_1 + R_2)}\right)$

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### Problem III: [20 pts.] Active Filter

Given the following circuit:



and assuming that the values for the passive elements are known, first redraw the circuit in the frequency domain, specifically using Laplace transforms. Then,

- (a) Determine an expression for the transfer function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

- (b) Re-cast the transfer function as a Fourier transform (i.e.,  $H(j\omega)$ ). Then determine the value for the maximum magnitude of the transfer function,  $H_{max}$ , and the frequency at which it occurs.

- (c) What kind of filter is this?

- (d) What is the natural frequency of the response?

- (e) What is the damping ratio of the response?

- (f) The following depends on your answer regarding the filter type:

- (a) If you believe this is a low-pass filter, determine the cutoff frequency and bandwidth.

- (b) If you believe this is a band-pass filter, determine the center frequency and quality.

- (c) If you believe this is a high-pass filter, determine the cutoff frequency and then wallow in self-pity, now secure in the knowledge that this is *not* a high-pass filter.

(a)

$$V_{out} = -\frac{Z_f}{Z_n} V_{in} \quad Z_f = R_2 \parallel \frac{1}{sC_2} = \frac{R_2}{sC_2} = \frac{R_2}{1+sR_2C_2} \quad Z_n = R_1 + \frac{1}{sC_1} = \frac{1+sR_1C_1}{sC_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{1+sR_2C_2} \times \frac{sC_1}{1+sR_1C_1} = -\frac{sR_2C_1}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2) + 1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{s \left( \frac{1}{R_1C_1} \right)}{s^2 + s \left( \frac{R_1C_1 + R_2C_2}{R_1R_2C_1C_2} \right) + \frac{1}{R_1R_2C_1C_2}}$$

$$(b) H(j\omega) = \frac{j\omega(R_2C_1)}{(j\omega)^2 R_1R_2C_1C_2 + j\omega(R_1C_1 + R_2C_2) + 1}$$

$$\omega_n = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$2\beta\omega_n = \frac{R_1C_1 + R_2C_2}{R_1R_2C_1C_2}$$

$$\zeta = \frac{1}{2} \quad \frac{R_1C_1 + R_2C_2}{\sqrt{R_1R_2C_1C_2}}$$

Looks like BANDPASS;  
get into  $H = \frac{K}{1 + jQ(\frac{\omega}{\omega_n} + \frac{\omega_n}{\omega})}$

$$Q = \frac{1}{2\zeta} = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2}$$

$$H(j\omega) = \frac{R_2C_1}{R_1C_1 + R_2C_2} =$$

$$j\omega \left( \frac{R_1R_2C_1C_2}{R_1C_1 + R_2C_2} \right) + 1 + \frac{1}{j\omega(R_1C_1 + R_2C_2)}$$

$$+ \frac{j\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2} \left( \frac{\omega}{\sqrt{R_1R_2C_1C_2}} - \frac{1/\sqrt{R_1R_2C_1C_2}}{\omega} \right)$$

$$K = \frac{R_2C_1}{R_1C_1 + R_2C_2}$$

$$\omega_n = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

(c) BANDPASS

$$(d) \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$(e) \zeta = \frac{R_1C_1 + R_2C_2}{2\sqrt{R_1R_2C_1C_2}}$$

$$(f) Q = \frac{1}{2\zeta} = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2}$$

$$\omega_c = \frac{\sqrt{1+4Q^2}}{2Q} = \frac{\sqrt{1 + \frac{4R_1R_2C_1C_2}{R_1C_1 + R_2C_2}}}{\frac{2\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2}} \quad \dots$$

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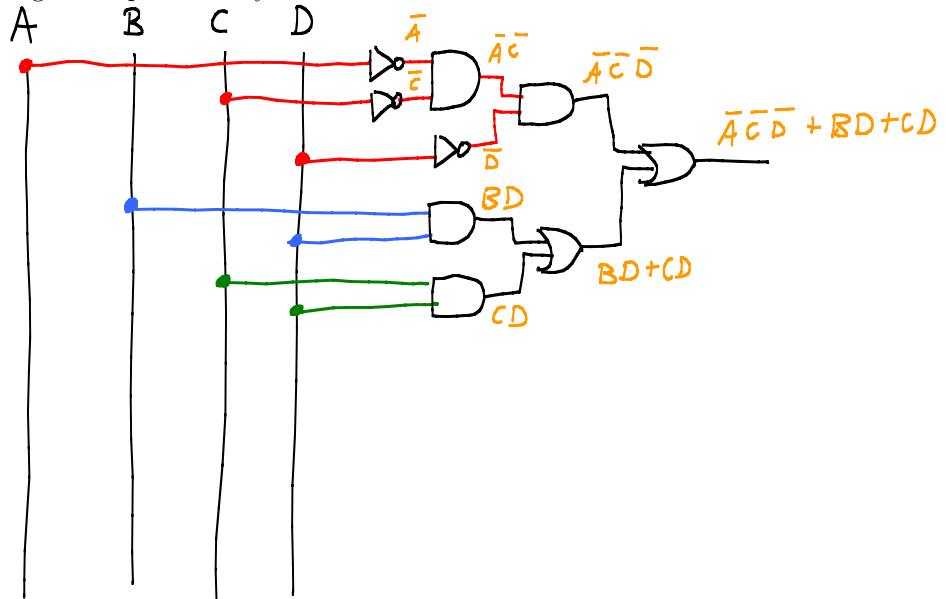
### Problem IV: [20 pts.] Digital Logic

- Given the following logical expression:

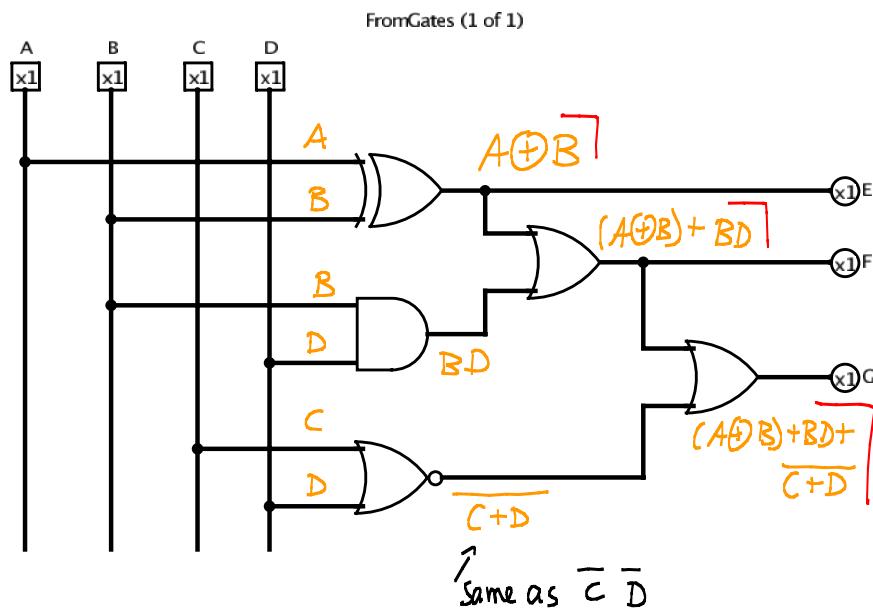
$$E = \overline{A} \overline{C} \overline{D} + \overline{B} D + C \overline{D}$$

complete the truth table and then generate a logic circuit that represents the expression using only one- or two-input gates. Be sure your gate shapes are very clear.

A	B	C	D	E
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1



- Given the logical circuit below, write out logical expressions for each of the three outputs and then complete the truth table. Note - the expressions are most easily written by interpreting the circuit; writing the expressions based on the truth table will give you many more and longer terms than are necessary.



A	B	C	D	E	F	G
0	0	0	0	0	0	1
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	1	1	1
0	1	0	1	1	1	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	0	0	1
1	1	0	1	0	1	1
1	1	1	0	0	0	0
1	1	1	1	0	1	1

Name (please print):

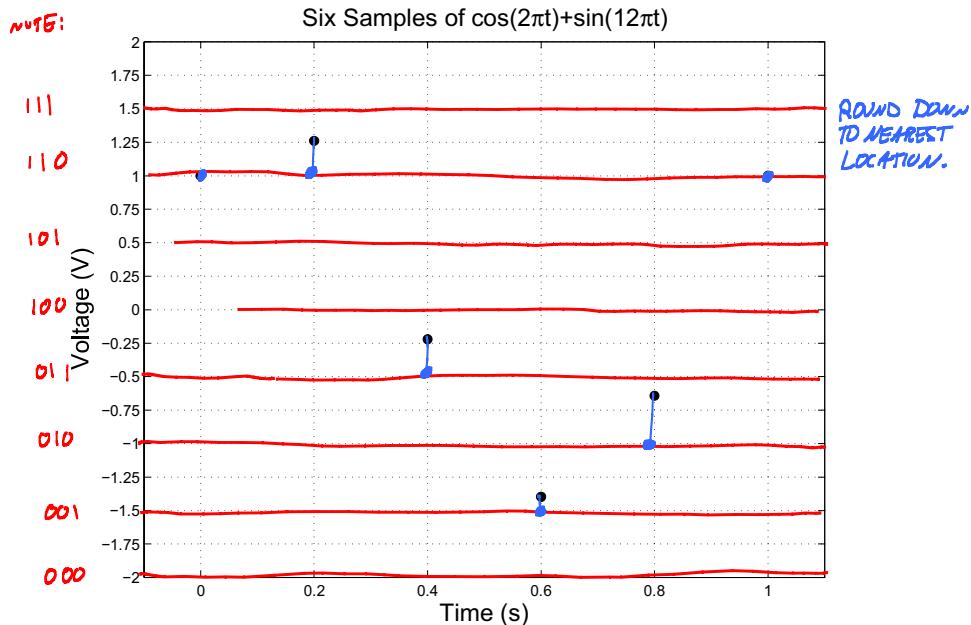
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### Problem V: [20 pts.] A/D D/A Sampling Aliasing Quantizing

Make sure your calculator is set to radians. The signal:

$$x(t) = \cos(2\pi t) + \sin(12\pi t)$$

is sampled every 0.2 sec, starting at 0 sec and lasting for one second, yielding a total of 6 samples as shown:



A 3-bit flash A/D converter with a range from -2 V to 2 V is used to sample the signal.

- For each of the six samples taken, what is the binary representation of the value sampled?
- For each of the six samples taken, what voltage does the binary representation translate to?
- What is the quantization size?
- Is there aliasing? Why or why not?

$t$	$k$	$v(k)$	$\lfloor \frac{v+2}{4} \times 8 \rfloor$	$d_2$	$d_1$	$d_0$	$\left(\frac{B_{IN}}{8} \times 4\right) - 2$
0	1	1.00	6	1	1	0	-1.00
.2	2	1.26	6	1	1	0	-1.00
.4	3	-0.22	3	0	1	1	-0.50
.6	4	-1.40	1	0	0	1	-1.50
.8	5	-0.64	2	0	1	0	-1.00
1.0	1	1.00	6	1	1	0	-1.00

$$(3) Q = \frac{\Delta V}{2^n} = \frac{4}{8} = \frac{1}{2}$$

(4) Aliasing occurs.  $T_s = 0.2$  so  $f_s = 5 \text{ Hz} / w_s = 10\pi \text{ rad/s}$

The  $2\pi \text{ rad/s}$  cosine is fine, the  $\sin(12\pi t)$  looks like  $\sin(2\pi t)$

$$(12\pi - \lfloor \frac{1}{2} + \frac{12\pi}{10\pi} \rfloor 10\pi = 12\pi - \lfloor 1.2 \rfloor 10\pi = 2\pi)$$