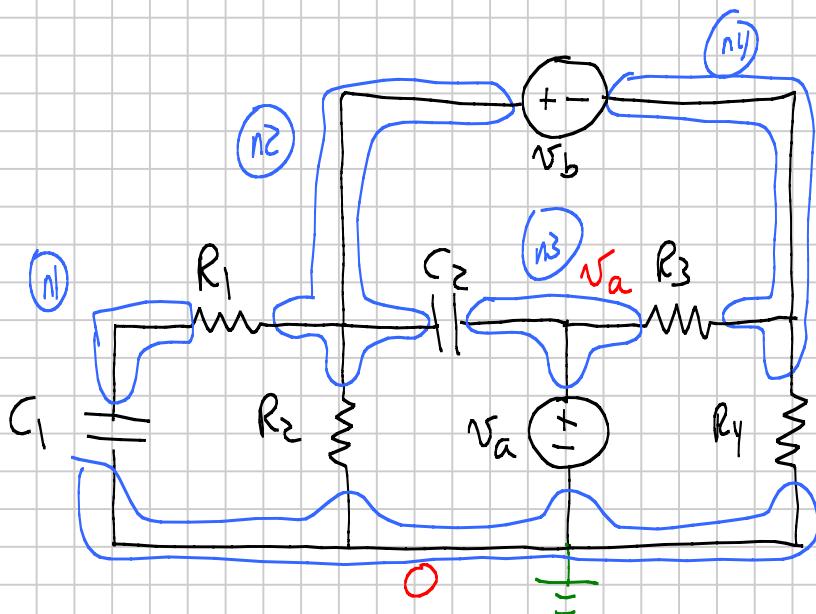


Test 1 Spring 2008 (Long Answers)

Note Title

3/6/2008

I)

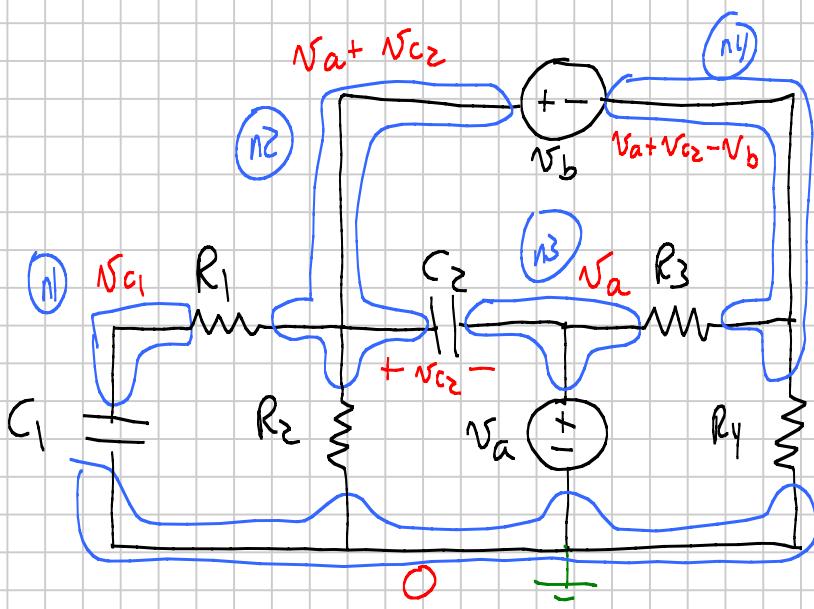


- 1) LABEL GND
- 2) LOCATE NODES
- 3) LABEL USING:
 - a) KNOWN VALUES
 - b) LABELED UNKNOWN
 - c) NEW UNKNOWN

n3) At first, only ground is known. V_a is connected to it so

$$V_{n_3} = 0 + V_a = V_a.$$

LABELS
I At this stage, no more node voltages can be found and there are no labeled unknowns to work with so pick a new one. One common practice would be to label the capacitor voltage V_{C_2} from which



$$V_{n_2} = V_{n_3} + V_{C_2}$$

$$\text{yields } V_{n_2} = V_a + V_{C_2}.$$

Then, note that

$$V_{n_2} - V_{n_1} = V_b \text{ so}$$

$$V_{n_1} = V_{n_2} - V_b = V_a + V_{C_2} - V_b.$$

Finally, add V_{C_1} as an unknown.

Two unknowns means two equations KCL must avoid voltage sources, so valid nodes & supernodes include n1, n2, n4, n3g (among others). Pick two of these; I'll give you all 3:

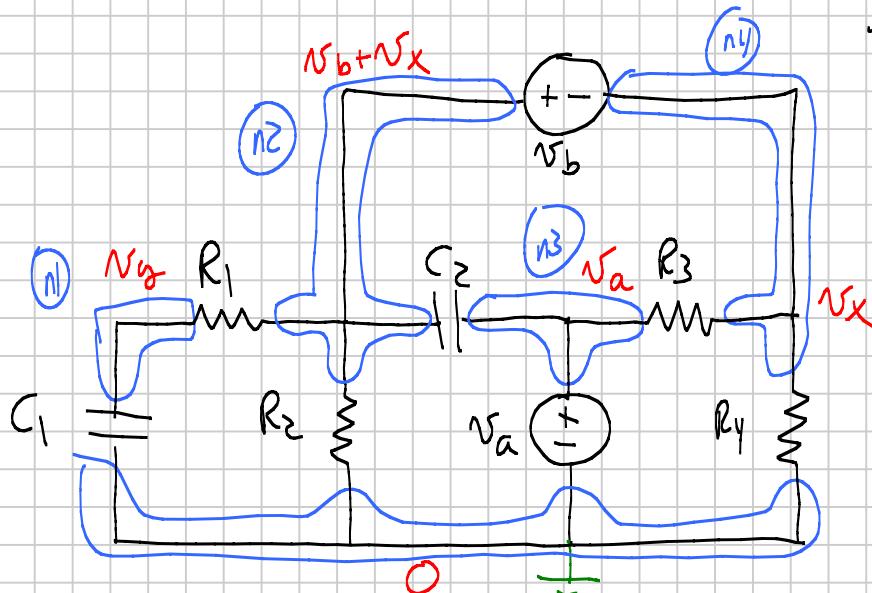
$$KCL, n_1: C_1 \frac{d}{dt} (V_{C_1} - 0) + \frac{(V_C) - (V_a + V_{C_2})}{R_1} = 0$$

$$KCL, n_2n_4: \frac{(V_a + V_{C_2}) - V_C}{R_1} + \frac{(V_a + V_{C_2}) - 0}{R_2} + C_2 \frac{d}{dt} (V_{C_2}) + \frac{(V_a + V_{C_2} - V_b) - V_a}{R_3} + \frac{(V_a + V_{C_2} - V_b) - 0}{R_4} = 0$$

$$KCL, n_3g: C_1 \frac{d}{dt} (0 - V_{C_1}) + \frac{(0 - V_a + V_{C_2})}{R_2} + C_2 \frac{d}{dt} (-V_{C_2}) + \frac{V_a - (V_a + V_{C_2} - V_b)}{R_3} + 0 - \frac{(V_a + V_{C_2} - V_b)}{R_4} = 0$$

NOTE THAT $n_1 + n_2n_4 + n_3g = 0$, so these are not independent.

Another way to label would be to just pick one of the nodes as an unknown voltage; for example, the far right is V_X



Then use $V_{n_2} = V_{n_4} + V_b$ to get $V_{n_2} = V_b + V_x$. V_{n_1} still needs a value, this time I'll call it V_y . The same nodes provide useful KCL:

$$KCL, n_1: C_1 \frac{d}{dt} (V_y) + \frac{(V_y) - (V_b + V_x)}{R_1} = 0$$

$$KCL, n_2n_4: \frac{(V_b + V_x) - (V_y)}{R_1} + \frac{(V_b + V_x) - 0}{R_2} + C_2 \frac{d}{dt} (V_b + V_x - V_a) + \frac{(V_x) - (V_a)}{R_3} + \frac{V_x - 0}{R_4} = 0$$

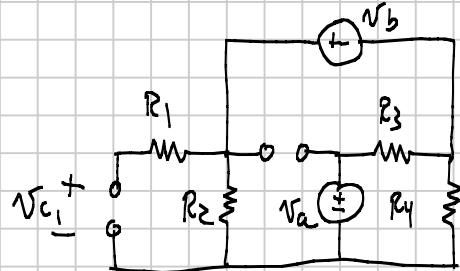
$$KCL, n_3g: C_1 \frac{d}{dt} (0 - V_y) + 0 - \frac{(V_b + V_x)}{R_2} + C_2 \frac{d}{dt} (V_a - (V_b + V_x)) + \frac{V_a - V_x}{R_3} + \frac{0 - V_x}{R_4} = 0$$

Again, $n_1 + n_2n_4 + n_3g = 0$; only two of these equations are required.

DC voltage: $i = C \frac{dv}{dt}$ and in the long term, $i_C = 0$

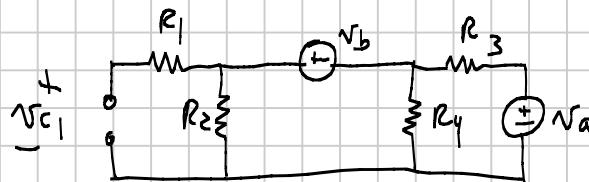
Exceptions: C in series w/ current source or no resistors.

Redrawn:



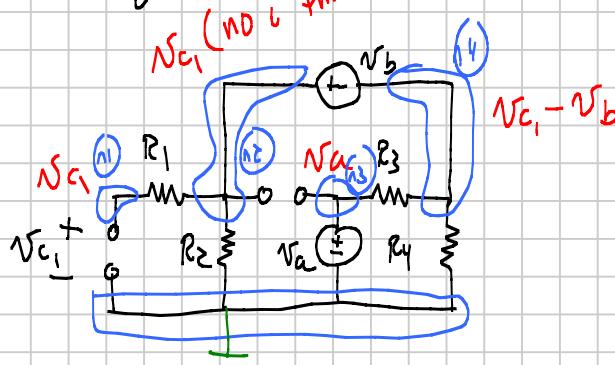
Notes: 1) no current through R_1 , so
no voltage drop across R_1 .
2) You cannot do voltage division -
no resistors are in series.

Here's another way to draw this, ignoring the "C₂" gap:



no series!

NVM on original DC-equivalent:



One unknown; KCL valid on n2n4, n3n5, among others.

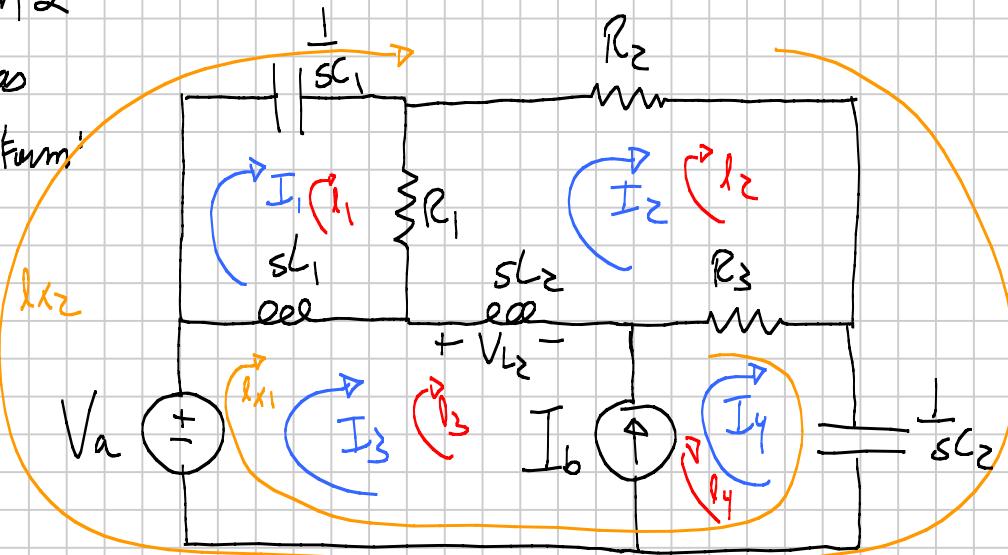
$$KCL, n2n4: \frac{V_{C1} - 0}{R_2} + \frac{(V_{C1} - V_b) - V_a}{R_3} + \frac{V_{C1} - V_b}{R_4} = 0$$

$$V_{C1} \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = V_a \left(\frac{1}{R_3} \right) + V_b \left(\frac{1}{R_3} + \frac{1}{R_4} \right)$$

$$V_{C1} = \frac{V_a \frac{1}{R_3} + V_b \left(\frac{1}{R_3} + \frac{1}{R_4} \right)}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

Problem 2

Redraw as
Tague XForm



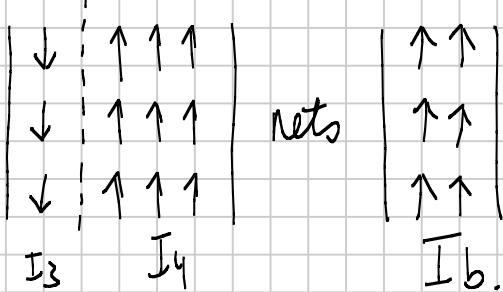
1) Label Mesh currents (I use \pm 's since I_b already taken)

4 meshes means 4 equations;

Possible loops: $l_1, l_2, l_1l_2, l_3l_4, l_1l_3l_4, l_2l_3l_4, l_1l_2l_3l_4$

Not all independent. Also must use aux. equation w/ I_b

So: aux: $I_b = I_4 - I_3$ note directions. I_b is the
actual current in the branch; I_4 and I_3 are components
of it:



$$\text{KVL}, l_1: \frac{1}{SC_1} (I_1) + R_1 (I_1 - I_2) + sL_1 (I_1 - I_3) = 0$$

$$\text{KVL}, l_2: R_1 (I_2 - I_1) + R_2 (I_2) + R_3 (I_2 - I_4) + sL_2 (I_2 - I_3) = 0$$

Third equation can be one of $l_3l_4, l_1l_2l_3l_4$

I'll present both but you only need one:

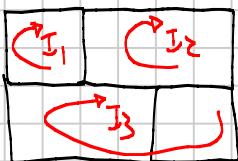
$$\text{KVL}, l_3: -V_a + sL_1 (I_3 - I_1) + sL_2 (I_3 - I_2) + R_3 (I_4 - I_2) + \frac{1}{SC_2} (I_4) = 0$$

OR

$$\text{KVL}, l_4: -V_a + \frac{1}{SC_1} (I_1) + R_2 (I_2) + \frac{1}{SC_2} (I_4) = 0$$

Note that $l_1 + l_2 + l_3 = l_{12}$. Also use the correct mesh current.

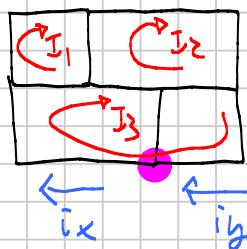
That is, some people write:



which is WRONG! Mesh currents should be the individual meshes; superloops will just happen to traverse multiple meshes.

Proof: the drawing above would require that

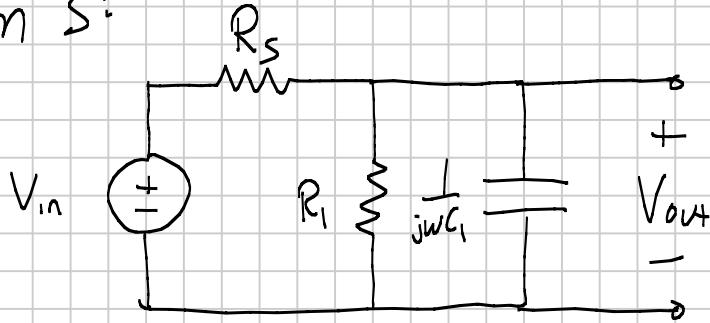
$i_x = i_y = i_3$ in the circuit ... but KCL



at the indicated node would then be $i_3 - i_3 + i_b = 0$; $i_b = 0$...
BUT i_b is a source!

The way V_{L2} is labeled above, $V_{L2} = sL_2(I_3 - I_2)$

Problem 3:



(a) Using voltage division,

$$V_{out} = \frac{R_1 \parallel \frac{1}{jwC_1}}{R_s + (R_1 \parallel \frac{1}{jwC_1})} V_{in} ; R_1 \parallel \frac{1}{jwC_1} = \frac{R_1}{jwC_1} = \frac{R_1}{R_1 + \frac{1}{jwC_1}} = \frac{R_1}{1 + jwR_1C_1}$$

so, $\frac{V_{out}}{V_{in}} = \frac{\frac{R_1}{1 + jwR_1C_1}}{R_s + \frac{R_1}{1 + jwR_1C_1}} \times \frac{1 + jwR_1C_1}{1 + jwR_1C_1} = \frac{R_1}{R_1 + R_s + jwR_1R_sC_1}$

(b) Note: for a fraction, must get form $\frac{a+jb}{c+jd}$ to take magnitude:

Magnitude is then $\frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$ so $|H| = \frac{\sqrt{R_1^2 + 0^2}}{\sqrt{(R_1+R_s)^2 + (wR_1R_sC_1)^2}}$

$$|H| = \frac{R_1}{\sqrt{(R_1+R_s)^2 + w^2 R_1^2 R_s^2 C_1^2}}$$

(c) Numerator is constant, denominator decreases as w increases so

Low-PASS

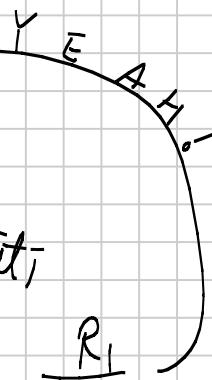
(d) Smallest denominator when $w=0$, so

$$|H(j0)| = \frac{R_1}{\sqrt{(R_1+R_s)^2}} = \frac{R_1}{R_1+R_s} = \frac{2000}{2000+1000} = \frac{2}{3}$$

Note when $w=0$, DC circuit so $C \rightarrow$ open circuit;



is just a voltage divider, $\frac{R_1}{R_1+R_s}$



$$(e) \omega_{co} \text{ when } |H(j\omega)| = \frac{|H(j\omega)|_{\max}}{\sqrt{\sum}}$$

$$\frac{R_1}{(R_1+R_s)^2 + \omega^2 R_1^2 R_s^2 C_1^{-2}} = \left(\frac{2}{3}\right) \left(\frac{1}{\sqrt{\sum}}\right) \quad \text{SQUARE BOTH SIDES}$$

$$\frac{R_1^2}{(R_1+R_s)^2 + \omega^2 R_1^2 R_s^2 C_1^{-2}} = \frac{2}{9}$$

$$9R_1^2 = 2(R_1^2 + 2R_1R_s + R_s^2) + 2\omega^2 R_1^2 R_s^2 C_1^{-2}$$

$$\omega^2 = \frac{7R_1^2 - 4R_1R_s - 2R_s^2}{2R_1^2 R_s^2 C_1^{-2}}$$

$$\omega_{co} = \sqrt{\frac{(7)(2000)^2 - 4(2000)(1000) - 2(1000)^2}{2(2000)^2 (1000)^2 (30 \cdot 10^{-6})^2}} = \sqrt{\frac{18 \cdot 10^6}{7.2 \cdot 10^3}} = \sqrt{2500} = 50 \text{ rad/s}$$

so BW = 50 rad/s

Problem 4

$|H|_{max} = 5$ and half-power frequencies are where $|H| = \frac{|H|_{max}}{\sqrt{2}} = 3.53$



$$(b) \omega_{coul} = 4 \text{ rad/s}$$

for ω_{com} estimate is 11 rad/s ; actual equation for

$$\text{is } |H| = 5 - \frac{1}{2}(\omega - 8) \text{ or } 9 - \frac{\omega}{2}$$

$$9 - \frac{\omega}{2} = \frac{5}{\sqrt{2}} \quad \omega = 2(9 - \frac{5}{\sqrt{2}}) = 10.929$$

$$BW = \omega_H - \omega_L = 10.929 - 4 = 6.929 \text{ rad/sec}$$

$$(c) \text{ If } T = \frac{2}{3}\pi, \omega_0 = \frac{2\pi}{\frac{2}{3}\pi} = 3. \text{ For } C_{0,out}, H(j0)=0 \text{ so } C_{0,out}=0$$

Yellow highlights above show possible frequencies.

k	ω (rad/s)	B_k	$H(j\omega_k)$	$B_{k,out}$
1	3	-1/π	2	-2/π
2	6	-1/2π	5	-5/2π
3	9	-1/3π	4.5	-3/2π
4	12	-1/4π	3	-3/4π

5 or greater are beyond filter's range

$$\text{Reconstructed } X_{out} = C_{0,out} + \sum_{k=1}^{\infty} A_{k,out} \cos(k\omega_0 t) + B_{k,out} \sin(k\omega_0 t)$$

$$X_{out} = -\frac{2}{\pi} \sin(3t) - \frac{5}{2\pi} \sin(6t) - \frac{3}{2\pi} \sin(9t) - \frac{3}{4\pi} \sin(12t)$$

Problem 5:

$$(a) \mathcal{U}_L \{ e^{-3t} \sin(4t) u(t) \} = \frac{4}{(s+3)^2 + (4)^2}$$

Note MOAT: $e^{-at}(A \cos(wt) + B \sin(wt)) = \frac{A(s+a) + B(w)}{(s+a)^2 + w^2}$

(b) Since $s(t) = \frac{d}{dt} u(t)$, $h(t) = \frac{d}{dt} s_r(t)$ and thus

$$H(s) = s S_r(s) = \frac{4s}{(s+3)^2 + (4)^2}$$

ASIDE: $\frac{4s}{(s+3)^2 + (4)^2} = \frac{4(s+3) - 3(4)}{(s+3)^2 + (4)^2}$

$$h(t) = (4e^{-3t} \cos(4t) - 3e^{-3t} \sin(4t)) u(t)$$

taking $\frac{ds_r}{dt} = \frac{d}{dt} (e^{-3t} \sin(4t) u(t)) = -3e^{-3t} \sin(4t) u(t) + 4e^{-3t} \cos(4t) u(t) + e^{-3t} \sin(4t) \delta(t)$

The last term = 0 since $\delta(t)$ only "turns on" at $t=0$ and $\sin(4(0))=0$

$$(c) Y = HX \quad X(s) = \mathcal{U}_L \{ (e^{-t} - e^{-2t}) u(t) \} = \frac{1}{s+1} - \frac{1}{s+2} = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2 + 3s + 2}$$

$$Y = \frac{4s}{(s+3)^2 + (4)^2} \cdot \frac{1}{(s+1)(s+2)}$$

(d) Easy Way: if $u(t) \rightarrow s_r(t)$ for an LTI system,
 $a u(t-t_0) \rightarrow a s_r(t-t_0)$ so

$$x_2 = u(t) - u(t-2) \longrightarrow s_r(t) - s_r(t-2) \text{ or}$$

$$e^{-3t} \sin(4t) u(t) - e^{-3(t-2)} \sin(4(t-2)) u(t-2)$$

Or Harder Way $\mathcal{U}_L \{ u(t) - u(t-2) \} = \frac{1}{s} - \frac{e^{-2s}}{s} = \frac{1 - e^{-2s}}{s}$

$$HX = \frac{4s}{(s+3)^2 + (4)^2} \cdot \frac{1 - e^{-2s}}{s} = \frac{4(1 - e^{-2s})}{(s+3)^2 + (4)^2} = \frac{4}{(s+3)^2 + (4)^2} - \frac{4e^{-2s}}{(s+3)^2 + (4)^2}$$

First one is a MOAT: $e^{-3t} \sin(4t) u(t)$

Second: see time shift so $- (e^{-3t} \sin(4t) u(t))$ shifted so $t=t-2$, or
 $- e^{-3(t-2)} \sin(4(t-2)) u(t-2)$

all told, $y_2(t) = e^{-3t} \sin(4t) u(t) - e^{-3(t-2)} \sin(4(t-2)) u(t-2)$