

Duke University  
Edmund T. Pratt, Jr. School of Engineering

EE 61 Section 2, Spring 2001  
**Test I**

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Name (please print) Answer Key 2/2009

In keeping with the Honor Code, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will fail the class and will be brought before the Undergraduate Judicial Board.

Signature: 

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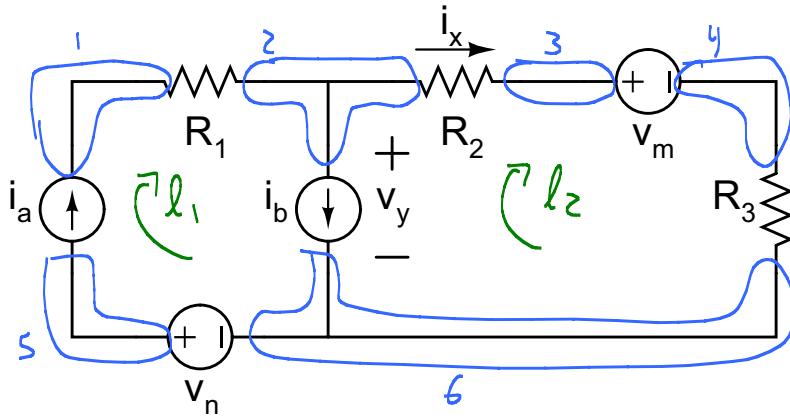
**Problem I: [15 pts] Element Table**

Fill in the table below. For the **Equation** column, you can put *any* equation for the given variable in terms of other variables.

Name	Variable	Units	Equation
charge	$q, Q$	C	(blank)
current	$i$	A	$dq/dt$
work	w	J	(blank)
voltage	v	V	$dw/dq$
power	P	W	$vi$
resistance	R	$\Omega$	$v/i$
conductance	G	$\Omega^{-1}$ or S	(blank)

## Problem II: [15 pts] Basic Circuit Relationships

Given the following circuit:



and known values  $i_a$ ,  $i_b$ ,  $v_m$ ,  $v_n$ ,  $R_1$ ,  $R_2$ , and  $R_3$ , find the following quantities in terms of the known values:

$$(1) i_x \quad KCL_{l_2}: -\bar{i}_a + \bar{i}_b + i_x = 0 \quad i_x = \underline{\bar{i}_a - \bar{i}_b}$$

$$(2) v_y \quad KVL_{l_2}: -v_y + R_2 i_x + v_m + R_3 i_x = 0$$

$$v_y = \underline{(R_2 + R_3) i_x + v_m} \quad i_x \text{ as above}$$

$$\text{or} \quad \underline{(R_2 + R_3)(\bar{i}_a - \bar{i}_b) + v_m}$$

$$(3) p_{abs, v_n} \quad p_{abs, v_n} = -\bar{i}_a v_n$$

$\nwarrow$  active sign convention

$$(4) p_{del, R_1} \quad p_{del, R_1} = \underline{-\bar{i}_a^2 R_1}$$

$\nwarrow$  wants power delivered

$$(5) p_{abs, R_3} \quad p_{abs, R_3} = \underline{\bar{i}_x^2 R_3} \quad i_x \text{ as above}$$

$$\text{or} \quad (\bar{i}_a - \bar{i}_b)^2 R_3$$

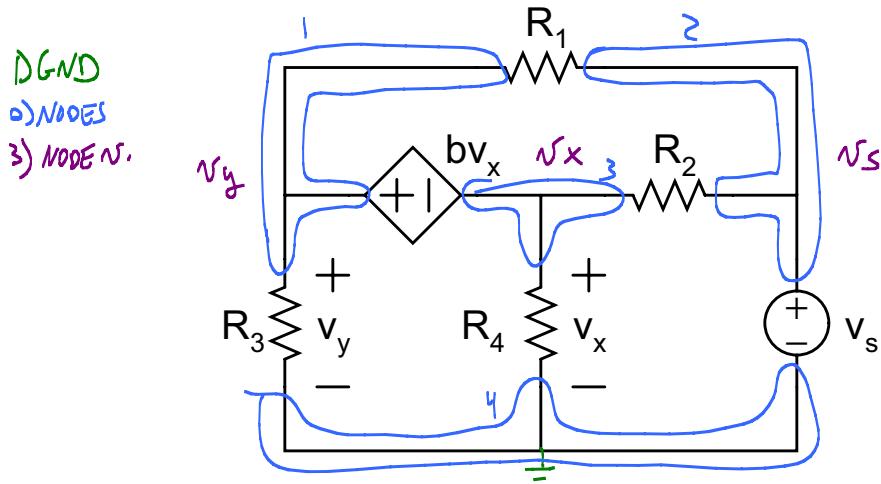
$$(6) p_{del, i_b} \quad p_{del, i_b} = -v_y \bar{i}_b \quad v_y \text{ as above}$$

$\nwarrow$  wants delivered; labeled w/ passive sign conv.

$$= -((R_2 + R_3)(\bar{i}_a - \bar{i}_b) + v_m) \bar{i}_b$$

### Problem III: [30 pts] Node Voltage Method

Given the following circuit:



and known values  $v_s$ ,  $b$ ,  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ , find  $v_y$  in terms of the known values using the Node Voltage Method.

$$\text{AUX: } VCVS \rightarrow b v_x = v_y - v_x \\ v_x = \frac{v_y}{b+1}$$

$$\text{KCL at } 1: \frac{v_y}{R_3} + \frac{v_y - v_s}{R_1} + \frac{v_x}{R_4} + \frac{v_x - v_s}{R_2}$$

$$\left(\frac{1}{R_1} + \frac{1}{R_3}\right)v_y + \left(\frac{1}{R_2} + \frac{1}{R_4}\right)v_x = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_s$$

$$\left(\frac{1}{R_1} + \frac{1}{R_3}\right)v_y + \left(\frac{1}{R_2} + \frac{1}{R_4}\right)\left(\frac{1}{b+1}\right)v_y = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_s$$

$$v_y = \frac{\frac{1}{R_1} + \frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{b+1}\left(\frac{1}{R_2} + \frac{1}{R_4}\right)} v_s$$

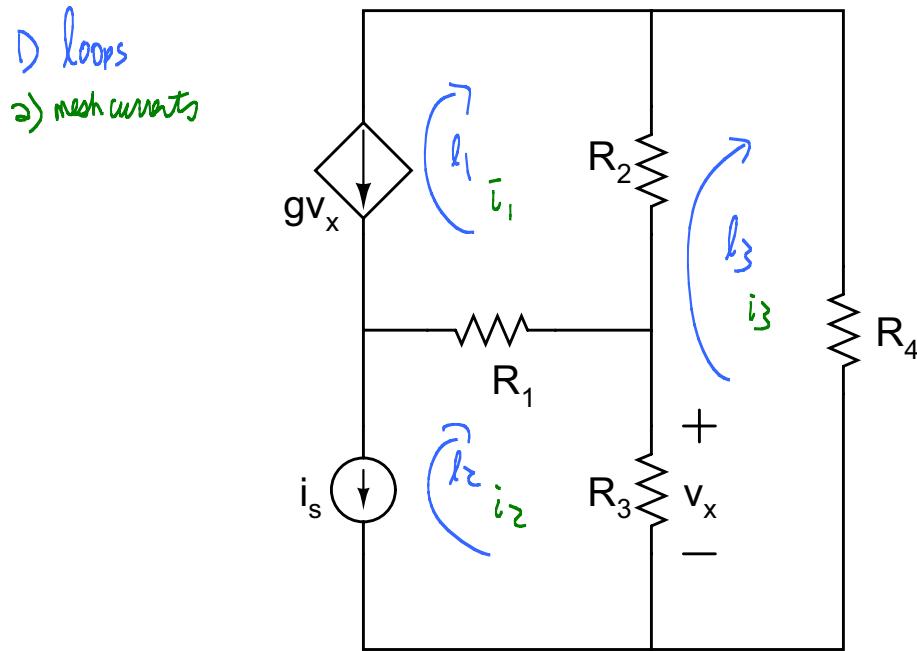
$$v_y = \frac{(b+1)(R_2 R_3 R_y + R_1 R_3 R_y)}{(b+1)(R_2 R_3 + R_4 + R_1 R_2 R_y) + R_1 R_3 R_y + R_1 R_2 R_3} v_s$$

Units:  $b \approx [1]$  so

$$\frac{[1] \text{ } \Omega^3}{[1] \Omega^3 + \Omega^3} V \rightarrow V \quad \checkmark$$

### Problem IV: [30 pts] Mesh Current Method

Given the following circuit:



and known values  $i_s$ ,  $g$ ,  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ , find  $p_{\text{abs}, R_4}$  in terms of the known values using the Mesh Current Method. Hint: use the two simple-source equations first to get a single equation for the unknown current in the tallest mesh.

$$\text{CONTROL VARIABLE: } N_x = R_3(i_2 - i_3)$$

$$\text{AUX 1: } i_1 = -gV_x = -gR_3(i_2 - i_3) = gR_3(i_3 - i_2)$$

$$\text{AUX 2: } i_2 = -i_s$$

$$\text{KVL, } i_3 : R_3(i_3 - i_2) + R_2(i_3 - i_1) + R_4 i_3 = 0$$

If required to solve, can substitute on:

$$\begin{bmatrix} 1 & gR_3 & -gR_3 \\ 0 & 1 & 0 \\ -R_2 & -R_3 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -i_s \\ 0 \end{bmatrix}$$

$$i_3 = \frac{\begin{vmatrix} 1 & gR_3 & 0 \\ 0 & 1 & -i_s \\ -R_2 & -R_3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & gR_3 & -gR_3 \\ 0 & 1 & 0 \\ -R_2 & -R_3 & R_2 + R_3 + R_4 \end{vmatrix}} = \frac{i_s g R_2 R_3 - i_s R_3}{R_2 + R_3 + R_4 - g R_2 R_3}$$

$$p_{\text{abs}, R_4} = i_3^2 R_4$$

### Problem V: [10 pts] Cramer's Rule

Given the following set of three linear equations:

$$\begin{aligned}x + 5y - 7z &= 10 \\-8x - 2y + 3z &= 11 \\6x + 9y - 4z &= 12\end{aligned}$$

- (1) Write the system as a matrix equation

$$\begin{bmatrix} 1 & 5 & -7 \\ -8 & -2 & 3 \\ 6 & 9 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

- (2) Set up, but do not solve, what you would need to do to find the value of the variable  $y$

$$y = \frac{\begin{vmatrix} 10 & -7 \\ 11 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 5 & -7 \\ -8 & -2 & 3 \\ 6 & 9 & -4 \end{vmatrix}}$$