

Duke University
Edmund T. Pratt, Jr. School of Engineering

EE 61L Section 2, Fall 2001
Test I

Michael R. Gustafson II

Name (please print) _____

Answer Key

In keeping with the Honor Code, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will fail the class and will be brought before the Undergraduate Judicial Board.

Signature: _____

[Signature] *P*

Problem I: [15 pts] Element Table

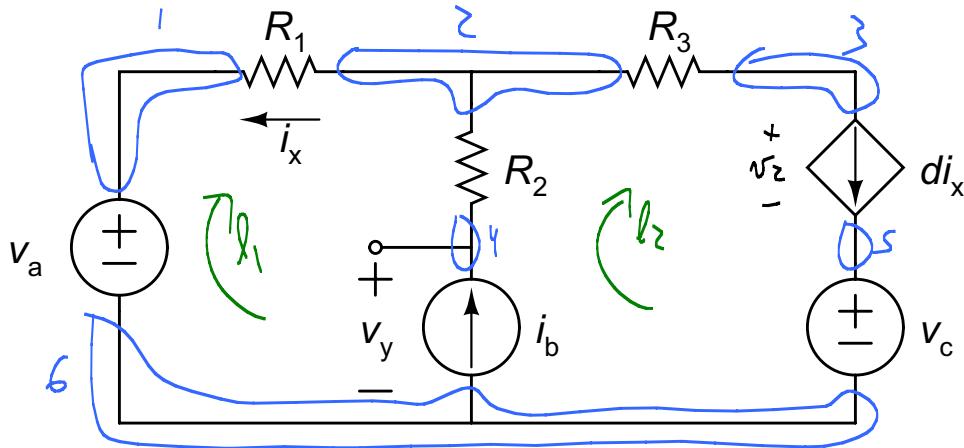
Fill in the table below. For the **Equation** column, you can put *any* equation for the given variable in terms of other variables *except* you may only use Ohm's Law **once**.

Name	Variable	Units	Equation
charge	q	C	(blank)
current	i	A	dq/dt
work	w	J	(blank)
voltage	v	V	dq/dw
power	p	W	$dw/dt, vi$
resistance	R	Ω	v/i
conductance	G	Ω^{-1} or S	(blank)

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Problem II: [15 pts] Basic Circuit Relationships

Given the following circuit:



and known values v_a , i_b , v_c , d , R_1 , R_2 , and R_3 , find the following quantities in terms of the known values:

$$(1) i_x \text{ KCL at } 234: i_x - i_b + di_x = 0 \quad i_x = \frac{i_b}{1+d}$$

$$(2) v_y \text{ KVL at } l_1: -v_a - R_1 i_x - R_2 i_b + v_y = 0$$

$$v_y = v_a + R_2 i_b + R_1 \underline{i_x} = v_a + R_2 i_b + \frac{R_1 i_b}{1+d}$$

$$(3) p_{abs,R_2} = \bar{i}_b^2 R_2$$

$$(4) p_{del,i_b} = v_y \bar{i}_b = (v_a + R_2 \bar{i}_b + \frac{R_1 \bar{i}_b}{1+d})(\bar{i}_b)$$

active signs conv.

$$(5) p_{abs,CCCS} \text{ KVL at } l_2: -v_y + R_2 \bar{i}_b + d \bar{i}_x R_3 + v_z + v_c = 0$$

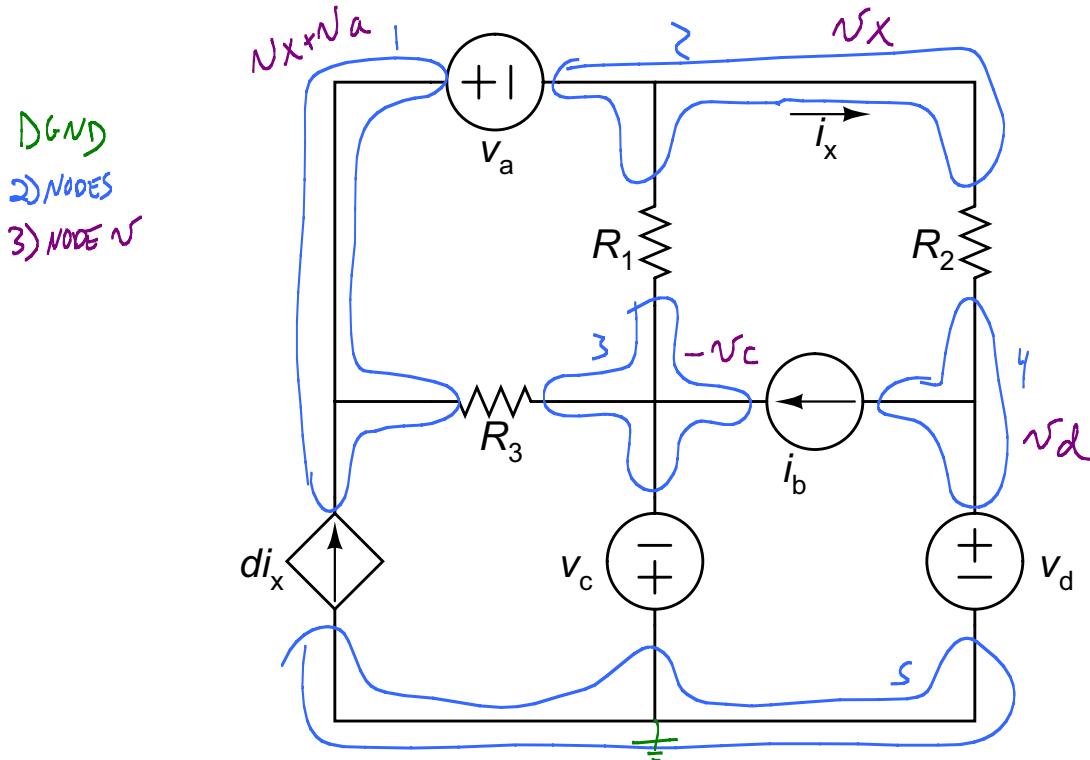
$$v_z = v_y - R_2 \bar{i}_b - d \bar{i}_x R_3 - v_c$$

$$p_{abs,cccs} = v_z d \bar{i}_x \quad v_z, \bar{i}_x \text{ as above.}$$

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Problem III: [30 pts] Node Voltage Method

Given the following circuit:



and known values v_a , i_b , v_c , v_d , d , R_1 , R_2 , and R_3 , find i_x in terms of the known values using the Node Voltage Method.

$$\begin{aligned}
 & \text{CONTROL} \quad i_x = \frac{v_x - v_d}{R_2} \\
 & KCL, s_{n_{12}}: -d i_x + \frac{(v_x + v_a) - (-v_c)}{R_3} + \frac{v_x - (-v_c)}{R_1} + \frac{v_x - v_d}{R_2} = 0 \\
 & -d \left(\frac{v_x - v_d}{R_2} \right) + \frac{v_x + v_a + v_c}{R_3} + \frac{v_x + v_c}{R_1} + \frac{v_x - v_d}{R_2} = 0 \\
 & v_x \left(\frac{1}{R_1} + \frac{1-d}{R_2} + \frac{1}{R_3} \right) = -\frac{v_a}{R_3} - \frac{v_c}{R_1} - \frac{v_c}{R_3} + \frac{(1-d)v_d}{R_2} \\
 & v_x = - \frac{\left(\frac{v_a}{R_3} + \frac{v_c}{R_1} + \frac{v_c}{R_3} - \frac{(1-d)v_d}{R_2} \right)}{\left(\frac{1}{R_1} + \frac{1-d}{R_2} + \frac{1}{R_3} \right)} = - \left(\frac{R_1 R_2 v_a + R_2 (R_1 + R_3) v_c - (1-d) R_1 R_3 v_d}{R_1 R_2 + (1-d) R_1 R_3 + R_2 R_3} \right) \\
 & i_x = \frac{v_x - v_d}{R_2} = \frac{-R_1 R_2 v_a - R_2 (R_1 + R_3) v_c + (1-d) R_1 R_3 v_d - (R_1 R_2 + (1-d) R_1 R_3 + R_2 R_3) v_d}{R_2 (R_1 R_2 + (1-d) R_1 R_3 + R_2 R_3)}
 \end{aligned}$$

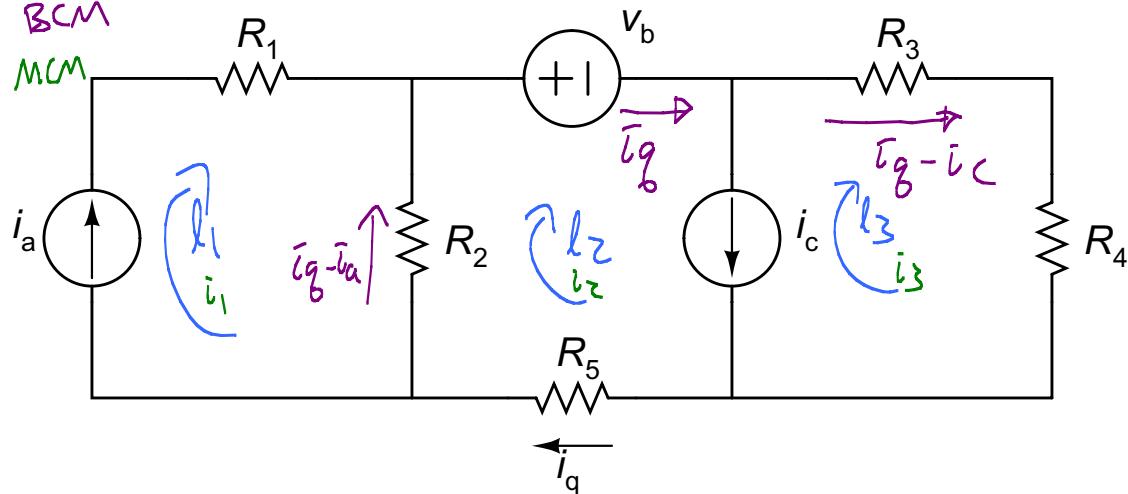
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Problem IV: [30 pts] Current Methods

Given the following circuit:

• FOR BCM

• FOR MCM



and known values i_a , v_b , i_c , R_1 , R_2 , R_3 , R_4 , and R_5 , find i_q and p_{del,i_c} in terms of the known values using either the Mesh Current Method or the Branch Current Method.

BCM:

$$\text{KVL}_{1, \text{SL}_2} : R_2(\bar{i}_g - \bar{i}_a) + v_b + R_3(\bar{i}_g - \bar{i}_c) + R_4(\bar{i}_g - \bar{i}_c) + R_5(\bar{i}_g)$$

$$\bar{i}_g(R_2 + R_3 + R_4 + R_5) = \bar{i}_a R_2 + \bar{i}_c(R_3 + R_4) - v_b$$

$$\bar{i}_g = \frac{\bar{i}_a R_2 + \bar{i}_c(R_3 + R_4) - v_b}{R_2 + R_3 + R_4 + R_5}$$

MCM:

$$\text{Aux 1: } \bar{i}_1 = \bar{i}_a$$

$$\text{Aux 2: } \bar{i}_2 - \bar{i}_3 = \bar{i}_c$$

$$\text{KVL}_{1, \text{SL}_{23}} : R_2(\bar{i}_2 - \bar{i}_1) + v_b + R_3 \bar{i}_3 + R_4 \bar{i}_3 + R_5 \bar{i}_2 = 0$$

$$\bar{i}_g = \bar{i}_2 \text{ or find } \bar{i}_2$$

$$R_2(\bar{i}_2 - \bar{i}_a) + v_b + R_3(\bar{i}_2 - \bar{i}_c) + R_4(\bar{i}_2 - \bar{i}_c) + R_5 \bar{i}_2 = 0$$

$$(R_2 + R_3 + R_4 + R_5) \bar{i}_2 = \bar{i}_a R_2 + R_3 \bar{i}_c + R_4 \bar{i}_c - v_b$$

$$\bar{i}_g = \bar{i}_2 = \frac{\bar{i}_a R_2 + \bar{i}_c(R_3 + R_4) - v_b}{R_2 + R_3 + R_4 + R_5}$$

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Problem V: [10 pts] Cramer's Rule

Given the following set of three linear equations:

$$\begin{aligned}y + 2z &= 2 \\2x - y &= 1 \\3x - 2y + 3z &= 2\end{aligned}$$

- (1) Write the system as a matrix equation.

$$\left[\begin{array}{ccc|c} 0 & 1 & 2 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} 2 \\ 1 \\ 2 \end{array} \right]$$

- (2) Use Cramer's Rule to solve for y . Be sure to show your work.

$$y = \frac{\begin{vmatrix} 0 & 2 & 2 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & 2 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{vmatrix}} = \frac{0+0+8-6-0-12}{0+0-8+6-0-6} = \frac{-16}{-8} = 1.25$$