Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 54L Spring 2010

Test The Second

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

For this test, you should not leave unevaluated convolution sums/integrals. Unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$

Name (please print):
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Problem I: [20 pts.] Fourier Series Analysis and Synthesis

(a) For the following periodic functions of time, determine its Fourier Series representation. Be sure to indicate the fundamental frequency or period of the signal as it is a part of the Fourier Series representation.

(1) $a(t) = \cos(2\pi t)$

(2) $b(t)$, a signal of period 2 where the first period (from $t = 0$ to $t = 2$) is given by $b(t) = 1 - u(t-1)$.

(3) $c(t) = a(t) \cdot b(t)$

(b) For the following Fourier Series representations of period signals, determine the signal in the time domain. In each case, assume that the fundamental period is $T_0 = 2$:

(1) $X[k] = \begin{cases} 1+j & k=3 \\ 3 & k=2 \\ 5 & k=0 \\ 3 & k=-2 \\ 1-j & k=-3 \end{cases}$

(2) $Y[k] = 3$

(3) $Z[k] = \frac{1}{k}\sin(5k)\cdot e^{-jk}$ *Note: this is a poorly made problem...should have been more like* $Z[k] = \frac{1}{k}\sin(2k)\cdot e^{-jk}$

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Problem II: [20 pts.] System Analysis 1

An LTI system has an impulse response $h(t)$ of

$$h(t) = \frac{\sin(4t)}{\pi t}$$

(a) Is this system causal? To get any credit, you must provide a brief explanation for your answer.

(b) An input $x(t) = \cos(t) \cdot \cos(\pi t)$ is applied to the system. Is this input periodic in time? If it is, find its Fourier Series representation; if not, state why you believe that.

(c) The output of this system given the input $x(t)$ above is called $y(t)$. Find $y(t)$. Note that you will not receive credit for any unevaluated convolution integrals.

(d) Is this output periodic in time? If it is, find its Fourier Series representation; if not, state why you believe that.

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Problem III: [20 pts.] System Analysis 10

An LTI system with an impulse response of $h(t)$ is subjected to an input $x(t)$ whose Fourier Transform is found to be:

$$X(j\omega) = 4\frac{\sin(\omega)}{\omega}e^{j\omega}$$

The output $y(t)$ in response to this input is found to be:

$$y(t) = e^{-2(t+1)}u(t+1) - e^{-2(t-1)}u(t-1)$$

(a) Find an expression for the transfer function of the system, $H(j\omega)$.

(b) Find an expression for the impulse response of the system, $h(t)$.

(c) Is this system causal? To get any credit, you must provide a brief explanation for your answer.

(d) Is the input $x(t)$ periodic in time? If it is, find its Fourier Series representation; if not, state why you believe that.

(e) Is the output $y(t)$ periodic in time? If it is, find its Fourier Series representation; if not, state why you believe that.

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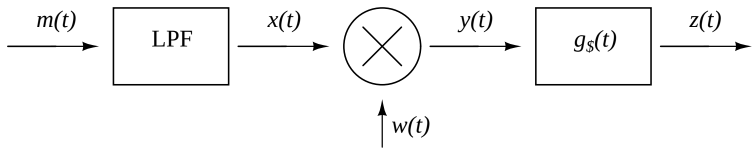
Problem IV: [20 pts.] System Analysis 11

A message signal $m(t)$ is sent through an ideal low-pass filter in order to produce a band-limited signal $x(t)$ which has energy distributed among all the frequencies between 0 rad/s and 40 rad/s. After that, several things happen as indicated in the following block diagram:



(a) The signal $x(t)$ is multiplied by a signal $w(t)$ known as the Triplex Processor of Cosine-y Goodness[1] which is

$$w(t) = \cos(50t) + \cos(100t) + \cos(200t)$$

Determine an expression for $W(j\omega)$, the Fourier Transform $w(t)$. Then, make a sketch of magnitude of the transform that clearly indicates the nature of the frequency content of $w(t)$. Be sure to label the axes.

(b) Call the output of the multiplier $y(t)$. Determine if the frequency domain of $y(t)$ is band-limited. If it is, indicate the band-limit.

(c) $y(t)$ is passed through an LTI filter to produce a signal $z(t)$. The filter has an impulse response $g_\$(t)$ as follows:

$$g_\$(t) = \frac{1}{t}\left(\sin(300t) - \sin(100t)\right)$$

Determine an expression for $G_\$(j\omega)$, the Fourier Transform of $g_\$(t)$, and sketch its magnitude. Be sure to label the axes.

(d) Determine if the frequency domain of $z(t)$ is band-limited. If it is, indicate the band-limit.

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[1] Yes, I just made that up...

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Problem V: [20 pts.] Applications

For this problem, assume you are always using the same message signal $m(t)$ with a maximum amplitude in the time domain of $\frac{20000}{\pi}$ and a Fourier Transform of

$$M(j\omega) = 200u(\omega + 200) - r(\omega + 200) + 2r(\omega) - r(\omega - 200) - 200u(\omega - 200)$$

First, sketch $M(j\omega)$ - be sure to label your axes. Next, determine if the frequency domain of $m(t)$ band-limited. If you believe it is, indicate the band-limit. If you do not believe it is, you may need to adjust your belief system.

(a) SAMPLING: Determine the minimum sampling rate that would be required in order to have any chance of perfectly recovering the signal if it is sampled by an impulse train operating at a sampling rate of T_s. Be sure to indicate units on your answer.

(b) FULL AMPLITUDE MODULATION: Sketch the magnitude of the frequency spectrum which would result if $m(t)$ were sent through a Full AM modulator with a carrier amplitude A_c of 2, a carrier frequency that is ten times the band-limit of $m(t)$, and an amplitude sensitivity factor of $\frac{\pi}{40000}$ (that is, 50% modulation). Be sure to label your axes and indicate important values. The drawing does not need to be perfectly to scale as long as the important points are labeled.

(c) FULL AMPLITUDE DEMODULATION: Draw the general circuit used to demodulate Full AM signals. What is this circuit called?

(d) DSB-SC MODULATION: Sketch the magnitude of the frequency spectrum which would result if $m(t)$ were sent through a DSB-SC modulator with a carrier amplitude of 4 and a carrier frequency that is five times the band-limit of $m(t)$. Be sure to label your axes and indicate important values.