

ECE 54 Spring 2010 Test II

Note Title

I) a1) $a(t) = \cos(2\pi t)$ $\omega_0 = 2\pi$ $A[1] = 1/2$
 $T_0 = 1$ $A[-1] = 1/2$

a2) $b(t) =$  $T_0 = 2$
 $\omega_0 = \pi$ $B[k] = \frac{\sin(k\pi/2)}{k\pi} e^{-jk\pi/2}$

rectangular pulse train,

$T_1 = 1/2$, time shift of $1/2$ sec.

a3) $a(t) \cdot b(t)$ has $T_0 = 2$ so
 rewrite $A[k]$ using $T_0 = 2, \omega_0 = \pi$ so $\begin{cases} "A[2]" = 1/2 \\ "A[-2]" = 1/2 \end{cases}$
 $a(t) \cdot b(t) \rightarrow \sum_{l=-\infty}^{\infty} A[l] B[k-l]$

$= A[2] B[k-2] + A[-2] B[k+2]$ all other $A[k] = 0$
 $= \frac{1}{2} B[k-2] + \frac{1}{2} B[k+2]$

b1) $T_0 = 2$ $\omega_0 = \pi$

$X[3] = 1+j = 2\left(\frac{1}{2}\right) + (-2)\left(\frac{1}{2j}\right) \rightarrow 2 \cos(3\pi t) - 2 \sin(3\pi t)$

$X[2] = 3 = 6\left(\frac{1}{2}\right) \rightarrow 6 \cos(2\pi t)$

$X[0] = 5 \rightarrow 5$

$x(t) = 5 + 6 \cos(2\pi t) + 2 \cos(3\pi t) - 2 \sin(3\pi t)$

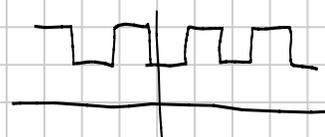
b2) $Y[k] = 3$

$\frac{1}{T} \rightarrow \sum_{n=-\infty}^{\infty} \delta(t-nT)$ and $T=2$ so

$y(t) = 6 \sum_{n=-\infty}^{\infty} \delta(t-2n)$

P
R
E
S
E
N
T
S

b3) $\frac{\sin(5k)}{k} e^{-jk}$... test fact. If $\omega_0 = \pi$,
 $5k = k \omega_0 T_1$ so $T_1 = 5/\pi \approx 1.6$
 And so $2T_1 > T$; ends up as



II)

$$h(t) = \frac{\sin(4t)}{\pi t}$$

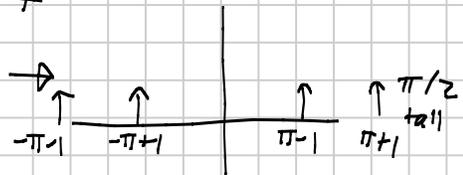
a) non causal; $h(t) \neq 0$ for all $t < 0$

$$b) x(t) = \cos(4t) \cdot \cos(\pi t) = \frac{\cos((\pi+1)t)}{2} + \frac{\cos((\pi-1)t)}{2}$$

Not periodic; $\frac{\pi+1}{\pi-1}$ is not rational

No Fourier Series

$$c) X(j\omega) = \frac{\pi}{2} \left(\delta(\omega + \pi + 1) + \delta(\omega - \pi - 1) \right) + \frac{\pi}{2} \left(\delta(\omega + \pi - 1) + \delta(\omega - \pi + 1) \right)$$



$H(j\omega)$ cutoff $\omega > 4$ rad

$$Y(j\omega) = \frac{\pi}{2} \left(\delta(\omega + \pi - 1) + \delta(\omega - \pi + 1) \right)$$

$$y(t) = \frac{1}{2} \cos((\pi-1)t)$$

d) Yes! $\omega = \pi - 1$ $Y[1] = \frac{1}{4}$
 $Y[2] = \frac{1}{4}$

III)

$$a) Y(j\omega) = \frac{e^{j\omega} - e^{-j\omega}}{j\omega + 2} = \frac{2j \sin(\omega)}{j\omega + 2}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2j \sin(\omega)}{j\omega + 2} \cdot \frac{\omega e^{j\omega}}{4 \sin(\omega)} = \frac{j\omega e^{j\omega}}{2(j\omega + 2)}$$

$$b) h(t) = \frac{d}{dt} \left(\frac{1}{2} e^{-2(t-1)} u(t-1) \right)$$

$$= \frac{1}{2} \left(-2 e^{-2(t-1)} u(t-1) + e^{-2(t-1)} \delta(t-1) \right)$$

$$= -e^{-2(t-1)} u(t-1) + \frac{1}{2} \delta(t-1)$$

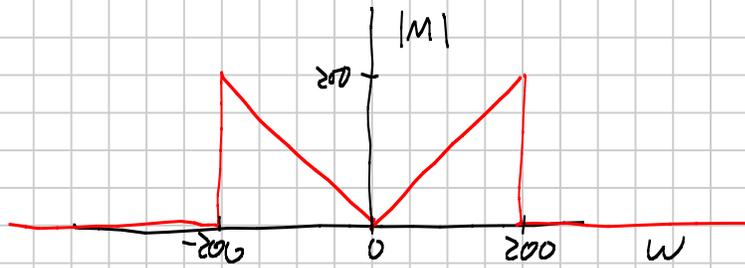
$\frac{1}{2} \uparrow$ scale
 $j\omega \uparrow$ $\frac{d}{dt}$
 $e^{-j\omega} \uparrow$ shift
 $\frac{1}{j\omega + 2} \uparrow$ $e^{-2t} u(t)$

c) causal, $h(t) = 0$ all $t < 0$

d) input not periodic; no δ in $X(j\omega)$

e) output not periodic; $y(t) = 0$ for all $t < -1$

V)

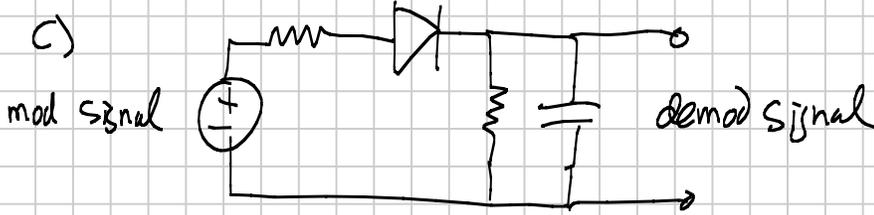


a) $\omega_s = 2\omega_m = 400 \text{ rad/s}$ | $F_s = \frac{\omega_s}{2\pi} = \frac{200}{\pi} \text{ samples/sec}$
 $T_s = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_m} = \frac{\pi}{200} \text{ sec}$

b) $2 \left(1 + \frac{\pi}{40000} m(t) \right) \cos(2000t)$

$2 \cos(2000t) + m(t) \cos(2000t)$

$\pi A_c = \pi \cdot 2 = 2\pi$



Envelope Detector

d) $4 m(t) \cos(1000t)$

$\frac{1}{2} A_c \max |M| = \frac{1}{2} (4)(200) = 400$

