In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: ________________________________

Instructions

First - please turn off any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do not write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a different problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name and NET ID are clearly written at the top of every page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem’s work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or the convolution integral. Also, unless otherwise specified:

- The \( \cdot \) symbol means multiplication
- The \( * \) symbol means convolution
- \( \delta(t) \) is the unit impulse function
- \( u(t) \) is the unit step
- \( r(t) \) is the unit ramp \( t \cdot u(t) \)
- \( q(t) \) is the “unit” quadratic \( \frac{1}{2}t^2 \cdot u(t) \)
Problem I: [12 pts.] Signal Classifications

(a) State whether each of the signals below is a power (P) or energy (E) signal and circle the appropriate letter. If the signal is a power signal, write but do not evaluate the formula to calculate the average power. If the signal is an energy signal, write but do not evaluate the formula to calculate the total energy.

\[ g(t) = \sum_{k=-\infty}^{\infty} (u(t - 5k) - u(t - 2 - 5k)) \cdot e^{-(t-5k)/4} \]

\[
\text{power: } P_{av} = \frac{1}{T} \int_{T} |g(t)|^2 \, dt = \frac{1}{5} \int_{0}^{5} \left( e^{-t/4} \right)^2 \, dt
\]
\[ \text{using limits from 0 to 5} \]

(2) (P or E): \( x(t) = (u(t) - u(t - 2)) \cdot \sin(2\pi t) \)

\[
\text{energy: } E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{0}^{2} \sin^2(2\pi t) \, dt
\]

(b) State whether each of the signals below is periodic or not. If the signal is periodic, state the period.

(1) \( a(t) = u(\cos(4t)) \)

\[ \text{periodic, } \omega = 4 \text{ and } T = \frac{2\pi}{4} = \frac{\pi}{2} \]

(2) \( b(t) = u(t) \cdot \cos(4t) \)

\[ \text{not periodic} \]

(3) \( c(t) = \cos(2\pi t) \cdot \sin(6\pi t) \cdot \cos(12\pi t) \)

\[ \text{periodic: frequencies of product of \cos/sin are equal to sum - difference, so } \cos(2\pi t) \sin(6\pi t) \text{ has } \omega \text{ of } 4\pi \text{ and } 8\pi \]
\[ \omega \text{ of } 4\pi \text{ and } 8\pi \times \omega \text{ of } 12\pi \text{ yields } \frac{8\pi}{4\pi} \text{ and } \frac{16\pi}{4\pi} \]
\[ \omega_0 = 4\pi \text{ and } T = \frac{2\pi}{\omega} = \frac{1}{2} \]
Problem II: [12 pts.] Signal Construction

(a) Given each of the signal graphs below, write an equation for the signal using singularity functions. Carefully note the $x$ and $y$ axis divisions. Also: $z(t)$ is periodic.

\[ x(t) = u(t+2) - 2u(t+1) + r(t) - 2r(t-1) + r(t-2) + u(t-3) \]

\[ y(t) = \frac{1}{2}r(t+1) - 2u(t+2) - \frac{1}{2}r(t) - u(t) \]

\[ z(t) = \text{period } = \frac{4}{3}; \text{ period from } 0^+ \text{ to } y^+ \text{ is } -u(t-1) + u(t-2) + u(t-3) - u(t-4) \text{ so } \]

\[ z(t) = \sum_{k=-\infty}^{\infty} -u(t-1-4k) + u(t-2-4k) + u(t-3-4k) - u(t-4-4k) \]

(b) Given the signals above, graph their transformed versions below.
Problem III: [16 pts.] System Classifications

For one point, complete the following statement using two different system properties:

“If a system is ______ than it is guaranteed to also be ______”

Then, for the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work below that table, but clearly indicate which system and system property you are working with.

<table>
<thead>
<tr>
<th>System</th>
<th>Linear?</th>
<th>Time Inv.?</th>
<th>Stable?</th>
<th>Memoryless?</th>
<th>Causal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t) = \frac{d^2}{dt^2} (x(2t))$</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>$y(t) = \int_{t-1}^{t+1} \sqrt{x(\tau)} , d\tau$</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>$y(t) = \frac{x(t-1)}{t+1}$</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$y(t) = \int_{-\infty}^{\infty} \delta(t - \beta + 2) \cdot x(\beta - 2) , d\beta$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$y(t) = \frac{x(t+1)}{x(t-1)}$</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>
Problem IV: [20 pts.] LTI Systems I

A linear, time-invariant system has an impulse response of:

\[ h(t) = u(t) - 2u(t-2) + u(t-3) \]

and an input signal:

\[ x_1(t) = u(t+4) - u(t+2) - \delta(t-1) \]

is applied to the system.

(a) Is the system stable?

(b) Is the system causal?

(c) Determine an expression for the output of this system to the input \( x_1(t) \) given above - call this output \( y_1(t) \).

You must evaluate any required integrals but are not required to simplify any algebra.

(d) Also determine an expression for the output of the system if the input is changed to:

\[ x_2(t) = e^{-t/3}u(t) \]

Call this output \( y_2(t) \). You must evaluate any required integrals but are not required to simplify any algebra.

(a) \( \int_{-\infty}^{\infty} |h(t)| \, dt = 3 < \infty \implies \text{stable} \)

(b) \( h(t) = 0 \) for \( t < 0 \) \implies \text{causal} \)

(c) \( y_1(t) = x_1(t) \ast h(t) \)

\[
= (u(t+4) - u(t+2) - \delta(t-1)) \ast (u(t) - 2u(t-2) + u(t-3)) \\
= r(t+4) - 2r(t+2) + r(t+1) \\
- (r(t+2) - 2r(t) + r(t-1)) \\
- u(t-1) + 2u(t-3) - u(t-1) \]

(d) \( y_2(t) = x_2(t) \ast h(t) \)

\[
= (e^{-t/3}u(t)) \ast (u(t) - 2u(t-2) + u(t-3)) \\
\text{General property: } e^{-t/3} u(t) \ast u(t) = \int_{-\infty}^{\infty} e^{-t/3} u(t) \, u(t-\tau) = u(t) \int_{0}^{\infty} e^{-\tau/3} \, d\tau = u(t) [3 - 3e^{-t/3}]_0^t = 3 - 3e^{-t/3}u(t) \\
\implies y_2(t) = 3(1-e^{-t/3})u(t) - 6(1-e^{-(t-2)/3})u(t-2) + 3(1-e^{-(t-3)/3})u(t-3) \]

using idea that \( 2u(t-2) = 2u(t) \ast \delta(t-2) \) and \( u(t-3) = u(t) \ast \delta(t-3) \)
Problem V: [20 pts.] LTI Systems II

You and your lab partner are given a box with three connections - an input, an output, and a ground - and are told that the system inside is an LTI system. When you apply an input voltage:

\[ v_i(t) = u(t - 1) - u(t - 5) \]

you measure an output response that - using general nonlinear fitting from EGR 53 - is best fit by the equation

\[ v_o(t) = (1 - e^{-3(t-2)})u(t - 2) - (1 - e^{-3(t-6)})u(t - 6) \]

(a) Is this system stable?
(b) Is this system causal?
(c) Determine the output \( v_{o1}(t) \) if the input voltage is:

\[ v_{11}(t) = 2u(t) - 2u(t - 8) \]

You must evaluate any required integrals but are not required to simplify any algebra.

(d) Determine the output \( v_{o2}(t) \) if the input voltage is:

\[ v_{12}(t) = e^{2t}u(t) \]

You must evaluate any required integrals but are not required to simplify any algebra.

\[ h(t) = \frac{ds(t)}{dt} = 3e^{-3(t-1)}u(t-1) \]

(a) \( \int_{-\infty}^{\infty} |h(t)| \, dt \) is finite so \( \text{stable} \)

(b) \( h(t) = 0 \) for \( t < 0 \) so \( \text{causal} \)

\[
\begin{align*}
N_{o1}(t) &= 2u(t) - 2u(t-8) \\
N_{o1}(t) &= 2s(t) - 2s(t-8) \\
&= 2(-e^{-2(t-1)}u(t-1)) - 2(1 - e^{-2(t-9)})u(t-9)
\end{align*}
\]

(c) \( N_{o2}(t) = e^{2t}u(t) \star 3e^{-3(t-1)}u(t-1) \]
\( = e^{2t}u(t) \star 3e^{-3t}u(t) \star \delta(t-1) \)
\( = 3 \int_{-\infty}^{\infty} e^{2t}u(t) \, e^{-3(t-n)}u(t-n) \, dt \star \delta(t-1) \)
\( = 3 \left[ \frac{e^{3t}e^{5t}}{5} \right]_0^t u(t) \star \delta(t-1) = \frac{3}{5} \left( e^{3t} - e^{3(t-1)} \right) u(t) \star \delta(t-1) \)
\( = \frac{3}{5} \left( e^{3(t-1)} - e^{3(t-1)} \right) u(t-1) \)
Problem VI: [20 pts.] Convolutions and Correlation Functions

Given continuous signals:

\[ x(t) = 2r(t + 3) - 2r(t + 1) \]
\[ y(t) = r(t + 2) - r(t - 2) - 4u(t - 2) \]
\[ z(t) = 2u(t + 4) - 3u(t - 2) \]

graph them here (using 0.5 per block is a good plan):

and then calculate the following:

- \( a(t) = x(t) \ast x(t) \)
- \( b(t) = x(t) \ast y(t) \)
- \( c(t) = y(t) \ast z(t) \)
- \( d(t) = \phi_{yy}(t) \)
- \( e(t) = \phi_{zz}(t) \)

\( u(t) \ast u(t) = \Gamma(t) = \Gamma(0) = t^u(t) \)
\( u(t) \ast c(t) = \delta(t) = \frac{1}{t^2} u(t) \)
\( c(t) \ast c(t) = \frac{1}{6} t^3 u(t) \)

**Note:** If you defined \( r(t) \) as \( c(t) \)

You must evaluate any required integrals but are not required to simplify any algebra.

(a) \( (2 \Gamma(t+3) - 2 \Gamma(t+1)) \ast (2 \Gamma(t+3) - 2 \Gamma(t+1)) = \)
\[ \int \frac{1}{6} (t+3)^3 u(t+3) - \frac{1}{6} (t+1)^3 u(t+1) \]
(b) \( (2 \Gamma(t+3) - 2 \Gamma(t+1)) \ast (\Gamma(t+2) - \Gamma(t-2) - 4u(t-2)) = \)
\[ 2 \frac{(t+5)^3}{6} u(t+5) - 2 \frac{(t+1)^3}{6} u(t+1) - 8 \delta(t+1) \]
\[ - 2 \frac{(t+3)^3}{6} u(t+3) + 2 \frac{(t-1)^3}{6} u(t-1) + 8 \delta(t-1) \]
(c) \( (\Gamma(t+3) - \Gamma(t-2) - 4u(t-2)) \ast (2 \delta(t+1) - 3u(t-2)) = \)
\[ 2 \delta(t+1) - 3 \delta(t) \]
\[ - 2 \delta(t+1) + 3 \delta(t-4) \]
\[ - 8 \Gamma(t+2) + 12 \Gamma(t-4) \]
\( (d) \quad \phi_{y_1} y_2 (t) = y_1(t) \ast y_2(-t) \)

Easiest to write \( y_2(-t) = \frac{4u(t+2) - r(t+2)}{r(t-2)} \)

\[
\begin{align*}
& (r(t+2) - r(t-2) - 4u(t-2)) \ast (4u(t+2) - r(t+2) + r(t-2)) : \\
& 4 \begin{aligned}
& g(t+4) - \frac{(t+4)^4 u(t+4)}{6} + \frac{(t+4)^2 u(t)}{6} \\
& - 4 \begin{aligned}
& g(t) + \frac{(t+4)^2 u(t)}{6} - \frac{(t-4)^3 u(t-4)}{6} \\
& - 16 r(t) + 4 g(t) - 4 g(t-4)
\end{aligned}
\end{aligned}
\end{align*}
\]

\( (c) \quad \phi_{y_2} y_1(t) = g(t) \ast g_2(t) \) as above, so

\[
\begin{align*}
& (2u(t+4) - 3u(t-2)) \ast (4u(t+2) - r(t+2) + r(t-2)) : \\
& 8 r(t+6) - 2g(t+6) + 2 g(t+2) \\
& - 12 r(t) + 3 g(t) - 3 g(t-4)
\end{align*}
\]