Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280L Summer 2018
Test II

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Name (please print)______________________________

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:______________________________

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

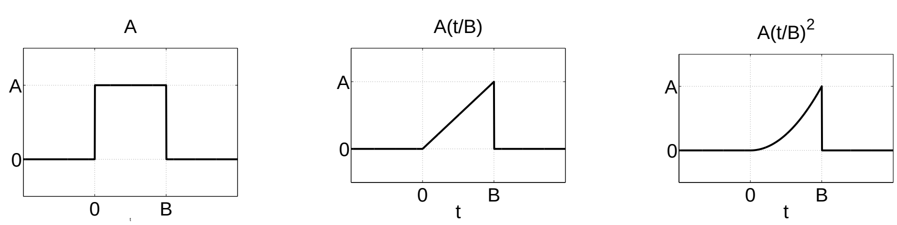
Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

For this test, you should not leave unevaluated convolution sums/integrals. Unless otherwise specified:

- The $\cdot$ symbol means multiplication while the $*$ symbol means convolution.
- $\delta(t)$ is the unit impulse function, $u(t)$ is the unit step, $r(t)$ is the unit ramp $t \cdot u(t)$, and $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$

For finite-duration constants, ramps, and quadratics, the area under the curves shown below are AB, $\frac{1}{2}AB$, and $\frac{1}{3}AB$, respectively.



Name (please print):
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Problem I: [16 *pts.*] Fourier Series

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below. You are not allowed to have complex exponentials or j's in any time domain representations either.

(1) An LTI system has a transfer function of:

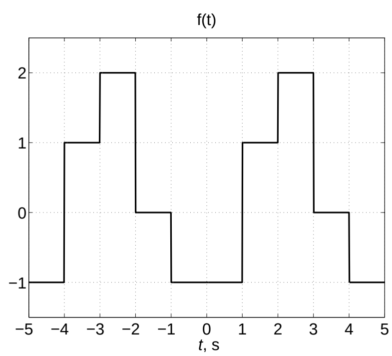
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega}{j\omega + 5}$$

Assuming a periodic input signal of:

$$x(t) = \cos(4t) + \sin(10t)$$

(a) Determine a differential equation that relates $y(t)$ and its derivatives to $x(t)$ and its derivatives.
(b) What kind of filter does this system represent? What is its cutoff frequency?
(c) Determine the fundamental frequency of and non-zero Fourier series coefficients $X[k]$ for $x(t)$.
(d) Determine the fundamental frequency of and non-zero Fourier series coefficients $Y[k]$ for $y(t)$.
(e) Determine $y(t)$. Your answer can have both sin and cos functions in it.
(f) Assume the filter represented by $H(j\omega)$ were replaced with an **ideal** version of that kind of filter. What is the output of this ideal filter if $x(t)$ is the input?

(2) Determine an expression for the Fourier series coefficients $F[k]$ of the periodic signal $f(t)$ graphed below. Remember - you are not allowed to have any unevaluated integrals, summations, or convolutions in your expression for the coefficients. Be sure to indicate the fundamental frequency of the signal.



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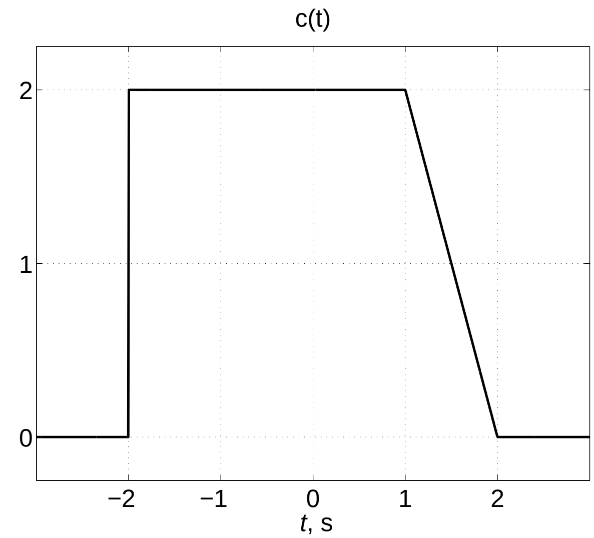
Problem II: [16 *pts.*] Continuous-Time Fourier Transforms

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below.

(1) Find the continuous time Fourier transform for each of the following signals:

- $a(t) = \sin(2t + 1)$
- $c(t)$ as shown below:



(2) Find the inverse Fourier transform for each of the following:

- $V(j\omega) = 10\frac{j\omega+3}{(j\omega)^2+10j\omega+26}$
- $W(j\omega) = 10\frac{j\omega+3}{(j\omega)^2+10j\omega+16}$
- $X(j\omega) = \frac{e^{-j\omega}}{(j\omega+6)^2}$

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Problem III: [12 *pts.*] Discrete-Time Fourier Transforms

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below.

(1) Find the Fourier transform for the following signals:

- $d[n] = \sin[2n + 1]$
- $f[n] = \left(\frac{1}{3}\right)^n u[n - 2]$

(2) Find the discrete time inverse Fourier transform for the following:

- $Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{20}e^{-j\omega} - \frac{1}{20}e^{-j2\omega}} = \frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)\left(1 + \frac{1}{5}e^{-j\omega}\right)}$
- $Z(e^{j\omega}) = \cos(\omega) + 5j\sin(3\omega)$

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Problem IV: [18 *pts.*] System Analysis I

A person clapping in a cavern can produce an echo that repeats with a particular time delay and which decays in amplitude over time. A discrete time model equation for the sound a person hears $y[n]$ when yelling into a cavern could be:

$$y[n] = x[n] + \frac{16}{25} y[n-2]$$

which means the sound heard at time n is a combination of the sound being made at time n - represented by $x[n]$ - and the sound echoing back to the person two periods later.

(1) What is the impulse response $h[n]$ of this system? Think of this as the sound you would hear if you clap at time 0. *Hint:* you will need to factor the denominator; remember that $a^2 - b^2 = (a-b)(a+b)$.

(2) Sketch the impulse response $h[n]$ for values of n starting at -1 and ending at 5.

(3) What is the step response $s_r[n]$ of this system? This is the sound you would hear if you clap every period.

(4) Sketch the step response $s_r[n]$ for values of n starting at -1 and ending at 5.

(5) Is this system causal?

(6) Is this system stable?

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Problem V: [18 *pts.*] System Analysis II

Assume you have some system S that has a transfer function of:

$$H(j\omega) = 1 + \frac{e^{j\omega} + e^{-j\omega}}{j\omega + 6}$$

(1) Find the impulse response $h(t)$ for the system.

(2) Find a differential equation that relates the output $y(t)$ and its derivatives the the input $x(t)$ and its derivatives.

(3) Find the response $y_1(t)$ of the system to an input $x_1(t) = e^{-6t}u(t)$.

(4) Is this system causal?

(5) Is this system stable?

Note: when I gave the test, there was a sign wrong in one of the exponents...which changed all the shifts and also the causality of the response. Oops.

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Problem VI: [20 *pts.*] Sampling and Modulation

Assume you have a band-limited signal $x(t)$ that has a Fourier transform

$$X(j\omega) = r(\omega + 500) - 2r(\omega) + r(\omega - 500)$$

where $r(\omega)$ is the ramp function, just with frequency as its argument.

(1) Sketch the magnitude of $X(j\omega)$. Note that $X(j\omega)$ is purely real here - what does that mean for $x(t)$?

(2) Is $x(t)$ an energy signal, a power signal, or neither? If it is an energy signal, also calculate the total energy E_∞. If it is a power signal, also calculate the overall average power, P_∞. If it is neither, no further calculation is necessary.

(3) Draw a block diagram showing a system that uses impulse sampling with a period of T_S to sample $x(t)$ above. That is, the signal $x(t)$ is multiplied by an impulse train $p(t)$ where $p(t)$ is

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$$

Then determine the minimum sampling rate ω_S in rad/s that would be required in order to have any chance of perfectly recovering the message signal.

(4) Sketch the Fourier transform of the output of the impulse sampler above assuming ω_S is three times the minimum rate you determined above.

(5) Sketch the Fourier transform of the output to the impulse sampler assuming ω_S is 75% of the minimum you determined above.

(6) Instead of sampling the signal, assume you multiply it by a pure sinusoid $\sin(2000t)$ to get a new signal $y(t) = x(t)\sin(2000t)$. Sketch the frequency content $Y(j\omega)$. Be sure to carefully label your axes and indicate important values in both the independent and dependent axes.

(7) What process would you use to recover $x(t)$ from $y(t)$? You can use block diagrams, frequency domain representations, mathematical expressions, or haiku to explain the process.

(8) If a person answered the previous question with "envelope detection,"

(a) What circuit would that person have drawn? Sketch it.
(b) Why would that person be wrong?
(c) How many points off should I take?