

ECE 280 Summer 2018 Test 2

Note Title

6/19/2018

I) a) $(j\omega + 5) Y(j\omega) = (j\omega) X(j\omega)$

$$\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt}$$

b) High-pass $\omega_{cutoff} = 5 \text{ rad/sec}$

c) $w_0 = 2 \text{ rad/sec}$ $X[2] = 1/2$ $X[5] = 1/2j$ all other $X[k] = 0$
 $X[-2] = 1/2$ $X[-5] = -1/2j$

d) $Y[2] = X[2] H(j4) = \frac{1}{2} \frac{j4}{j4+5} * \frac{-j4+5}{-j4+5} = \frac{16+j20}{82} \text{ or } \frac{1}{2} \frac{16}{41} - \frac{1}{2} \frac{j20}{41}$

$$Y[-2] = Y^*[2] = \frac{16-j20}{82}$$

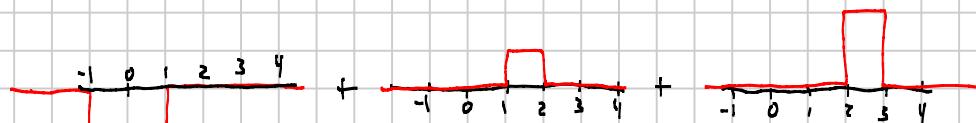
$$Y[5] = X[5] H(j10) = \frac{1}{2j} \frac{j10}{j10+5} * \frac{-j10+5}{-j10+5} = \frac{100+j50}{250j} = \frac{1-j2}{5} = \frac{1}{2} \frac{2}{5} + \frac{1}{2} \frac{j4}{5}$$

$$Y[-5] = Y^*[5] = \frac{1}{2} \frac{2}{5} - \frac{1}{2} \frac{j4}{5}$$

e) $y(t) = \frac{16}{41} \cos(2t) - \frac{20}{41} \sin(2t) + \frac{2}{5} \cos(10t) + \frac{4}{5} \sin(10t)$

f) IDEAL HPF w/ cutoff at 5 rad/sec means $y_{ideal}(t) = \sin(10t)$

2) Several possible answers; $T=5$, $w_0 = 2\pi/5$



$$F[k] = (-1)\left(\frac{2}{5}\right) \operatorname{sinc}\left(k\frac{2}{5}\right) + (1)\left(\frac{1}{5}\right) \operatorname{sinc}\left(k\frac{1}{5}\right) e^{-jk\left(\frac{2\pi}{5}\right)\left(\frac{3}{2}\right)} + (2)\left(\frac{1}{5}\right) \operatorname{sinc}\left(k\frac{1}{5}\right) e^{jk\left(\frac{2\pi}{5}\right)\left(\frac{5}{2}\right)}$$



$$F[k] = (-1)\left(\frac{2}{5}\right) \operatorname{sinc}\left(k\frac{2}{5}\right) + (1)\left(\frac{2}{5}\right) \operatorname{sinc}\left(k\frac{2}{5}\right) e^{-jk\left(\frac{2\pi}{5}\right)(2)} + (1)\left(\frac{1}{5}\right) \operatorname{sinc}\left(k\frac{1}{5}\right) e^{jk\left(\frac{2\pi}{5}\right)\left(\frac{5}{2}\right)}$$

Note: for I(1)(e), could use phasors:

$$\omega=4: X = | \angle 0^\circ, H[j^4] = \frac{j^4}{j^4+5} = 0.624 \angle 51.3^\circ \quad \Psi = 0.624 \angle 51.3^\circ$$

$$\omega=10: X = | \angle -90^\circ, H[j^{10}] = \frac{j^{10}}{j^{10}+5} = 0.894 \angle -26.6^\circ \quad \Psi = 0.894 \angle -63.4^\circ$$

$$y(t) = 0.624 \cos(4t + 51.3^\circ) + 0.894 \cos(10t - 63.4^\circ)$$

8

$$a) \quad \sin(2t+1) = \sin(2(t+\frac{1}{2}))$$

$$A(j\omega) = e^{j\omega\frac{1}{2}} \left(\frac{\pi}{j} \delta(\omega-2) - \frac{\pi}{j} \delta(\omega+2) \right)$$

c) Since average is 0, can use integral property

$$c(t) = 2u(t+2) - 2r(t-1) + 2r(t-2)$$

$$C(j\omega) = \frac{\sum e^{j\omega 2}}{j\omega} - \frac{\sum e^{j\omega 1}}{(j\omega)^2} + \frac{\sum e^{j\omega 2}}{(j\omega)^2}$$

$$\alpha(t) = \frac{d(c(t))}{dt} = \begin{array}{c} \uparrow^2 \\ -2 \end{array} \quad | \quad ?$$

$$B(j\omega) = e^{j2\omega} - (2) \frac{2 \sin(\omega/2)}{\omega} e^{-j3\omega/2} ; \quad \sin(\frac{\omega}{2}) = \frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}$$

$$= e^{j2\omega} - \frac{2}{j\omega} (e^{-j\omega} - e^{-j3\omega})$$

$$C(j\omega) = \frac{B(j\omega)}{j\omega} = \frac{e^{j2\omega}}{j\omega} - \frac{2e^{-j\omega}}{(j\omega)^2} + \frac{2e^{-j3\omega}}{(j\omega)^2}$$

$$v) \quad 10^2 - 4 \cdot 36 < 0 : \text{MOAT} \rightarrow \frac{10(j\omega + 3)}{(j\omega + 5)^2 + (1)^2} = \frac{10(j\omega + 5) - 20(1)}{(j\omega + 5)^2 + (1)^2}$$

$$w(t) = e^{-5t} (10 \cos(t) - 20 \sin(t)) u(t)$$

$$w) \quad 10^2 - 4 \cdot 16 > 0 : \text{FACTOR R} \rightarrow \frac{10(j\omega + 3)}{(j\omega + 2)(j\omega + 8)} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 8}$$

$$A = \lim_{j\omega \rightarrow -2} \frac{10(j\omega + 3)}{j\omega + 8} = \frac{10}{6} = \frac{5}{3} \quad B = \lim_{j\omega \rightarrow -8} \frac{10(j\omega + 3)}{j\omega + 2} = -\frac{50}{-6} = \frac{25}{3}$$

$$(\text{check: } A+B=10 \checkmark \quad 8A+2B=30 \checkmark)$$

$$w(t) = \frac{5}{3} e^{-2t} u(t) + \frac{25}{3} e^{-8t} u(t)$$

$$x) \quad x(t) = (t-1) e^{-6(t-1)} u(t-1)$$

(all $t \geq 1$ shift !)

D)

$$\sin[2n+1] = \sin[2(n+\frac{1}{2})]$$

$$D(e^{jw}) = e^{-jw \cdot \frac{1}{2}} \sum_{k=-\infty}^{\infty} (\delta(w-2-2\pi k) - \delta(w+2-2\pi k))$$

$$\text{f)} \quad \left(\frac{1}{3}\right)^n u[n-2] = \left(\frac{1}{3}\right)^{n-2+2} u[n-2] = \frac{1}{9} \left(\frac{1}{3}\right)^{n-2} u[n-2]$$

$$F(e^{jw}) = \frac{1}{9} \frac{1}{1 - \frac{1}{3}e^{-jw}} e^{-j2w}$$

2)

$$y) \quad \frac{1}{(1-\frac{1}{4}e^{jw})(1+\frac{1}{5}e^{-jw})} = \frac{A}{(1-\frac{1}{4}e^{jw})} + \frac{B}{(1+\frac{1}{5}e^{-jw})}$$

$$A = \lim_{e^{jw} \rightarrow 4} \frac{1}{(1+\frac{1}{5}e^{-jw})} = \frac{1}{1+\frac{4}{5}} = \frac{5}{9} \quad B = \lim_{e^{jw} \rightarrow -5} \frac{1}{(1-\frac{1}{4}e^{jw})} = \frac{1}{1+\frac{5}{4}} = \frac{4}{9}$$

(check: $A+B=1 \checkmark \quad \frac{1}{5}A - \frac{1}{4}B=0 \checkmark$)

$$y[n] = \frac{5}{9} \left(\frac{1}{4}\right)^n u[n] + \frac{4}{9} \left(-\frac{1}{5}\right)^n u[n]$$

$$z) \quad Z(e^{jw}) = \cos(w) + j \sin(3w) = \frac{e^{jw} + e^{-jw}}{2} + \sum_j (e^{j3w} - e^{-j3w})$$

$$z[n] = \frac{1}{2} \delta[n+1] + \frac{1}{2} \delta[n-1] + \frac{5}{2} \delta[n+3] - \frac{5}{2} \delta[n-3]$$

IV)

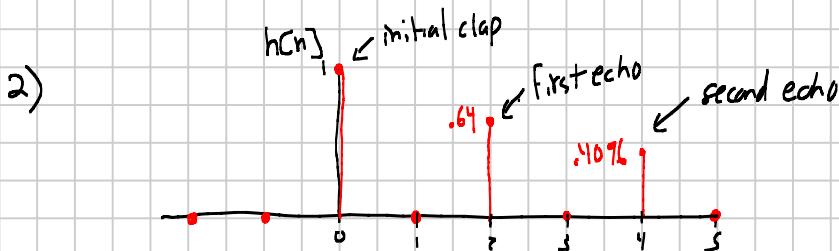
(1)

$$Y(e^{j\omega}) = X(e^{j\omega}) + \frac{16}{25} e^{-j2\omega} Y(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{16}{25} e^{-j2\omega}} = \frac{A}{1 - \frac{4}{5} e^{-j\omega}} + \frac{B}{1 + \frac{4}{5} e^{-j\omega}}$$

$$A = \lim_{e^{-j\omega} \rightarrow \infty} \frac{1}{1 + \frac{4}{5} e^{-j\omega}} = \frac{1}{2} \quad B = \lim_{e^{-j\omega} \rightarrow -\infty} \frac{1}{1 - \frac{4}{5} e^{-j\omega}} = \frac{1}{2}$$

$$h[n] = \frac{1}{2} \left(\frac{4}{5}\right)^n u[n] + \frac{1}{2} \left(-\frac{4}{5}\right)^n u[n]$$



$$3) s_r[n] = \sum_{v=-\infty}^n h[v] = \sum_{v=-\infty}^n \frac{1}{2} \left(\frac{4}{5}\right)^v u[n] + \frac{1}{2} \left(-\frac{4}{5}\right)^v u[n]$$

$$= \frac{1}{2} \left(\frac{1 - \left(\frac{4}{5}\right)^{n+1}}{1 - \frac{4}{5}} u[n] + \frac{1 - \left(-\frac{4}{5}\right)^{n+1}}{1 + \frac{4}{5}} u[n] \right)$$

4) accumulate $h[n]$ Note $s_r[\infty] = \frac{1}{2} \left(\frac{1}{1 - \frac{4}{5}} + \frac{1}{1 + \frac{4}{5}} \right) = \frac{1}{2} \left(5 + \frac{5}{9} \right) = \frac{25}{9}$



5) $\forall n \quad h[n] = 0 \text{ all } n < 0$

6) Yes, $\sum_{n=-\infty}^{\infty} |h[n]|$ in this case = $s_r[\infty]$ which is finite.

$$\text{IV) (1)} \quad H(j\omega) = 1 + \frac{2e^{-j\omega}}{j\omega+6} \quad h(t) = \delta(t) + 2 e^{6(t-1)} u(t-1)$$

$$(2) \quad \frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega+6 + 2e^{-j\omega}}{j\omega+6}$$

$$(j\omega+6) Y(j\omega) = (j\omega+6 + 2e^{-j\omega}) X(j\omega)$$

$$\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 6x(t) + 2x(t-1)$$

$$(3) \quad Y_1(j\omega) = \left(\frac{1}{j\omega+6} \right) \left(1 + \frac{2e^{-j\omega}}{j\omega+6} \right) = \frac{1}{j\omega+6} + \frac{2e^{-j\omega}}{(j\omega+6)^2}$$

$$y_1(t) = e^{-6t} u(t) + 2(t-1) e^{-6(t-1)} u(t-1)$$

as on
test given
in class

(4) Yes: $h(t) = 0$ for $t < 0$

(5) Yes: $\int_{-\infty}^{\infty} |h(t)| dt$ is finite

What I meant to have: $H(j\omega) = 1 + \frac{e^{j\omega} + e^{-j\omega}}{j\omega+6}$

$$(1) \quad h(t) = \delta(t) + e^{-6(t+1)} u(t+1) + e^{-6(t-1)} u(t-1)$$

$$(2) \quad (j\omega+6) Y(j\omega) = (j\omega+6 + e^{j\omega} + e^{-j\omega}) X(j\omega)$$

$$\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 6x(t) + x(t+1) + x(t-1)$$

$$(3) \quad Y_1(j\omega) = \left(\frac{1}{j\omega+6} \right) \left(1 + \frac{e^{j\omega}}{j\omega+6} + \frac{e^{-j\omega}}{j\omega+6} \right)$$

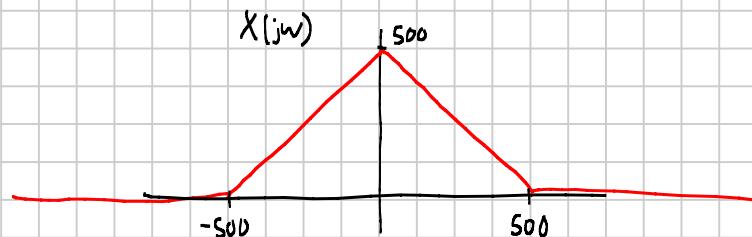
$$y_1(t) = e^{-6t} u(t) + (t+1) e^{-6(t+1)} u(t+1) + (t-1) e^{-6(t-1)} u(t-1)$$

(4) No $h(t) \neq 0$ between $-1 < t < 0$

(5) YES $\int_{-\infty}^{\infty} |h(t)| dt$ is finite

IV)

(1)

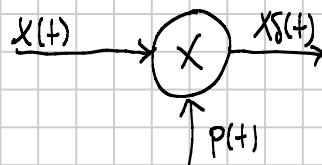


$X(jw)$ purely real means $x(t)$ is even

(2) Energy $\frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw =$ Area under two parabolas w/ $B=500$, $A=500^2$

$$= \frac{1}{2\pi} \left((2) \left(\frac{1}{3}\right) (500)(500)^2 \right) = \frac{500^3}{3\pi} = \frac{125000000}{3\pi}$$

(3)



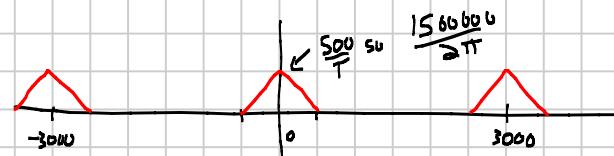
$$\omega_s > 2.500 \text{ rad/sec}$$

$$\omega_s > 1000$$

$$T_s = \frac{2\pi}{\omega_s} = \frac{\pi}{500}$$

(4) if $\omega_s = 3000$,

$$T = \frac{2\pi}{3000}$$



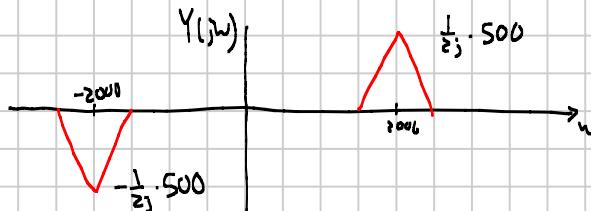
(5) if $\omega_s = 750$

note different scales:

$$T = \frac{2\pi}{750}$$



(6)

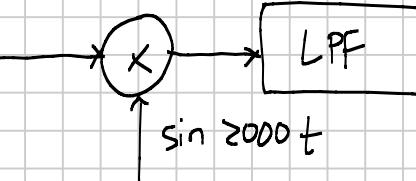


Gain = 2, Cutoff between 500 and 3500 rad/sec

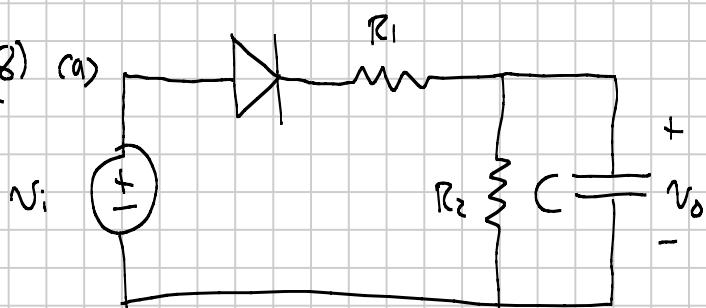
(7)

Component:

$$g(t)$$



(8) (a)



(b) Only works with Full AM

(c) SO many