Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280L Summer 2017
Test II

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

For this test, you should not leave unevaluated convolution sums/integrals. Unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$

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Problem I: [25 *pts.*] Fourier and Periodic Signals

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below. You are not allowed to have complex exponentials or j's in any time domain representations either.

(1) Determine the non-zero Fourier series coefficients $A[k]$ for the signal:

$$a(t) = (\sin(3t) + 1)^2$$

Be sure to clearly indicate the fundamental frequency.

(2) Determine the Fourier transform representation $B(j\omega)$ for the signal:

$$b(t) = (\sin(3t) + 1)^2$$

(3) Determine the time-domain representation $c(t)$ for a periodic signal with a period of 6 seconds and non-zero Fourier series coefficients $C[k]$ of:

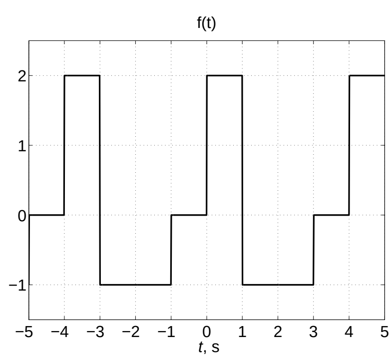
$$C[k] = \begin{cases} 4-2j & k=4 \\ 1 & k=1 \\ 2 & k=0 \\ 1 & k=-1 \\ 4+2j & k=-4 \end{cases}$$

(4) Determine the time-domain representation $d(t)$ for a periodic signal that has a Fourier transform $D(j\omega)$ of:

$$D(j\omega) = \sum_{k=-2}^{2} \delta(\omega - k)$$

Your function for $d(t)$ must *not* contain any j's.

(5) Determine an expression for the Fourier series coefficients $F[k]$ of the periodic signal $f(t)$ graphed below. Remember - you are not allowed to have any unevaluated integrals, summations, or convolutions in your expression for the coefficients. Be sure to indicate the fundamental frequency of the signal.



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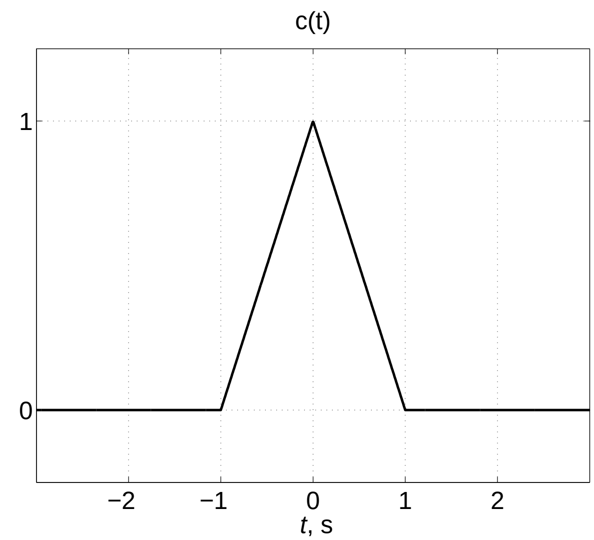
Problem II: [20 *pts.*] Fourier Transforms

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below.

(1) Find the Fourier transform for the following signals:

- $a(t) = e^{-5t}u(t-1)$
- $c(t)$ as shown below:



(2) Find the inverse Fourier transform for the following:

- $X(j\omega) = 10\frac{j\omega+2}{(j\omega)^2+8j\omega+12}$
- $Y(j\omega) = 10\frac{j\omega+2}{(j\omega)^2+8j\omega+20}$
- $Z(j\omega) = \frac{1-e^{-j\omega}}{j\omega+5}$

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Problem III: [30 pts.] System Analysis

(1) Assume you have some system S_1 and observe the following input-output pairing:

$$x_1(t) = e^{-t}u(t) \qquad\qquad y_1(t) = 2e^{-t}u(t) - 2e^{-2t}u(t)$$

(a) Find the transfer function $H_1(j\omega)$ for the system.
(b) Find the impulse response $h_1(t)$ for the system.
(c) Find a differential equation that relates $y_1(t)$ and its derivatives to $x_1(t)$ and its derivatives.

(2) Assume you have some system S_2 with a transfer function:

$$H_2(j\omega) = \frac{j\omega}{j\omega + 5}$$

(a) Find the impulse response $h_2(t)$ for the system.
(b) Find the step response $s_{r2}(t)$ for the system.
(c) Find the response of the system to an input $x_2(t) = e^{-5t}\cos(2t)u(t)$.

(3) System S_3 is a generic unity gain high pass filter and has a transfer function of $H_3(j\omega) = \frac{j\omega}{j\omega+a}, a > 0$. A generic right-sided exponential $x_3(t) = e^{-bt}u(t), b > 0$ is put through the system. Determine the generic form of the output from this system, $y_3(t)$. *Note:* there may be a special case to consider.[1]

(4) A system S_4 has an impulse response of:

$$h_4(t) = e^{-(t-1)}u(t-1)$$

(a) Find the step response $s_{r4}(t)$ for the system.
(b) Find the response to an input $x_4(t) = u(t+1) - u(t-1)$.
(c) Is the system S_4 causal?
(d) Is the system S_4 stable?

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[1] And by "may be," I mean "is."

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Problem IV: [25 pts.] Sampling and Modulation

Assuming you have some signal $x(t)$:

$$x(t) = \frac{\sin(1000t)}{1000t} = \text{sinc}\left(\frac{1000}{\pi}t\right)$$

(1) TIME DOMAIN: Sketch $x(t)$ for $|t| < \frac{3\pi}{1000}$. Clearly label the peak value of $x(t)$ and also the times at which $x(t) = 0$ over that domain.

(2) FREQUENCY DOMAIN: Find and sketch the magnitude spectrum of $x(t)$, $X(j\omega)$. Be sure to label your axes.

(3) BAND-LIMIT: Next, determine if the frequency domain of $x(t)$ band-limited. If you believe it is, indicate the band-limit. If you do not believe it is, you may need to adjust your belief system.

(4) SAMPLING: Draw a block diagram showing a system that uses impulse sampling with a period of T_s to sample $x(t)$ above. That is, the signal $x(t)$ is multiplied by an impulse train $p(t)$ where $p(t)$ is

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Then determine the minimum sampling rate ω_S in rad/s that would be required in order to have any chance of perfectly recovering the message signal.

(5) SAMPLING II: Draw the magnitude spectrum of the output of this sampling system if the input is $x(t)$ and the sampling rate ω_S is 1500 rad/s. Be sure to label your axes and include units where appropriate. *Note:* 1500 rad/s may or may not be a sampling rate that allows for perfect recovery. Call this output $y(t)$.

(6) FILTERING: If you put your signal $y(t)$ through an ideal low pass filter with a cutoff frequency of 2000 rad/s and a magnitude of 1 to get an output called $z(t)$, sketch the magnitude spectrum $Z(j\omega)$ and determine a formula for $z(t)$. *Hint:* there's an easier way to go about this and several harder ways.

(7) AMPLITUDE MODULATION: Assume your original signal $x(t)$ is multiplied by $a(t) = \cos(1500t)$ to produce a new signal $b(t) = a(t) \cdot x(t)$. Sketch and label the magnitude spectra $A(j\omega)$ and $B(j\omega)$.

(8) PUTTING IT ALL TOGETHER: If you were to want to sample this signal $b(t)$ with another impulse sampler $p_2(t)$, what is the minimum sampling rate ω_{S2} of $p_2(t)$ that would guarantee that $b(t)$ could be perfectly recovered?