Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280L Summer 2017

Test I

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Name (please print)____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or the convolution integral. Also, unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$
- $c(t)$ is the "unit" cubic $\frac{1}{6}t^3 \cdot u(t)$

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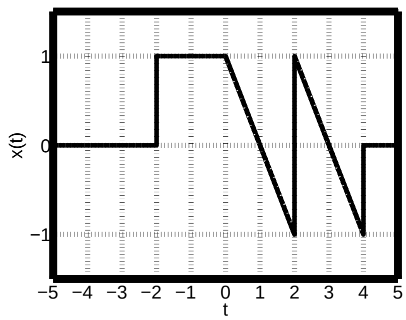
Problem I: [25 *pts.*] Signals

(1) Given the following graph of $x(t)$, and noting that $x(t) =$ for all $|t| \geq 5$, write an equation for the signal using singularity functions.



$x(t) =$

(2) Given $x(t)$, make a sketch of the even part of $x(t)$ and a sketch of the odd part of $x(t)$. Be sure to label your sketches.

(3) Given $x(t)$, accurately sketch the transformed versions below. Be sure to put labels and values on your axes.

(x) $y(t) = 2x(t/2 + 1)$
(y) $z(t) = x(1 - 2t)$

(4) For each of the following signals, determine if they are periodic or not. If they are periodic, state the period:

(a) $a(t) = \cos(3t) \cdot \sin(9t)$
(b) $b(t) = \cos(3\pi t) + \sin(9\pi t)$
(c) $c(t) = \cos(3t) + \sin(9\pi t)$

(5) For each of the following signals, determine if they are energy signals, power signals, or neither. If a signal is an energy signal, also give its total energy E_∞; if a signal is a power signal, calculate its overall average power P_∞.

(k) $k(t) = 1 + \sin(t)$
(l) $l(t) = e^{-2t}u(t)$
(m) $m(t) = e^{-2|t|}$

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Problem II: [25 *pts.*] System Classifications

(1) For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
$y(t) = x^2(t)$					
$y(t) = x(t^2)$					
$y(t) = \int_0^t x(\tau)\, d\tau$					
$y(t) = \frac{1}{2}\left(x(t) - x(t-2)\right)$					
$y(t) = \mathrm{atan}(x(t))$					

(2) Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response $h_i(t)$ or the step response $s_{r,i}(t)$. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Stable?	Memoryless?	Causal?
$h_1(t) = e^{-\lvert t\rvert}$			
$h_2(t) = \frac{1}{t+1}u(t)$			
$s_{r,3}(t) = tu(t)$			
$s_{r,4}(t) = 2u(t)$			

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Problem III: [25 *pts.*] Basic Convolution and Correlation

(1) Write an integral formula to calculate the convolution of signals $x(t)$ and $y(t)$:

$x(t) * y(t) =$

(2) Write an integral formula to calculate the correlation function of signals $x(t)$ and $y(t)$:

$\phi_{xy}(t) =$

(3) Write a formula to calculate the correlation function of signals $x(t)$ and $y(t)$ using the convolution operator:

$\phi_{xy}(t) =$

(4) Given the following functions:

$$x(t) = u(t) - u(t-2) \qquad\qquad y(t) = r(t+1) - r(t) - r(t-2) + r(t-3)$$

- Make labeled sketches of $x(t)$ and $y(t)$, then determine formulas for the following convolutions and correlations but do not make sketches of them:
- $A(t) = x(t) * x(t)$
- $B(t) = x(t) * y(t)$
- $C(t) = \phi_{xx}(t)$
- $D = \phi_{yy}(0)$ (note this is the autocorrelation, not the autocorrelation function!)
- What is the range of valid values for the measure of correlation between any two real signals?
- Determine the measure of correlation between $x(t)$ and $y(t)$.

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Problem IV: [25 *pts.*] System Analysis

A linear, time-invariant system S has an impulse response of:

$$h_1(t) = u(t+1)$$

and an input signal:

$$x(t) = e^{-t}(u(t) - u(t-1))$$

is applied to the system.

(1) Make a sketch of both $x(t)$ and $h_1(t)$. Note that $e \approx 2.72$ and also that $e^{-1} = 1/e \approx 0.368$.

(2) Is the system stable? Why do you believe that to be the case?

(3) Is the system causal? Why do you believe that to be the case?

(4) Determine an expression for the output of this system to the input $x_1(t)$ given above - call this output $y_1(t)$. You must evaluate any required integrals but are not required to simplify any algebra.

(5) Make a sketch of $y_1(t)$. Be sure to label your axes.

(6) Determine an expression for the output of different system to that same input if the impulse response for System 2 is:

$$h_2(t) = u(t)$$

Call this output $y_2(t)$. You must evaluate any required integrals but are not required to simplify any algebra.

(7) Determine an expression for the output of a third system if the impulse response is for System 3 is:

$$h_3(t) = e^{-t}u(t)$$

Call this output $y_3(t)$. You must evaluate any required integrals but are not required to simplify any algebra.