ECE 280
Summer 2017 Test II

Problem 1

Tuesday, June 20, 2017 9:38 AM

I) (1) $(\sin(3t)+1)^2 = \sin^2(3t) + 2\sin(3t) + 1$

$$= \frac{1}{2} - \frac{\cos(6t)}{2} + 2\sin(3t) + 1$$

$$= \frac{3}{2} + 2\sin(3t) - \frac{\cos(6t)}{2}$$

$$w_0 = 3 \quad A[k] = \begin{cases} k=0, \ 3/2 & k=1 \ \ 1/j \quad k=2 \ \ -1/4 \\ & k=-1 \ \ -1/j \quad k=-2 \ \ -1/4 \end{cases}$$

(2) $b(t) = a(t)$; $B(jw) = \left(\frac{3}{2}\right)\left(2\pi\delta(w)\right) + \frac{2\pi}{j}\delta(w-3) - \frac{2\pi}{j}\delta(w+3) - \frac{\pi}{2}\delta(w-6) - \frac{\pi}{2}\delta(w+6)$

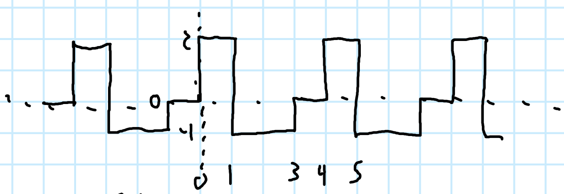
(3) $T=6,\ w_0 = \frac{\pi}{3}$ $\qquad c(t) = 8\cos(4\tfrac{\pi}{3}t) + 4\sin(4\tfrac{\pi}{3}t) + 2\cos(\tfrac{\pi}{3}t) + 2$

(4) $D(jw) = \delta(w-2) + \delta(w-1) + \underbrace{\delta(w)}_{1/2\pi} + \delta(w+1) + \delta(w+2)$

$\frac{1}{\pi}\cos(2t)$ $\qquad \frac{1}{\pi}\cos(t)$

so $d(t) = \frac{1}{2\pi} + \frac{1}{\pi}\cos(t) + \frac{1}{\pi}\cos(2t)$

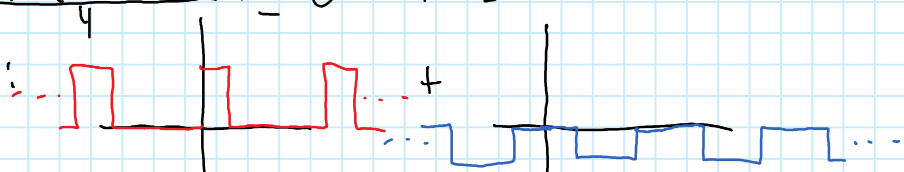
(5)



$T = 4,\ w_0 = \pi/2$

$$\text{average} = \frac{(2)(1) + (-1)(2) + (0)(1)}{4} = 0 \qquad F[0] = 0$$

Mult. options; here's one:



$T_1 = 1/2 \quad t_0 = 1/2 \quad A = 2$ $\qquad\qquad$ $T_1 = 1 \quad t_0 = 2 \quad A = -1$

$$F_1[k] = 2\frac{\sin(kw_0T_1)}{k\pi}e^{-jkw_0t_0} = 2\frac{\sin(k\pi/4)}{k\pi}e^{-jk\pi/4}$$

$$F_2[k] = -1\frac{\sin(kw_0T_1)}{k\pi}e^{-jkw_0t_0} = -\frac{\sin(k\pi/2)}{k\pi}e^{-jk\pi}$$

$$F[k] = \frac{2\sin(k\pi/4)e^{-jk\pi/4}}{k\pi} - \frac{\sin(k\pi/2)e^{-jk\pi}}{k\pi}$$

or $\frac{1}{2}\operatorname{sinc}\left(\frac{k}{4}\right)e^{-jk\pi/4} - \frac{1}{2}\operatorname{sinc}\left(\frac{k}{2}\right)e^{-jk\pi}$

Problem 2

Tuesday, June 20, 2017 9:57 AM

II (1) $a(t) = e^{-5t}u(t-1) = e^{-5(t-1+1)}u(t-1) = e^{-5}e^{-5(t-1)}u(t-1)$

$$A(j\omega) = \boxed{\frac{e^{-5}e^{-j\omega}}{j\omega+5}}$$

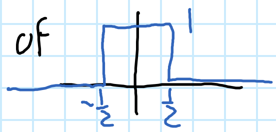
(2) $c(t)$ note $f(t) = \frac{dc(t)}{dt} =$

$$F(j\omega) = \frac{2\sin(\omega/2)}{\omega}e^{j\omega/2} - \frac{2\sin(\omega/2)}{\omega}e^{-j\omega/2} = \frac{4j\sin(\omega/2)\sin(\omega/2)}{\omega}$$

$$= \frac{4j\sin^2(\omega/2)}{\omega}$$

$$C(j\omega) = \frac{1}{j\omega}F(j\omega) + \pi\delta(\omega)F(j0) = \boxed{\frac{4\sin^2(\omega/2)}{\omega^2} + 0 \quad \text{since } F(j0)=0}$$

alt: $c(t)$ is convolution of $g(t)=$



with itself

$c(t) = g(t) * g(t)$ $\quad C(j\omega) = G(j\omega)\cdot G(j\omega)$

$G(j\omega) = \frac{2\sin(\omega/2)}{\omega}$ $\quad \boxed{C(j\omega) = \frac{4\sin^2(\omega/2)}{\omega^2}}$

(3) $X(j\omega) = \frac{10(j\omega+2)}{(j\omega+2)(j\omega+6)} = \frac{10}{(j\omega+6)}$ $\quad x(t) = 10e^{-6t}u(t)$

(4) $Y(j\omega) = \frac{10(j\omega+2)}{(j\omega)^2+8j\omega+20} \rightarrow \frac{10(j\omega+2)}{(j\omega+4)^2+(2)^2} = \frac{A(j\omega+4)+B(2)}{(j\omega+4)^2+(2)^2}$

$8^2-4\cdot 20<0$

MUAT $\quad A=10 \quad \cancel{10j\omega}+40+2B = \cancel{10j\omega}+20$

$B=-10$

$$y(t) = e^{-4t}\left(10\cos(2t) - 10\sin(2t)\right)u(t)$$

(5) $Z(j\omega) = \frac{1-e^{-j\omega}}{j\omega+5}$ $\quad \boxed{z(t) = e^{-5t}u(t) - e^{-5(t-1)}u(t-1)}$

Problem 3

Tuesday, June 20, 2017 9:58 AM

(1) $x_1(t) = e^{-t}u(t)$ $\quad y_1(t) = 2e^{-t}u(t) - 2e^{-2t}u(t)$

$X_1(j\omega) = \frac{1}{j\omega+1}$ $\quad Y_1(j\omega) = \frac{2}{j\omega+1} - \frac{2}{j\omega+2} = \frac{2}{(j\omega+1)(j\omega+2)}$

(a) $H = \frac{Y_1}{X_1} = \frac{2}{(j\omega+1)(j\omega+2)} \frac{(j\omega+1)}{1} = \frac{2}{j\omega+2}$

(b) $h_1(t) = 2e^{-2t}u(t)$

(c) $(j\omega+2)Y_1 = 2X_1 \longrightarrow \frac{dy_1(t)}{dt} + 2y_1(t) = 2x_1(t)$

(2) $H_2(j\omega) = \frac{j\omega}{j\omega+5} = \frac{j\omega+5-5}{j\omega+5} = 1 - \frac{5}{j\omega+5}$

(a) $h_2(t) = \delta(t) - 5e^{-5t}u(t)$

(b) $s_r(t) = \mathcal{F}^{-1}\left\{\frac{1}{j\omega} H_2(j\omega)\right\} = \mathcal{F}^{-1}\left\{\left(\frac{1}{j\omega+5}\right)\right\} = e^{-5t}u(t)$

(c) $x_2 = e^{-5t}\cos(2t)u(t)$ $\quad X_2(j\omega) = \frac{(j\omega+5)}{(j\omega+5)^2+(2)^2}$

$Y_2 = H_2 X_2 = \frac{j\omega}{(j\omega+5)^2+(2)^2} = \frac{1(j\omega+5) - \frac{5}{2}(2)}{(j\omega+5)^2+(2)^2}$

$y_2 = e^{-5t}\left(\cos(2t) - 5/2\sin(2t)\right)u(t)$

(3) $H_3(j\omega) = \frac{j\omega}{j\omega+a}$ $\quad X_3(j\omega) = \frac{1}{j\omega+b}$ $\quad Y(j\omega) = \frac{j\omega}{(j\omega+a)(j\omega+b)}$

if $a \neq b$, $Y(j\omega) = \frac{\frac{-a}{-a+b}}{j\omega+a} + \frac{\frac{-b}{a-b}}{j\omega+b} = \frac{1}{a-b}\left(\frac{a}{j\omega+a} - \frac{b}{j\omega+b}\right)$

$y(t) = \frac{1}{a-b}\left(ae^{-at} - be^{-bt}\right)u(t)$

if $a=b$, $Y(j\omega) = \frac{j\omega}{(j\omega+a)^2} = \frac{j\omega+a}{(j\omega+a)^2} - \frac{a}{(j\omega+a)^2} = \frac{1}{j\omega+a} - \frac{a}{(j\omega+a)^2}$

$y(t) = e^{-at}u(t) - ate^{-at}u(t)$

(4) $h_4(t) = e^{-(t-1)}u(t-1)$ $\quad H_4(j\omega) = \frac{e^{-j\omega}}{j\omega+1}$

$s_{r4}(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = \int_{-\infty}^{t} e^{-(\tau-1)}u(\tau-1)\,d\tau = u(t-1)\int_{1}^{t} e^{-(\tau-1)}\,d\tau =$
$u(t-1)\left[-e^{-(\tau-1)}\right]_1^t = u(t-1)\left(-e^{-(t-1)}+1\right) = \left(1-e^{-(t-1)}\right)u(t-1)$

(b) $y_4(t) = s_{r4}(t+1) - s_{r4}(t-1)$

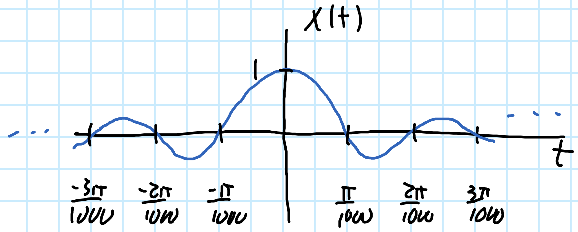
(c) yes $h(t) = 0 \quad t<0$

(d) yes $\int_{-\infty}^{\infty} |h(t)|\,dt$ is finite.

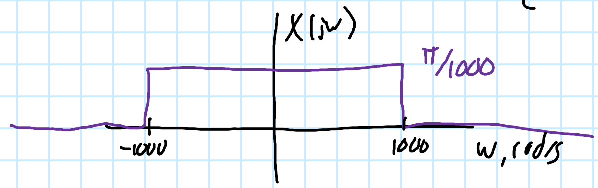
Problem 4

Tuesday, June 20, 2017 10:08 AM

(1) max of 1 @ $t=0$,
0 $t = \frac{n\pi}{1000}$

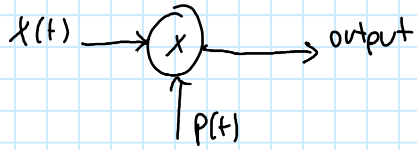


(2) $x(t) = \frac{\sin(1000t)}{1000t} = \frac{\pi}{1000} \frac{\sin(1000t)}{\pi t}$ $\quad X(j\omega) = \begin{cases} \pi/1000 & |\omega| < 1000 \\ 0 & |\omega| > 1000 \end{cases}$



(3) Band-limited; limit is 1000 rad/s

(4)

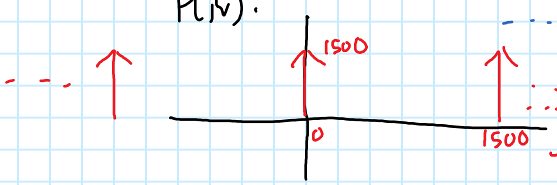


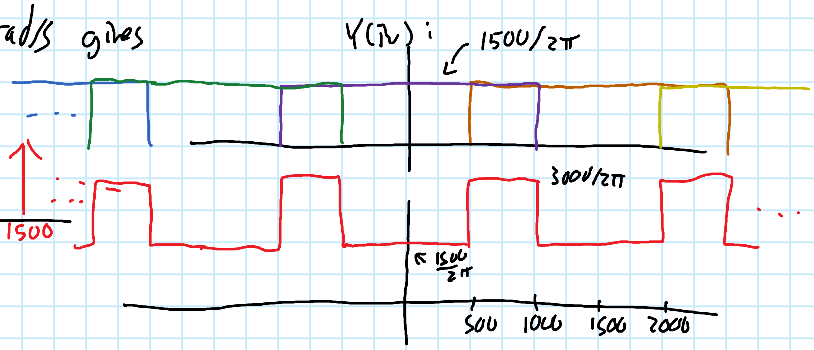
$\omega_s > 2 \cdot \omega_x$

$\omega_s > 2000$ rad/s

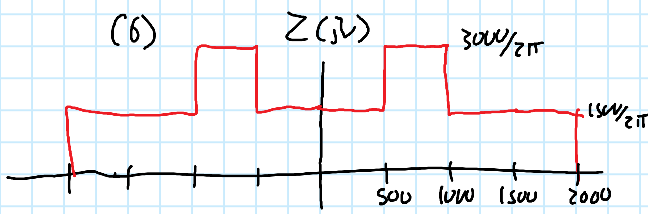
(5) $p(t)$ w/ $\omega_s = 1500$ rad/s gives

P(jω):



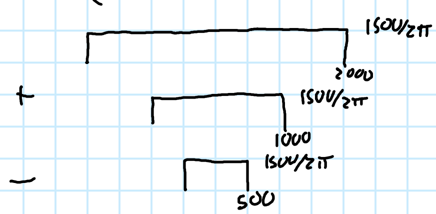


(6)

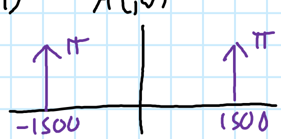


many ways to handle

$$\frac{1500}{2\pi}\left(\frac{\sin(2000t)}{\pi t} + \frac{\sin(1000t)}{\pi t} - \frac{\sin(500t)}{\pi t}\right)$$



(7) New Scale!



(8) $\omega_s > 5000$ rad/s