Problem 1

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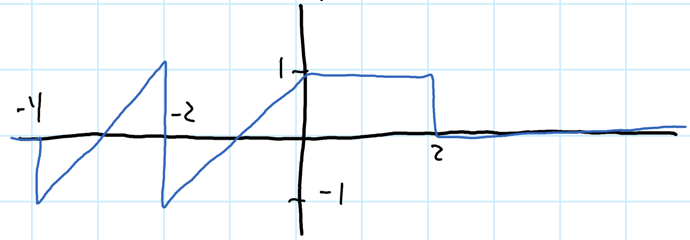
Problem I:

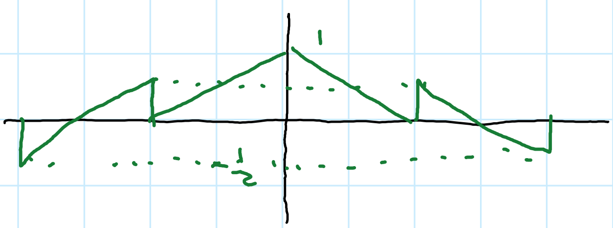
(1) $x(t) = u(t+2) - r(t) + 2u(t-2) + r(t-4) + u(t-4)$

(2) $x(t)$

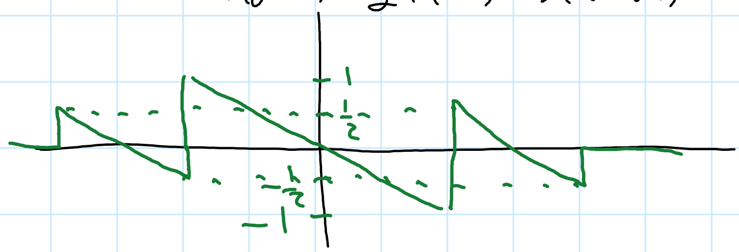
$x(-t)$



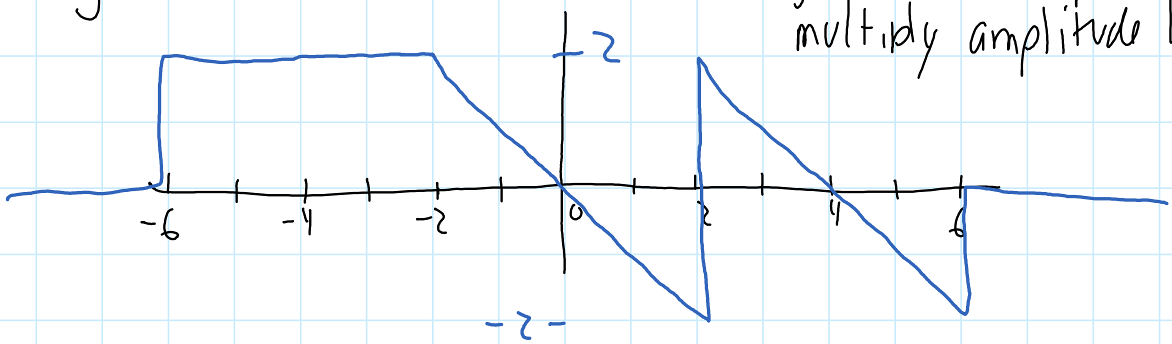
$x_e(t) = \frac{1}{2}(x(t) + x(-t))$



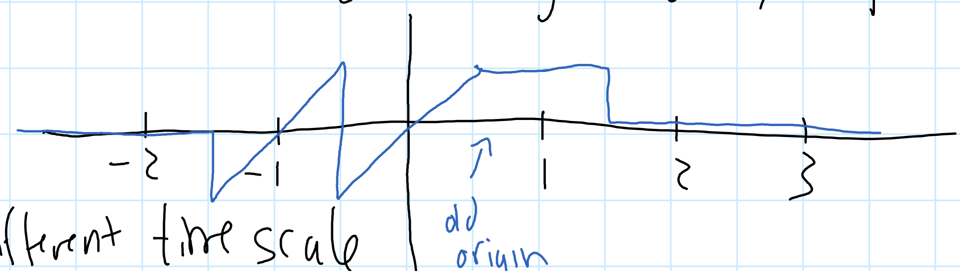
$x_o(t) = \frac{1}{2}(x(t) - x(-t))$



(3) $y(t) = 2x\left(\frac{1}{2}(t+2)\right)$ shift origin to -2, expand by $2\times$, multiply amplitude by 2



$z(t) = x\left(-2\left(t - \frac{1}{2}\right)\right)$ shift origin to $\frac{1}{2}$, compress by $2\times$, reverse



* note different time scale

* note different time scale | dd origin

(4) (a) periodic; components are periodic w/ frequencies 9 ± 3 so 6 and 12, $\omega_0 = 6$ $T = \pi/3$

(b) periodic; components are periodic w/ frequencies 3π and 9π $\omega_0 = 3\pi$ $T = 2/3$

(c) Not periodic; $9\pi/3$ not rational

(5) $k(t)$ is periodic, so not energy; $T = 2\pi$

$$P_\infty = \frac{1}{2\pi}\int_0^{2\pi} \underbrace{(1+\sin(t))^2}_{\text{always non-negative}} dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left(\sin^2(t) + 2\sin(t) + 1\right) dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{1-\cos(2t)}{2} + 2\sin(t) + 1 \, dt$$

$$= \left(\frac{1}{2\pi}\right)\left(\frac{t}{2} - \frac{\sin(4t)}{4} + 2\cos(t) + t\right)\Big|_0^{2\pi}$$

$$= \left(\frac{1}{2\pi}\right)\left(\frac{3}{2}\right)(2\pi) = \boxed{\frac{3}{2}}$$

$l(t) = e^{-2t}u(t)$ try E_∞:

$$E_\infty = \int_{-\infty}^{\infty} |e^{-2t}u(t)|^2 dt = \int_{-\infty}^{\infty} e^{-4t}u(t)\,dt = \int_0^{\infty} e^{-4t}u(t)$$

$= 1/4$ $\boxed{\text{Energy}}$

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$m(t) = e^{-2|t|} = e^{-2t}u(t) + e^{2t}u(-t)$

twice the area under the square of $l(t)$ so

$E_{\infty} = \frac{1}{2}$ Energy

Problem 2

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Sys	L	TI	S	M	C
$x^2(t)$	N	Y	Y	Y	Y
$x(t^2)$	Y	N	Y	N	N
$\int_0^t x(\tau)\,d\tau$	Y	N	N	N	N
$\frac{x(t)-x(t-2)}{2}$	Y	Y	Y	N	Y
$\operatorname{atan}(x(t))$	N	Y	Y	Y	Y

Sys Fun	h	S	M	C
$h=e^{-\lvert t\rvert}$	$e^{-\lvert t\rvert}$	Y	N	N
$\frac{1}{t+1}u(t)$	$\frac{1}{t+1}u(t)$	N	N	Y
$s_r = t\,u(t)$	$h=u(t)$	N	N	Y
$s_r = 2u(t)$	$h=2\delta(t)$	Y	Y	Y

Problem 3

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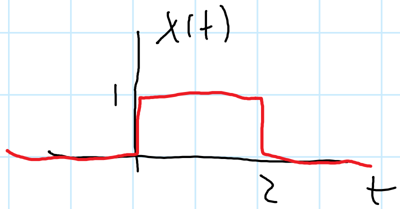
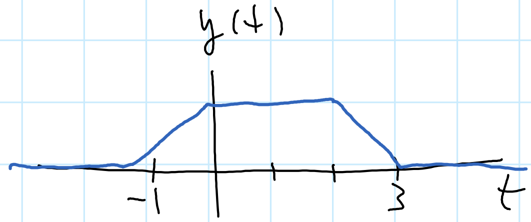
(1) $x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$ or $\int_{-\infty}^{\infty} x(t-\tau)\, y(\tau)\, d\tau$

(2) $\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$

(3) $\phi_{xy}(t) = x(t) * y(-t)$

(4)

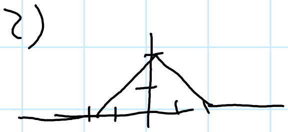




$$A = x(t) * x(t) = (u(t) - u(t-2)) * (u(t) - u(t-2))$$
$$= r(t) - r(t-2) - r(t-2) + r(t-4)$$
$$= r(t) - 2r(t-2) + r(t-4)$$

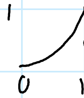
B: $$x(t) * y(t) = (u(t) - u(t-2)) * (r(t+1) - r(t) - r(t-2) + r(t-3))$$
$$= q(t+1) - q(t) - q(t-2) + q(t-3) - q(t-1) + q(t-2) + q(t-4) - q(t-5)$$
$$= q(t+1) - q(t) - q(t-1) + q(t-3) + q(t-4) - q(t-5)$$

C: note $x(-t) = u(t+2) - u(t)$
$$\phi_{xx}(t) = x(t) * x(-t) = (u(t) - u(t-2)) * (u(t+2) - u(t))$$
$$= r(t+2) - r(t) - r(t) + r(t-2)$$
$$= r(t+2) - 2r(t) + r(t-2)$$



D: $$\phi_{yy}(0) = \int_{-\infty}^{\infty} y^2(t)\, dt = \int_{-1}^{0} (t+1)^2 dt + \int_{0}^{2} (1)^2 dt + \int_{2}^{3} (-t+3)^2 dt$$
$$= \left.\frac{(t+1)^3}{3}\right]_{-1}^{0} + \left. t \right]_{0}^{2} + \left.\frac{-(-t+3)^3}{3}\right]_{2}^{3}$$
$$= \frac{1}{3} + 2 + \frac{1}{3} = \frac{8}{3}$$

$$= \frac{1}{3} + 2 + \frac{1}{3} = \boxed{\frac{8}{3}}$$

(Also recall area under  $\leftarrow t^2 = 1/3$ so

$= 8/3$!

Measure of Correlation $= \dfrac{(\phi_{xy}(t))^2_{max}}{\phi_{xx}(0)\ \phi_{yy}(0)}$ → happens when x is centered on y; $\phi_{xy,max} = 2$

can be [0,1]

$\phi_{xx}(0) = 2$, $\phi_{yy}(0) = 8/3$ from D

$$= \frac{4}{2 \cdot \frac{8}{3}} = \frac{12}{16} = \boxed{0.75}$$

Problem 4

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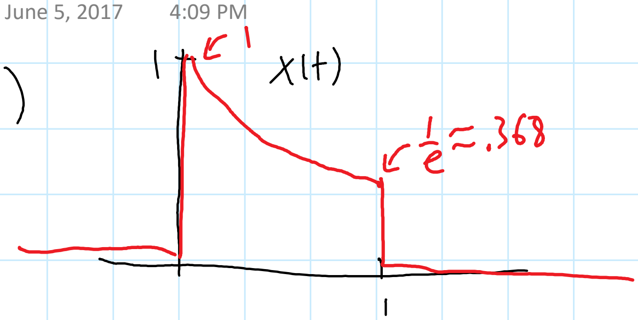
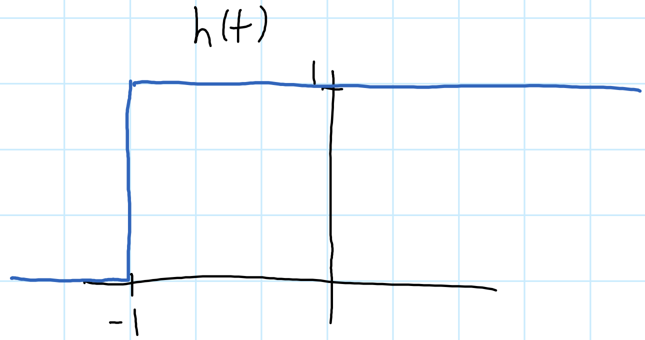
(1)





(2) unstable: $\int_{-\infty}^{\infty} |h(t)|\,dt \to \infty$

(3) non-causal; $h(t) \neq 0$ for all $t < 0$

(4) $y_1(t) = \int_{-\infty}^{\infty} x(\tau)\,h_1(t-\tau)\,d\tau$ or $\int_{-\infty}^{\infty} x(t-\tau)\,h_1(\tau)\,d\tau$

$$\int_{-\infty}^{\infty} e^{-\tau}\left(u(\tau)-u(\tau-1)\right) u(t-\tau+1)\,d\tau =$$

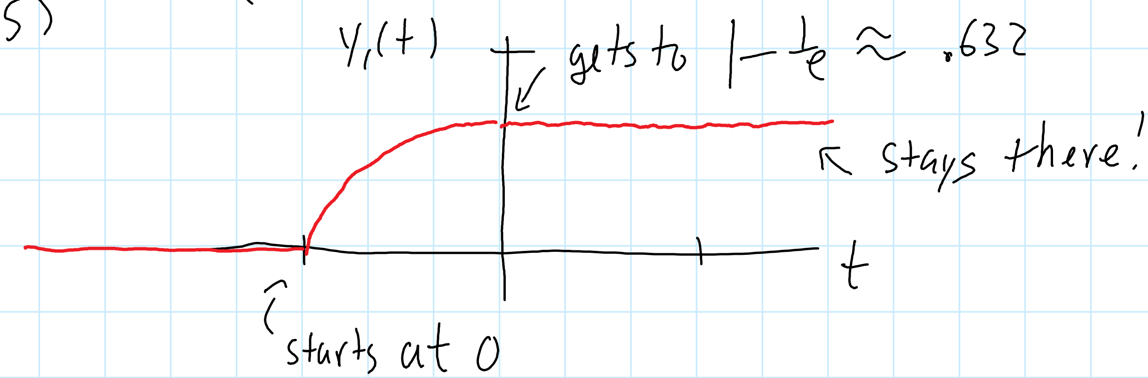
$$\int_{-\infty}^{\infty} e^{-\tau}\,u(\tau)\,u(t-\tau+1)\,d\tau - \int_{-\infty}^{\infty} e^{-\tau}u(\tau-1)\,u(t-\tau+1)\,d\tau$$

$$u(t+1)\int_{0}^{t+1} e^{-\tau}\,d\tau - u(t)\int_{1}^{t+1} e^{-\tau}\,d\tau$$

$$u(t+1)\left[-e^{-\tau}\right]_0^{t+1} - u(t)\left[-e^{-\tau}\right]_1^{t+1}$$

$$u(t+1)\left(-e^{-(t+1)}+1\right) - u(t)\left(-e^{-(t+1)}+e^{-1}\right)$$

(5)



(6) since $h_2(t) = h_1(t-1)$, $y_2(t) = y_1(t-1)$

$$= u(t)(-e^{-t}+1) - u(t-1)(-e^{-t}+e^{-1})$$

(7) $x(t) = e^{-t}(u(t)-u(t-1))$ $\quad h_3(t) = e^{-t}u(t)$

$$y_3(t) = \int_{-\infty}^{\infty} e^{-\tau}(u(\tau)-u(\tau-1))\, e^{-(t-\tau)}\, u(t-\tau)\, d\tau$$

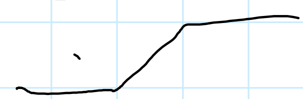
$$= e^{-t}\left(\int_{-\infty}^{\infty} u(\tau)u(t-\tau)\,d\tau - \int_{-\infty}^{\infty} u(\tau-1)u(t-\tau)\,d\tau\right)$$

$$= e^{-t}\left(u(t)\int_0^t d\tau - u(t-1)\int_1^t d\tau\right)$$

$$= e^{-t}\left(u(t)\left[\tau\right]_0^t - u(t-1)\left[\tau\right]_1^t\right)$$

$$= e^{-t}\left(u(t)(t-0) - u(t-1)(t-1)\right)$$

$$\boxed{= e^{-t}\left(t\,u(t) - (t-1)\,u(t-1)\right)}$$

Note: this ends up as a 1 second ramp: 
a decaying exponential