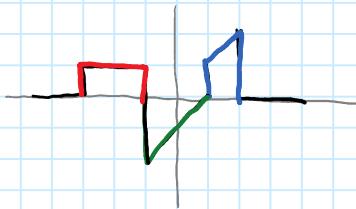


1

$$1) X(t) = u(t+3) - 3u(t+1) + r(t+1) + u(t-1) - 2u(t-2) - r(t-2)$$

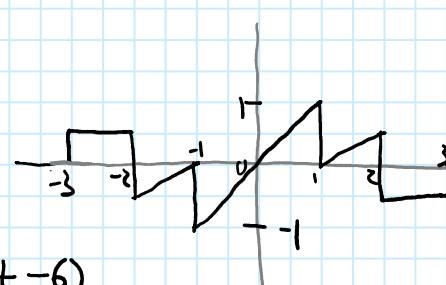
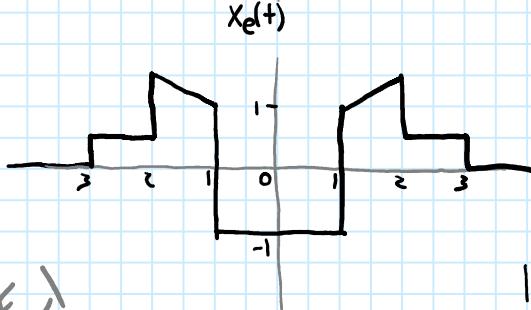
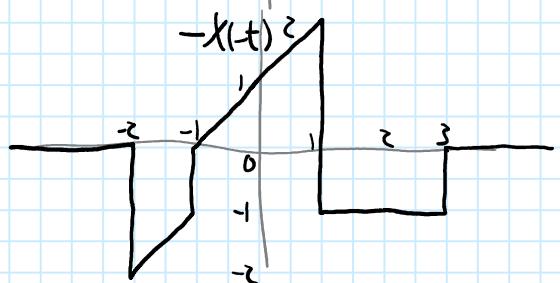
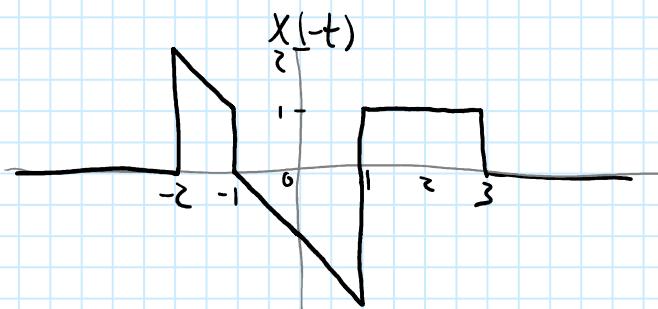
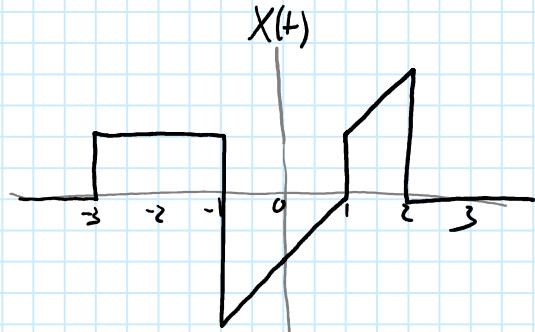
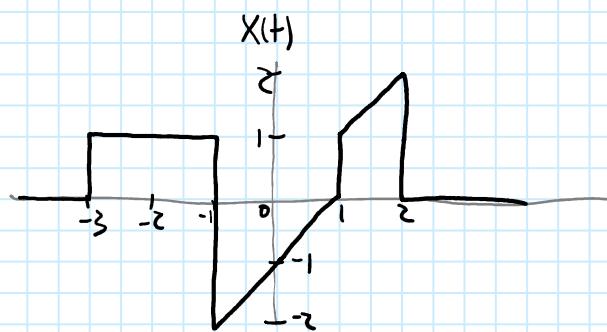
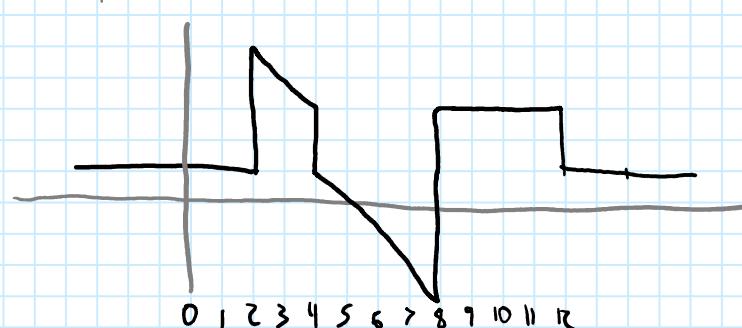
2) ENERGY



$$(2)(1)^2 + \frac{1}{3}(2)(2)^2 + \frac{1}{3}(1)(1.1 + 1.2 + 2 \cdot 2)$$

$$2 + \frac{8}{3} + \frac{7}{3} = 7$$

3)

NOTE
SCALE.S
4
3
2
1
0
-1
-2
-3

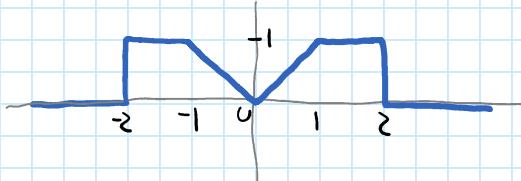
$$1 + 2x(-\frac{1}{2}(t-6))$$

2

1. (a) $\sin(7\pi t) \cdot \cos(3t) = \frac{1}{2} (\sin((7\pi - 3)t) + \sin(7\pi + 3)t)$
 $\frac{7\pi - 3}{7\pi + 3} \neq \text{RATIONAL; NOT PERIODIC}$

1. (b) $\cos(6t) + \sin(15t)$ $\frac{6}{15} = \text{RATIONAL} = \text{PERIODIC}$
 $\omega_1 = 6 \quad \omega_2 = 15 \quad \omega_0 = 3 \quad T = \frac{2\pi}{\omega_0} = \frac{2\pi}{3}$

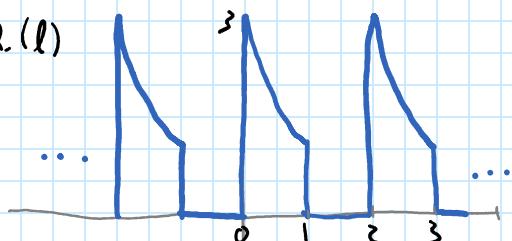
2. (k)



$$E: (1)(1)^2 + \frac{1}{3} ((-1))^2 + \frac{1}{3} (1)(1)^2 + (1)(1)^2$$

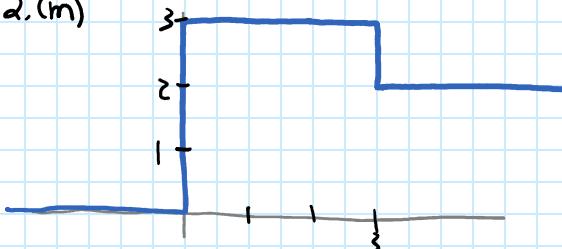
$$E = 2 \frac{2}{3} = \frac{8}{3}$$

2. (l)



$$P: \frac{1}{T} \int_T |x(t)|^2 dt \\ = \frac{1}{2} \int_0^1 9 e^{-2t} dt = \frac{9}{2} \left[-\frac{e^{-2t}}{2} \right]_0^1 = \frac{9}{4}(1 - e^{-2})$$

2. (m)



$$P: \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \left(\int_{-\infty}^0 0 dt + \int_0^3 9 dt + \int_3^{T/2} 4 dt \right)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} (0 + 27 + 2T - 12) = 2$$

YOU CAN IGNORE "EXTRA"
ENERGY PARTS OF POWER SIGNALS, SO

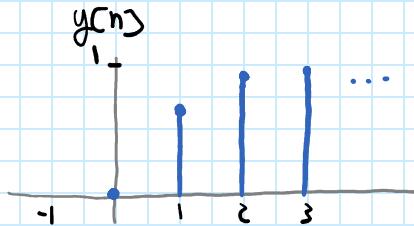
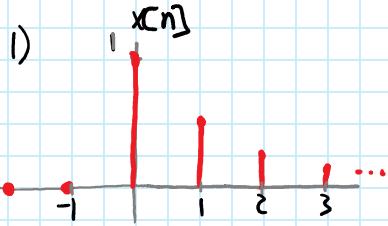
$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_3^{T/2} 4 dt = \lim_{T \rightarrow \infty} \frac{1}{T} (2T - 12) = 2$$

System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
$y(t) = t x(t) + 1$	N	N	N	Y	Y
$y(t) = \int_{-\infty}^t x^2(\tau) d\tau$	N	Y	N	N	Y
$y(t) = \int_{-\infty}^t x(\tau^2) d\tau$	Y	N	N	N	N
$y[n] = x[n - 1] u[n + 1]$	Y	N	Y	N	Y
$y[n] = x[n + 1] u[n - 1]$	Y	N	Y	N	N
$y[n] = \sqrt{\frac{1}{5} \sum_{k=-2}^2 x^2[n - k]}$	N	Y	Y	N	N

System	Stable?	Memoryless?	Causal?
$h_1(t) = 2u(t + 1) - u(t - 1)$	N	N	N
$h_2[n] = (2)^n u[-n]$	Y	N	N
$s_{r,3}(t) = (1 - e^{-t}) u(t)$	Y	N	Y
$s_{r,4}[n] = \frac{1}{2}u[n]$	Y	Y	Y

$$h_{r,3}(t) = e^{-t} u(t)$$

$$h_{r,4}[n] = \frac{1}{2} \delta[n]$$



2) SHORTCUT: $\alpha^n u[n] * \beta^n u[n] = \left(\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right) u[n]$ IF $\alpha \neq \beta$

$$x[n] * y[n] = \left(\frac{1}{2} \right)^n u[n] * \left(u[n] - \left(\frac{1}{3} \right)^n u[n] \right)$$

$$A = \left(\frac{1}{2} \right)^n u[n] * u[n] = \frac{1^{n+1} - \left(\frac{1}{2} \right)^{n+1}}{1 - \frac{1}{2}} u[n] = 2 \left(1 - \left(\frac{1}{2} \right)^{n+1} \right) u[n]$$

$$B = \left(\frac{1}{2} \right)^n u[n] * \left(\frac{1}{3} \right)^n u[n] = \frac{\left(\frac{1}{3} \right)^{n+1} - \left(\frac{1}{2} \right)^{n+1}}{\frac{1}{3} - \frac{1}{2}} = -6 \left(\left(\frac{1}{3} \right)^{n+1} - \left(\frac{1}{2} \right)^{n+1} \right) u[n]$$

$$z[n] = A - B$$

$$= 6 \left(\left(\frac{1}{2} \right)^{n+1} - \left(\frac{1}{3} \right)^{n+1} \right) u[n]$$

LONG WAY:

SIMPLER

$$z[n] = x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[n-k] y[k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\left(\frac{1}{2} \right)^{n-k} u[n-k] \right) \left(u[k] - \left(\frac{1}{3} \right)^k u[k] \right)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2} \right)^{n-k} u[n-k] u[k] - \sum_{k=-\infty}^{\infty} \left(\frac{1}{2} \right)^{n-k} u[n-k] \left(\frac{1}{3} \right)^k u[k]$$

$$= u[n] \left(\frac{1}{2} \right)^n \sum_{k=0}^n \left(\frac{1}{2} \right)^k - \left(\frac{1}{2} \right)^n u[n] \sum_{k=0}^n \left(\frac{1}{3} \right)^k$$

$$= u[n] \left(\frac{1}{2} \right)^n \frac{1 - 2^{n+1}}{1 - 2} - u[n] \left(\frac{1}{2} \right)^n \frac{1 - \left(\frac{1}{3} \right)^{n+1}}{1 - \frac{1}{3}}$$

$$= \left(\frac{1}{2} \right)^n \left(2^{n+1} - 1 \right) u[n] + 3 \left(\frac{1}{2} \right)^n \left(\left(\frac{1}{3} \right)^{n+1} - 1 \right) u[n]$$

$$= \underbrace{\left(2 - \left(\frac{1}{2} \right)^n \right) u[n]}_A + \underbrace{\left(2 \left(\frac{1}{3} \right)^n - 3 \left(\frac{1}{2} \right)^n \right) u[n]}_{-B \text{ FROM ABOVE}}$$

1)



2) $\int_{-\infty}^{\infty} |h(t)| dt$ is $\int_0^{\infty} (1-e^{-t}) dt = [t + e^{-t}]_0^{\infty} \rightarrow \infty$ UNSTABLE

3) $h(t)=0$ FOR $t<0$ CAUSAL

4) $S_r(t) = \int_{-\infty}^t h(\tau) d\tau = \int_{-\infty}^t (1-e^{-\tau}) d\tau = u(t) \int_0^t (1-e^{-\tau}) d\tau$
 $= u(t) \left[\tau + e^{-\tau} \right]_0^t = (t + e^{-t} - 1) u(t)$

5) $X_1(t) = 2u(t) - u(t-3)$ $y_1(t) = 2S_r(t) - S_r(t-3)$
 $= 2(t + e^{-t} - 1) u(t) - ((t-3) + e^{-(t-3)} - 1) u(t-3)$

6) $y_2(t) = X_2(t) * h(t)$

$$= e^{-at} u(t) * (1 - e^{-t}) u(t)$$

SHORT WAY: $e^{-at} u(t) * e^{-bt} u(t) = \frac{e^{-bt} - e^{-at}}{a-b} u(t)$ or $\frac{e^{-at} - e^{-bt}}{b-a}$ if $a \neq b$

$$A = e^{-at} u(t) * u(t) = \frac{1 - e^{-2t}}{2-0} u(t) = \left(\frac{1 - e^{-2t}}{2} \right) u(t)$$

$$B = e^{-at} u(t) * e^{-t} u(t) = \frac{e^{-t} - e^{-2t}}{2-1} u(t) = (e^{-t} - e^{-2t}) u(t)$$

$$y_2(t) = A - B$$

LONG WAY: $y_2(t) = \int_{-\infty}^{\infty} X_2(t-\tau) h(\tau) d\tau$

$$= \int_{-\infty}^{\infty} (e^{-a(t-\tau)} u(t-\tau)) (1 - e^{-\tau}) u(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} e^{-a(t-\tau)} u(t-\tau) u(\tau) - \int_{-\infty}^{\infty} e^{-a(t-\tau)} u(t-\tau) e^{-\tau} u(\tau) d\tau$$

$$= e^{-at} u(t) \int_0^t e^{a\tau} - e^{-at} u(t) \int_0^t e^{\tau} d\tau$$

$$= e^{-at} \left[\frac{e^{a\tau}}{a} \right]_0^t u(t) - e^{-at} \left[e^{\tau} \right]_0^t u(t)$$

$$= e^{-2t} \left[\frac{e^{2t}-1}{2} \right] u(t) - e^{-2t} \left[e^t - 1 \right] u(t) = \overbrace{\left(\frac{1}{2} - \frac{e^{-2t}}{2} \right) u(t)}^A - \overbrace{\left(e^{-t} - e^{-2t} \right) u(t)}^B$$

$$= \left(\frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \right) u(t)$$