

ECE 280L Spring 2017

Test II

Michael R. Gustafson II

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Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

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## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NetID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

## Notes

For this test, you should not leave unevaluated convolution sums/integrals. Unless otherwise specified:

- The  $\cdot$  symbol means multiplication
- The  $*$  symbol means convolution
- $\delta(t)$  is the unit impulse function
- $u(t)$  is the unit step
- $r(t)$  is the unit ramp  $t \cdot u(t)$
- $q(t)$  is the "unit" quadratic  $\frac{1}{2}t^2 \cdot u(t)$

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### Problem I: [20 pts.] Fourier and Periodic Signals

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below. You are not allowed to have complex exponentials or  $j$ 's in any time domain representations either.

- (1) Determine the non-zero Fourier series coefficients  $A[k]$  for the signal:

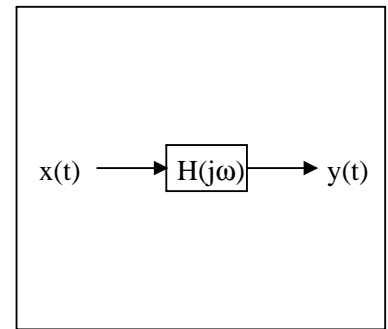
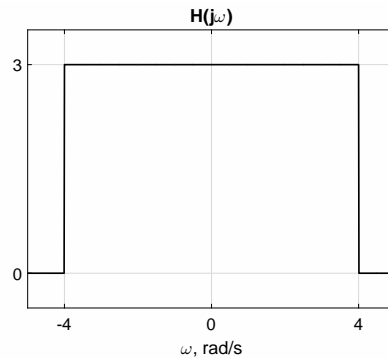
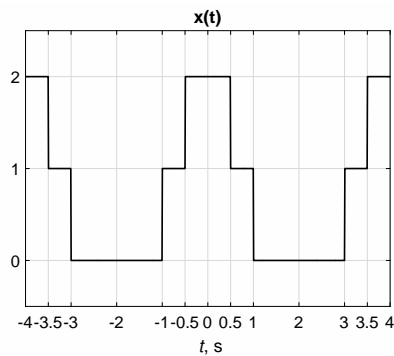
$$a(t) = 1 + \cos(3t) \cdot \sin(5t)$$

Be sure to clearly indicate the fundamental frequency.

- (2) Determine the Fourier transform representation  $A(j\omega)$  for the signal:

$$a(t) = 1 + \cos(3t) \cdot \sin(5t)$$

- (3) The next three problems involve a periodic signal  $x(t)$ , a filter with a transfer function  $H(j\omega)$ , and the block diagram below:



- (a) Determine an expression for the Fourier series coefficients  $X[k]$  of the periodic signal  $x(t)$ . Remember - you are not allowed to have any unevaluated integrals, summations, or convolutions in your expression for the coefficients.
- (b) Given the transfer function of a filter  $H(j\omega)$ , find the time-domain representation of the impulse response of that filter,  $h(t)$ .
- (c) Given the block diagram above, determine the Fourier series representation of the output ( $Y[k]$ ) and the time domain representation of the output ( $y(t)$ ). Note, you are not allowed to have any unevaluated integrals, summations, or convolutions in your expression for  $y(t)$ , nor can there be complex exponentials or  $j$ 's. You may leave  $\pi$  or any square roots as-is, however.

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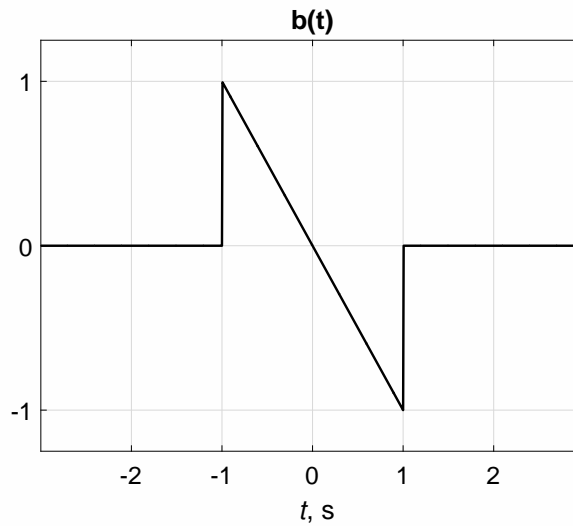
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## Problem II: [20 pts.] Fourier Transforms

Note: you are not allowed to have any unevaluated integrals, summations, or convolutions in any of your final answers below.

(1) Find the Fourier transform for the following signals:

- $a(t) = te^{-3t} \cos(4t)u(t)$
- $b(t)$  as shown below:



(2) Find the inverse Fourier transform for the following:

- $X(j\omega) = 5 \frac{j\omega+1}{(j\omega+2)(j\omega+4)}$
- $Y(j\omega) = 3e^{-4j\omega} \left( \frac{\sin(2\omega)}{\omega} \right)$

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### Problem III: [30 pts.] Sampling and Reconstruction

- (1) (Inspired by OW 7.21) Assume a signal is going to be sampled by multiplying it with a periodic impulse train  $p(t)$  where  $p(t)$  is

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

and  $T$  is the period of the impulse train. If  $T$  is 1 ms, such that  $f_s = 1/T$  is 1000 Hz, for which of the signals below does the sampling theorem guarantee that a signal, thus sampled, can be recovered exactly using the appropriate low-pass filter:

- $a(t)$  if  $A(j\omega) = 0$  for  $|\omega| > 500\pi$ .
  - $b(t)$  if  $B(j\omega) = 0$  for  $|\omega| > 1500\pi$ .
  - $c(t)$  if  $c(t)$  is real and  $C(j\omega) * C(j\omega) = 0$  for  $\omega > 500\pi$ .
  - $d(t)$  if  $d(t)$  is real and  $D(j\omega) * D(j\omega) = 0$  for  $\omega > 1500\pi$ .
  - $e(t)$  if  $e(t)$  is real and  $\mathfrak{F}\{e(t) \cos(1500\pi t)\} = 0$  for  $|\omega| < 1000\pi$  if  $\mathfrak{F}$  represents the Fourier transform.
  - $f(t)$  if  $f(t)$  is real and  $\mathfrak{F}\{f(t) \cos(1500\pi t)\} = 0$  for  $\omega > 2000\pi$ .
- (2) (Inspired by OW 7.23) Assume a band-limited signal  $x(t)$  is going to be sampled by multiplying it with an alternating periodic impulse train  $q(t)$  where  $q(t)$  is

$$q(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) - \delta(t - nT - T/2)$$

The sampled signal is  $x_q(t) = x(t) \cdot q(t)$ .

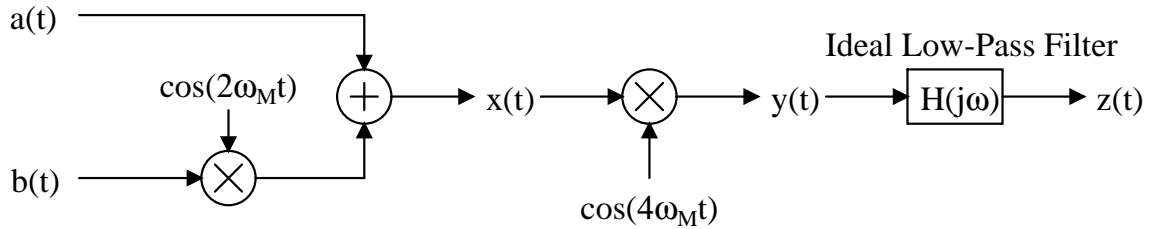
- (a) Sketch  $q(t)$  in the time domain for at least three periods - be sure to label your axes and tick marks.
- (b) Determine and sketch  $Q(j\omega)$ , the Fourier transform of  $q(t)$ . Be sure to label your axes and tick marks.
- (c) Assuming  $X(j\omega) = 0$  for  $|\omega| > \omega_M$ , what is the relationship between  $\omega_M$  and  $T$  that guarantees that  $x(t)$  could be perfectly recovered from  $x_q(t)$ ?
- (d) Assuming that the relationship between  $\omega_M$  and  $T$  is such that  $x(t)$  can be perfectly recovered from  $x_q(t)$ , design a system that will do just that. Draw a block diagram of the system and clearly specify any amplitudes, frequencies, etc. Simpler systems will get more points.

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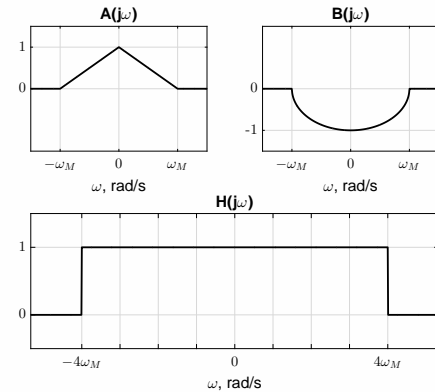
### Problem IV: [30 pts.] Communication Systems

- (1) MIXED MESSAGES: Real messages  $a(t)$  and  $b(t)$  are both band-limited such that they have no frequency content at or above  $\omega_M$ . They are put through the following system to create new signals  $x(t)$ ,  $y(t)$ , and  $z(t)$ :



Assuming the spectra and transfer function at right:

- Sketch the spectrum of  $x(t)$ .
- Sketch the spectrum of  $y(t)$ .
- Sketch the spectrum of  $z(t)$ .
- Design a system that can be used to recover exactly  $a(t)$  *only* from  $z(t)$ . Be sure to specify all amplitudes and frequencies of anything you use. You only need to provide either the impulse response *or* the transfer function for any block. For either, a mathematical description or a well-labeled sketch in the time or frequency domain will suffice. Simpler systems will get more points.
- Design a system that can be used to recover exactly  $b(t)$  *only* from  $z(t)$ . Be sure to specify all amplitudes and frequencies of anything you use. You only need to provide either the impulse response *or* the transfer function for any block. For either, a mathematical description or a well-labeled sketch in the time or frequency domain will suffice. Simpler systems will get more points.



- (2) FULL AMPLITUDE DEMODULATION: Draw the general circuit used to demodulate Full AM signals. What is this circuit called?
- (3) DSB-SC MODULATION: Assuming you have some message signal  $a(t)$  as given in the problem above that is band-limited by  $\omega_M$ , sketch a block diagram for DSB-SC modulation going from message signal  $a(t)$  to transmitted signal  $s(t)$ , then sketch the magnitude of the frequency spectrum which would result if  $a(t)$  were sent through a DSB-SC modulator with a carrier amplitude of 5 and a carrier frequency that is 3 times the band-limit of  $a(t)$ . Be sure to label your axes and indicate important values.
- (4) DSB-SC DEMODULATION: Sketch the block diagram for the system that you would use to recover  $a(t)$  from the DSB-SC modulated signal  $s(t)$  above. The recovered signal must have the same amplitude as the original message  $a(t)$ . Be sure to clearly indicate either the precise impulse response or transfer function of any system blocks. You may either provide mathematical expressions or labeled magnitude sketches for these. Also, you only need to provide either the impulse response *or* the transfer function for any block. For either, a mathematical description or a well-labeled sketch in the time or frequency domain will suffice.