

ECE 280 Test II Spring 2017

Note Title

I) $a(t) = 1 + \cos(3t) \cdot \sin(5t)$

Using trig, $a(t) = 1 + \frac{1}{2} (\sin(2t) + \sin(8t))$

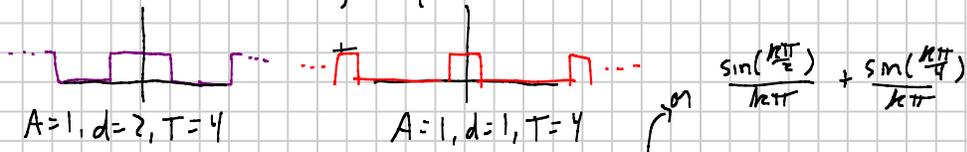
$A[k] = \begin{cases} k=-4 & -1/4j \\ k=-1 & -1/4j \\ k=0 & 1 \\ k=1 & 1/4j \\ k=4 & 1/4j \end{cases}$ $\omega_0 = 2$, so

Alt: $1 + \cos(3t) \cdot \sin(5t) = 1 + \left(\frac{e^{j3t} + e^{-j3t}}{2} \right) \left(\frac{e^{j5t} - e^{-j5t}}{2j} \right)$
 $= 1 + \frac{e^{j8t}}{4j} + \frac{e^{j2t}}{4j} - \frac{e^{-j2t}}{4j} - \frac{e^{-j8t}}{4j}$

2) $a(t) = 1 + \frac{1}{2} (\sin(2t) + \sin(8t))$

$A(j\omega) = 2\pi \delta(\omega) + \frac{1}{2} \left(\frac{\pi}{j} \delta(\omega-2) - \frac{\pi}{j} \delta(\omega+2) + \frac{\pi}{j} \delta(\omega-8) - \frac{\pi}{j} \delta(\omega+8) \right)$

3) $x(t)$ is sum of two rectangular pulse trains:



$X[k] = \frac{(1)(2)}{(4)} \text{sinc}\left(\frac{k(2)}{(4)}\right) + \frac{(1)(1)}{(4)} \text{sinc}\left(\frac{k(1)}{(4)}\right) = \frac{1}{2} \text{sinc}\left(\frac{k}{2}\right) + \frac{1}{4} \text{sinc}\left(\frac{k}{4}\right)$

$\mathcal{F}^{-1}\{H(j\omega)\} = \frac{3 \sin(4t)}{\pi t}$ from sinc function in table. $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$H(j\omega)$ is a low-pass filter w/ cutoff of 4 rad/s.

once $k\omega_0 = \frac{|k\pi|}{2} > 4$, output amplitude is zero, so $|k| > \frac{8}{\pi}$

only k 's that matter are $-2, -1, 0, 1, 2$ $\omega_0 = \pi/2$

Recall: $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$Y[k] = \begin{cases} k=-2 & \frac{1}{2} \text{sinc}(-1) + \frac{1}{4} \text{sinc}(-\frac{1}{2}) = +\frac{1}{4} \frac{\sin(-\pi/2)}{-\pi/2} = \frac{1}{2\pi} \\ k=-1 & \frac{1}{2} \text{sinc}(-\frac{1}{2}) + \frac{1}{4} \text{sinc}(-\frac{1}{4}) = \frac{1}{2} \frac{\sin(-\pi/2)}{-\pi/2} + \frac{1}{4} \frac{\sin(-\pi/4)}{\pi/4} = \frac{1}{\pi} + \frac{\sqrt{2}}{2\pi} \\ k=0 & \frac{1}{2} \text{sinc}(0) + \frac{1}{4} \text{sinc}(0) = \frac{3}{4} \\ k=1 & \frac{1}{2} \text{sinc}(\frac{1}{2}) + \frac{1}{4} \text{sinc}(\frac{1}{4}) = \frac{1}{2} \frac{\sin(\pi/2)}{\pi/2} + \frac{1}{4} \frac{\sin(\pi/4)}{\pi/4} = \frac{1}{\pi} + \frac{\sqrt{2}}{2\pi} \\ k=2 & \frac{1}{2} \text{sinc}(1) + \frac{1}{4} \text{sinc}(\frac{1}{2}) = \frac{1}{4} \frac{\sin(\pi/2)}{\pi/2} = \frac{1}{2\pi} \end{cases}$

$\text{sinc}(\pm 4) = \frac{-\sqrt{2}}{2}$

$y(t) = 3 \cdot \left(\frac{3}{4} + \frac{(2+\sqrt{2})}{\pi} \cos(\pi t) + \frac{1}{\pi} \cos(2\pi t) \right)$ ✓
 From Filter

II a: $a(t) = t e^{-3t} (\cos(4t) u(t))$ | alt: $t e^{-3t} u(t) \cdot \cos(4t) \rightarrow$

use freq. diff. property

$$j \frac{d}{d\omega} \left(\frac{(j\omega+3)}{(j\omega+3)^2 + 16} \right) = j \left(\frac{(j\omega+3)^2 + 16(j) - (j\omega+3)2(j\omega+3)}{(j\omega+3)^2 + 16)^2} \right)$$

$$= \frac{2(j\omega+3)^2 - (j\omega+3)^2 + 16j}{((j\omega+3)^2 + 16)^2} = \frac{(j\omega+3)^2 - 16}{((j\omega+3)^2 + 16)^2}$$

Similar to OW 4.21 g

$$\frac{1}{2\pi} \left(\frac{1}{(j\omega+3)^2} * \pi(\delta(\omega-4) + \delta(\omega+4)) \right)$$

$$= \frac{1}{2} \left(\frac{1}{(j(\omega-4)+3)^2} + \frac{1}{(j(\omega+4)+3)^2} \right)$$

$$= \frac{1}{2} \left(\frac{1}{(j\omega+3-4j)^2} + \frac{1}{(j\omega+3+4j)^2} \right)$$

$$= \frac{(j\omega+3)^2 - 16}{((j\omega+3)^2 + 16)^2}$$



Note: cannot use integral property to get $\mathcal{D}\{r(t)\}$; only works if $X(0)$ is defined.

$b(t) = -t \cdot \text{rectangular pulse from } -1 \text{ to } 1$

$$B(j\omega) = -j \frac{d}{d\omega} \frac{2 \sin(\omega)}{\omega} = -j \left(\frac{\omega(2 \cos(\omega)) - 2 \sin(\omega)}{\omega^2} \right)$$

$$= 2j \left(\frac{\sin(\omega) - \omega \cos(\omega)}{\omega^2} \right)$$

If your answer has a $2 + \delta(\omega)$, that has to be wrong since that implies an average value of 1!

$$X(j\omega) = \int \frac{j\omega+1}{(j\omega+2)(j\omega+4)} = \frac{A}{j\omega+2} + \frac{B}{j\omega+4}$$

$$A = \lim_{j\omega+2 \rightarrow 0} \frac{5(j\omega+1)}{(j\omega+4)} = \frac{-5}{2} \quad B = \lim_{j\omega+4 \rightarrow 0} \frac{5(j\omega+1)}{(j\omega+2)} = \frac{+15}{2}$$

$$x(t) = \left(\frac{-5}{2} e^{-2t} + \frac{15}{2} e^{-4t} \right) u(t)$$

From used d/dt , note that it's



$$Y(j\omega) = \int e^{-4j\omega} \frac{2 \sin(2\omega)}{\omega} \rightarrow \frac{3}{2} (u(t+2) - u(t-2)) * \delta(t-4)$$

(time shift) (rectangular pulse width 4)

$$= \frac{3}{2} (u(t-2) - u(t-6))$$



III $T = 1 \text{ms}, f_s = 1000 \text{Hz}, \omega_s = 2000\pi \text{ rad/s}$

To keep from aliasing, $X(j\omega) = 0$ for $|\omega| > 1000\pi$

- a) guaranteed
- b) not guaranteed
- c) guaranteed; if $c(t)$ is real and $C(j\omega)X(j\omega) = 0$ for $\omega > 500\pi$, $C(j\omega) = 0$ for $|\omega| > 250\pi$!
- d) guaranteed; if $d(t)$ is real and $D(j\omega)X(j\omega) = 0$ for $\omega > 1500\pi$, $D(j\omega) = 0$ for $|\omega| > 750\pi$
- e) tricky! This only means $F(j\omega) = 0$ for $500\pi < |\omega| < 2500\pi$ but there may be higher freq parts \rightarrow no guarantee.
- f) guaranteed! This means $F(j\omega) = 0$ for $|\omega| < 500\pi$

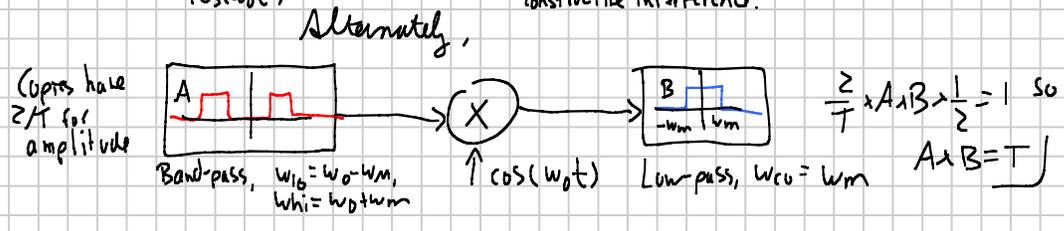
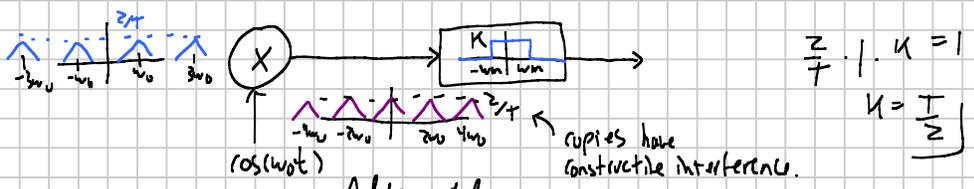


$$g(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) - \sum_{n=-\infty}^{\infty} \delta(t - T/2 - nT)$$

$$\begin{aligned} b) Q(j\omega) &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T}) - \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T}) e^{-j\omega T/2} \\ &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T}) - \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T}) e^{-j\pi k} \\ &= \frac{4\pi}{T} \sum_{k=-\infty, k \text{ odd}}^{\infty} \delta(\omega - \frac{2\pi k}{T}) \quad \omega_0 = \frac{2\pi}{T} \end{aligned}$$

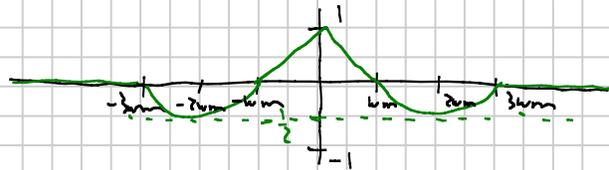


- c) In this case, $\omega_m < \omega_0$ works, so $\omega_m < \frac{2\pi}{T}$ or $T < \frac{2\pi}{\omega_m}$ (i.e. can sample half as fast as regular pulse train)
- d) Need to get a single copy back to origin; easiest may be:

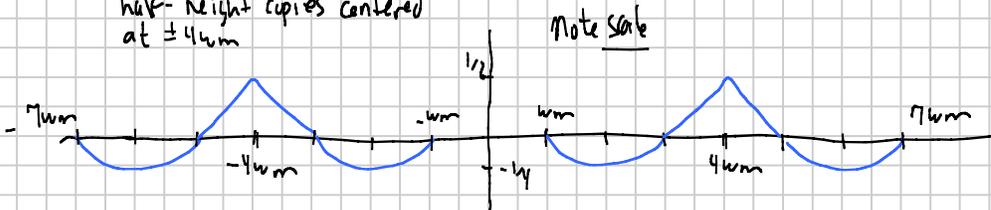


IV) $x(t) = a(t) + b(t) \cos(2\omega_m t)$

$X(j\omega) = A(j\omega) + \frac{1}{2} (B(-j(\omega - 2\omega_m)) + B(j(\omega + 2\omega_m)))$



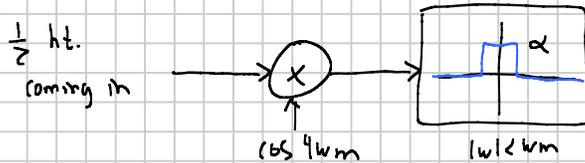
• after multiplication, half-height copies centered at $\pm 4\omega_m$



after LPF:

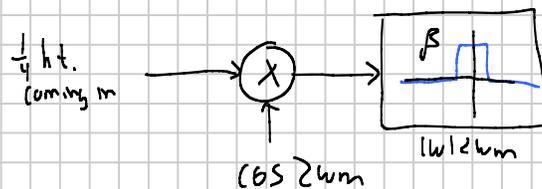


to get a: isolate single side bands, modulate to shift copies to origin, LPF



output amplitude is $\frac{1}{2} \cdot \frac{1}{2} \cdot 2 = \frac{\alpha}{4}$ so $\alpha = 4$
 ↳ loses half height since SSB

to get b: similar but different frequencies + DSB, so



output is $\frac{1}{4} \cdot 1 \cdot \beta = \frac{\beta}{4}$ so $\beta = 4$
 ↳ lost half w/ first modulation, half w/ second, but copies overlap in demod.

