Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280.2 Fall 2023 Test II

Name (please print): NetID (please print):

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure that your name *and* **NetID are** *clearly* **written at the top of** *every* **page**. If you need more space for a particular problem or want to show more work, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page and submit those extra pages in problem-order **after** all pre-printed pages of the test. Also, in the box for the problem, write a note that says "see extra page."

Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**) and put them in the box with the top left corner of the test going into the back left corner of the folder. You must turn in all the original pages of the test even if all you wrote on them is your name and NetID. You do *not* need to staple your test - just make sure your name and NetID are on every page!

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or unevaluated integrals or summations. Also, unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $h(t)$ is the impulse response and both $s_{\mathrm{r}}(t)$ and $y_{\mathrm{step}}(t)$ represent the step response
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$
- $c(t)$ is the "unit" cubic $\frac{1}{6}t^3 \cdot u(t)$

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Problem I: [18 *pts.*] LCCDDE Analysis

Find the impulse response $h_k[n]$ for the following three systems:

1. System k =1:

$$y[n] - \frac{2}{3}y[n-1] = 4x[n]$$

2. System k =2:

$$y[n] + \frac{1}{4}y[n-1] = x[n] - \frac{1}{2}x[n-2]$$

3. System k =3:

$$6y[n] - y[n-1] - y[n-2] = 18x[n]$$

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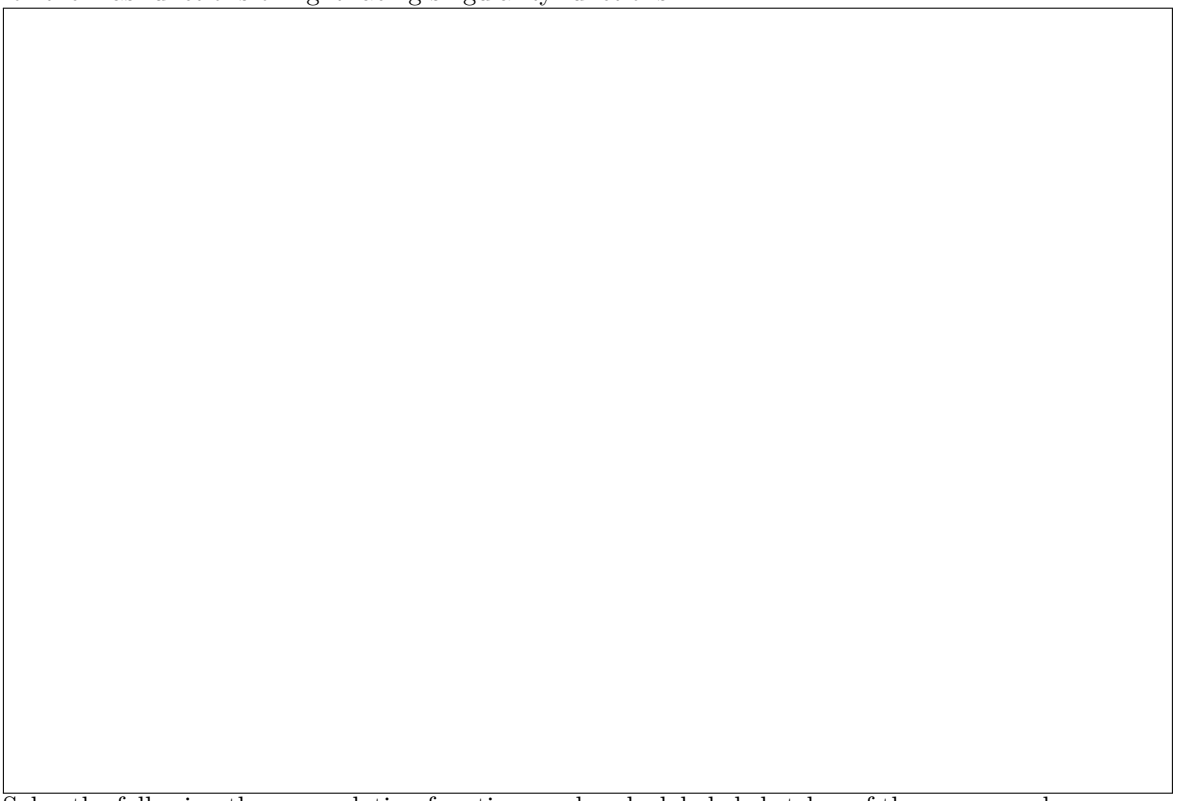
Problem II: [20 *pts.*] Correlation

1. Given the following specific functions:

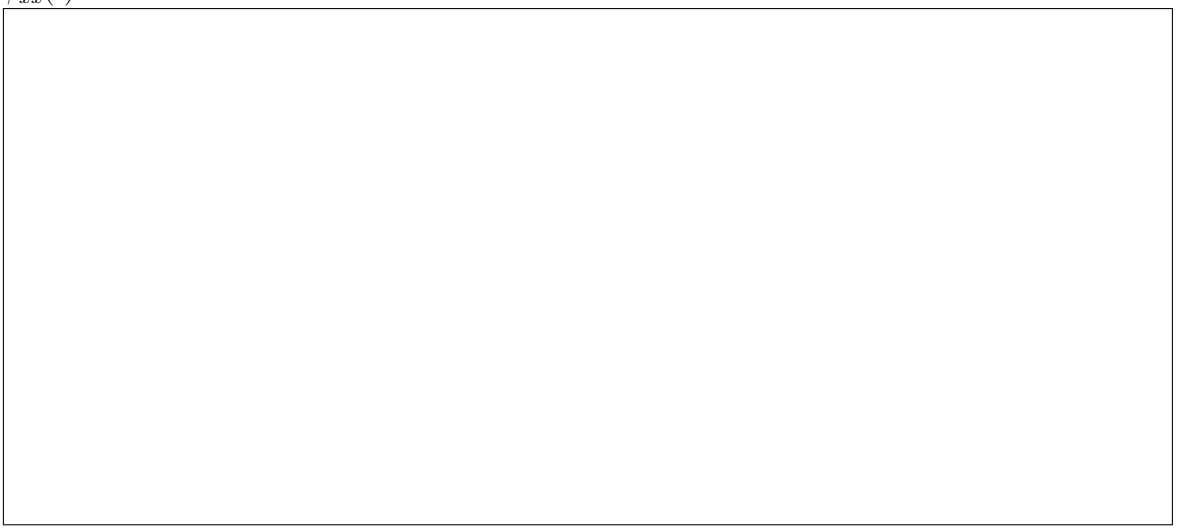
$$x(t) = 2u(t) - u(t-1) - u(t-2) \qquad\qquad y(t) = u(t+1) - u(t-2)$$

- Make labeled sketches of $x(t)$, $y(t)$, $x_m(t) = x(-t)$, $y_m(t) = y(-t)$ and, for the last two, write formulas for them as functions of right-facing singularity functions.



Solve the following three correlation functions and make labeled sketches of them - remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page:

- $\phi_{xx}(t)$



- $\phi_{yy}(t)$

- $\phi_{xy}(t)$

- Determine the measure of correlation between $x(t)$ and $y(t)$.

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Problem III: [14 *pts.*] Fourier Series

1. Determine the fundamental frequency ω_0 and the non-zero Fourier Series coefficients $W[k]$ for the periodic signal:

$$w(t) = -1 + 2\sin(6t) + 3\cos(15t)$$

2. Determine the time-domain representation for a periodic signal $x(t)$ with a period $T = 3$ s and the following non-zero Fourier Series coefficients:

$$X[k] = \begin{cases} k = 7, & 4 + j5 \\ k = 2, & 1 \\ k = 0, & 2 \\ k = -2 & 1 \\ k = -7 & 4 - j5 \end{cases}$$

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Problem IV: [24 *pts.*] Fourier Transform

1. Determine the Fourier Transform for the following signals, keeping in mind that there can be neither be unevaluated integrals nor convolutions in your final answer:

   (a) $a(t) = e^{-4t}\,u(t)$

   (b) $b(t) = te^{-4t}\,u(t-2)$

   (c) $c(t) = 2u(t) - r(t) + r(t-1) - u(t-2)$

   (d) $d(t) = \sin\left(\frac{\pi}{2}t\right) \cdot (u(t-1) - u(t+1))$

2. Determine the inverse Fourier Transform for the following signals, keeping in mind that there can be neither be unevaluated integrals nor convolutions in your final answer:

   (w) $W(j\omega) = \frac{1-e^{-2j\omega}}{j\omega+5}$

   (x) $X(j\omega) = \frac{5}{(j\omega)^2+8j\omega+16}$

   (y) $Y(j\omega) = \frac{12j\omega}{(j\omega)^2+7j\omega+10}$

   (z) $Z(j\omega) = \frac{15j\omega+20}{(j\omega)^2+6j\omega+13}$

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Problem V: [24 *pts.*] System Analysis

An LTI system has an input signal $x(t)$ and an output signal $y(t)$. In one experiment, we determine that an input of

$$x(t) = \left(3e^{-t} - 2e^{-4t}\right) u(t)$$

generates a response of:

$$y(t) = \left(2e^{-2t} - 2e^{-4t}\right) u(t)$$

1. What is the transfer function for the system, $H(j\omega)$?
2. What is the impulse response of the system, $h(t)$?
3. Clearly using Fourier Transforms and/or inverse Fourier Transforms, find the response of the system to an input $x_1(t) = e^{-t}u(t)$.
4. Clearly using Fourier Transforms and/or inverse Fourier Transforms, find the response of the system to an input $x_2(t) = e^{-2t}u(t)$.
5. Write a differential equation that models this system, relating $y(t)$ and its derivatives to $x(t)$ and its derivatives.