Duke University
Edmund T. Pratt, Jr. School of Engineering

**ECE 280.2 Fall 2023 Test I**

Name (please print): NetID (please print):

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

**Only write on one side of any given page and please be sure that your name and NetID are clearly written at the top of every page.** If an answer box is provided, **please be sure to put each answer in the correct box**. If you absolutely need more space for a particular problem, or want to show work, put that work **on one side** of its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page and submit those extra pages in problem-order **after** all preprinted pages of the test. Also, in the box for the problem, write a note that says "see extra page."

You will *not* be stapling your test but instead will be turning in your test in its original folder. Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**), put them in the folder you received with the test, and bring the folder to the front of the room.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

## Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution, the convolution integral, or the convolution sum. Also, unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $h(t)$ is the impulse response and both $s_{\mathrm{r}}(t)$ and $y_{\mathrm{step}}(t)$ represent the step response
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$
- $c(t)$ is the "unit" cubic $\frac{1}{6}t^3 \cdot u(t)$
- $\delta[n]$ is the unit impulse function
- $h[n]$ is the impulse response and both $s_{\mathrm{r}}[n]$ and $y_{\mathrm{step}}[n]$ represent the step response
- $u[n]$ is the unit step
- $r[n]$ is the unit ramp $(n+1) \cdot u[n]$
- $q[n]$ is the "unit" quadratic $\frac{(n+1)(n+2)}{2} \cdot u[n]$
- $c[n]$ is the "unit" quadratic $\frac{(n+1)(n+2)(n+3)}{6} \cdot u[n]$

# Additional Math Formulae

## Summation Formulae

$$\sum_{n=k}^{N-1} \alpha^n = \begin{cases} N-k & \alpha = 1 \\ \frac{\alpha^k - \alpha^N}{1-\alpha} & \alpha \neq 1 \end{cases} \qquad \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \text{ if } |\alpha| < 1$$

$$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \text{ if } |\alpha| < 1 \qquad \sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \text{ if } |\alpha| < 1$$

## Trigonometric Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \qquad \cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right) \qquad \sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right) \qquad \cos^2(\theta) + \sin^2(\theta) = 1$$

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2} \qquad \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin(\theta) = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

## Energy and Power

| | | |
|---|---|---|
| Total Energy | $E_\infty = \int_{-\infty}^{\infty} \|x(t)\|^2\, dt$ | $E_\infty = \sum_{n=-\infty}^{\infty} \|x[n]\|^2$ |
| Avg. Power; Integral/sum over $T/N+1$ | $P_\infty = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \|x(t)\|^2\, dt$ | $P_\infty = \lim_{N\to\infty} \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \|x[n]\|^2$ |
| Avg. Power; Integral/sum over $2T/2N+1$ | $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \|x(t)\|^2\, dt$ | $P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \|x[n]\|^2$ |
| Energy for one period | $E_{\text{period}} = \int_T \|x(t)\|^2\, dt$ | $E_{\text{period}} = \sum_{n=<N>} \|x[n]\|^2$ |
| Avg. Power | $P_{\text{period}} = \frac{1}{T} \int_T \|x(t)\|^2\, dt$ | $P_{\text{period}} = \frac{1}{N} \sum_{n=<N>} \|x[n]\|^2$ |

## Convolution

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)y(\tau)d\tau$$

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]y[k]$$

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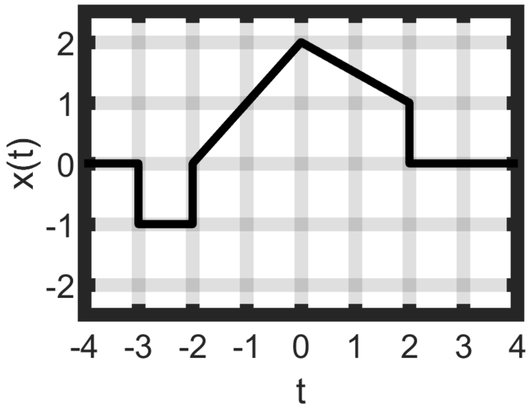
## Problem I: [10 *pts.*] Signals I

Given the following graph of $x(t)$, and noting that $x(t) = 0$ for all $t < -3$ and $t > 2$,




1. Write an equation for the signal using singularity functions:
$x(t) =$

2. Does $x(t)$ represent an energy signal, a power signal, or neither? If it is an energy signal, also give its total energy $E_\infty$; if it is a power signal, calculate its overall average power $P_\infty$.

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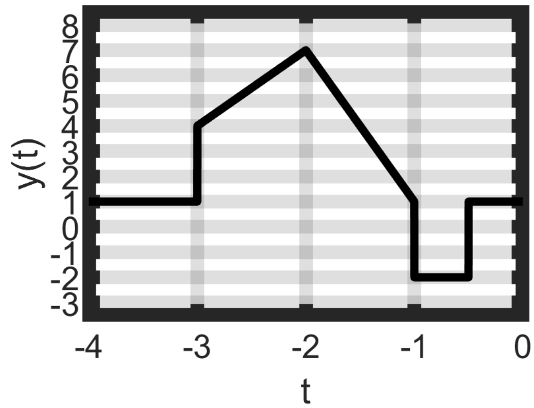
3. Given the following graph of $y(t)$, where $y(t) = A\,x(B\,t + C) + D$, determine $A$, $B$, $C$, and $D$. Note - the scales are different from the original graph.



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## Problem II: [10 *pts.*] Signals 2

1. For each of the following signals, determine if they are periodic or not. If they are periodic, state the period:
   (a) $a(t) = \cos(14\pi t) \cdot \sin(6\pi t)$
   (b) $b(t) = \cos(4t) + \sin(4\pi t)$
   (c) $c(t) = \sum_{n=-\infty}^{\infty} (u(t-1-3n) - u(t-2-3n))$

2. For each of the following signals, determine if they are energy signals, power signals, or neither. If a signal is an energy signal, also give its total energy $E_\infty$; if a signal is a power signal, calculate its overall average power $P_\infty$.
   (k) $k(t) = 2u(t) - u(t-1) - u(t-2)$
   (l) $l(t) = 2r(t) - r(t-1) - r(t-2)$
   (m) $m(t) = 5\,\cos(2t)$

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## Problem III: [25 *pts.*] System Classifications

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

| System | Linear? | Time Inv.? | Stable? | Memoryless? | Causal? |
|---|---|---|---|---|---|
| $y(t) = 2\,x(t) - 1$ | | | | | |
| $y(t) = x(2t) - 1$ | | | | | |
| $y(t) = 2\,x(t-1)$ | | | | | |
| $y[n] = \frac{x[n]+1}{x[n]-1}$ | | | | | |
| $y[n] = \sum_{k=0}^{5} x[n-k]$ | | | | | |
| $y[n] = \sum_{k=0}^{n} x[n-k]$ | | | | | |

2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response ($h_i(t)$ or $h_i[n]$) or the step response ($s_{\mathrm{r,i}}(t)$ or $s_{\mathrm{r,i}}[n]$). You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

| System | Stable? | Memoryless? | Causal? |
|---|---|---|---|
| $h_1(t) = e^{-2\|t\|}$ | | | |
| $h_2[n] = u[n-1]$ | | | |
| $s_{\mathrm{r,3}}(t) = 2u(t)$ | | | |
| $s_{\mathrm{r,4}}[n] = u[n-1]$ | | | |

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## Problem IV: [15 *pts.*] __r___h_c_l Convolution

Given the following functions:

$$x(t) = e^t u(-t) \qquad\qquad y(t) = u(t) - u(t-2)$$

1. Make labeled sketches of $x(t)$ and $y(t)$
2. Determine $z(t) = x(t) * y(t)$ but do not make a sketch of it - remember, you *cannot* leave unevaluated integrals. *Hint 1:* One of the homework assignments had a very specific recommendation for the method to use if convolving signals that start off facing in different directions. You should do that. *Hint 2:* You can leave your answer as a function that is defined piecewise. [1]

[1] Graphical...do graphical convolution.

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## Problem V: [20 *pts.*] System Analysis 1

A linear, time-invariant system $S$ has an impulse response of:

$$h(t) = r(t+1) - r(t) - u(t-2)$$

1. Make a sketch of $h(t)$.
2. Is the system stable? Why do you believe that to be the case?
3. Is the system causal? Why do you believe that to be the case?
4. Determine an expression for the step response of the system, $s_{\mathrm{r}}(t)$ (a.k.a. $y_{\mathrm{step}}(t)$)

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5. Determine an expression for the output of this system to the input

$$x_1(t) = u(t+1) - u(t-1)$$

and call this output $y_1(t)$. Remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page. You can also use any previously defined functions in this problem.

6. Determine an expression for the output of this system to the input

$$x_2(t) = r(t)$$

and call this output $y_2(t)$. Remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page.

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## Problem VI: [20 *pts.*] System Analysis 2

A linear, time-invariant system $S$ has an impulse response of:

$$h[n] = \left(1 - \left(\frac{1}{3}\right)^n\right) u[n] = u[n] - \left(\frac{1}{3}\right)^n u[n]$$

1. Make a sketch of $h[n]$.

2. Is the system stable? Why do you believe that to be the case?

3. Is the system causal? Why do you believe that to be the case?

4. Determine an expression for the step response of the system, $s_{\text{r}}[n]$ (a.k.a. $y_{\text{step}}[n]$)

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5. Determine an expression for the output of this system to the input

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

and call this output $y[n]$. Remember, you *cannot* leave unevaluated summations or convolutions but you can use the singularity functions defined on the cover page.