

1

$$1) \quad y[n] - \frac{2}{3}y[n-1] = 4x[n]$$

$$y^n - \frac{2}{3}y^{n-1} = 0$$

$$y^{n-1}(y - \frac{2}{3}) = 0 \quad y = \frac{2}{3}$$

$$h[n] = A\left(\frac{2}{3}\right)^n u[n] \quad h[0] = A = 4$$

$$h[n] = 4\left(\frac{2}{3}\right)^n u[n]$$

$$2) \quad y[n] + \frac{1}{4}y[n-1] = x[n] - \frac{1}{2}x[n-2]$$

solve for  $z[n] + \frac{1}{4}z[n-1] = x[n]$  & then  $y[n] = z[n] * (\delta[n] - \frac{1}{2}\delta[n-2])$

$$y^n + \frac{1}{4}y^{n-1} = 0$$

$$y^{n-1}(y + \frac{1}{4}) = 0 \quad y = -\frac{1}{4}$$

$$h_z[n] = A(-\frac{1}{4})^n u[n] \quad h_z[0] = A = 1$$

$$h_z[n] = (-\frac{1}{4})^n u[n]$$

$$h_y[n] = h_z[n] * (\delta[n] - \frac{1}{2}\delta[n-2]) = (-\frac{1}{4})^n u[n] - \frac{1}{8}(-\frac{1}{4})^{n-2} u[n-2]$$

$$3) \quad 6y[n] - y[n-1] - y[n-2] = 18x[n]$$

$$6y^n - y^{n-1} - y^{n-2} = 0$$

$$6y^{n-2}(6y^2 - y - 1) = 0 \quad y = \frac{1 \pm \sqrt{1+24}}{12} = \frac{1 \pm 5}{12} = \frac{1}{2}, -\frac{1}{3}$$

$$h[n] = A\left(\frac{1}{2}\right)^n u[n] + B\left(-\frac{1}{3}\right)^n u[n]$$

$$n=0: \quad 6h[0] - h[-1] - h[-2] = 18$$

$$6A + 6B = 18 \quad A+B=3$$

$$n=1 \quad 6h[1] - h[0] - h[-1] = 0$$

$$6A \cdot \frac{1}{2} + 6B \cdot \left(-\frac{1}{3}\right) - (A+B) + 0 = 0$$

$$3A - 2B - A - B = 2A - 3B = 0 \quad A = \frac{3}{2}B$$

$$\frac{3}{2}B + B = \frac{5}{2}B = 3 \quad B = \frac{6}{5}$$

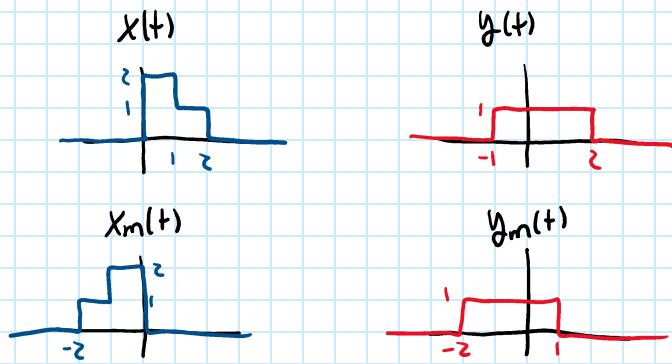
$$A = \frac{3}{2} \cdot \frac{6}{5} = \frac{9}{5}$$

$$h[n] = \frac{9}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{5}\left(-\frac{1}{3}\right)^n u[n]$$

$$(check: \quad h[0] = \frac{9}{5} + \frac{6}{5} = 3)$$

$$h[1] = \frac{9}{5} \cdot \frac{1}{2} + \frac{6}{5} \cdot \left(-\frac{1}{3}\right) = \frac{27}{30} - \frac{12}{30} = \frac{1}{2}$$

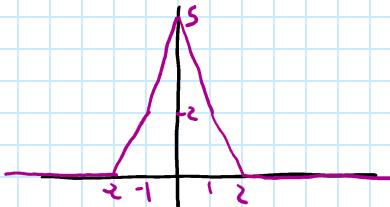
$$\textcircled{3} n=1 \quad 6y[1] - y[0] - y[-1] = (6)\left(\frac{1}{2}\right) - (3) - 0 = 0 \quad \checkmark$$



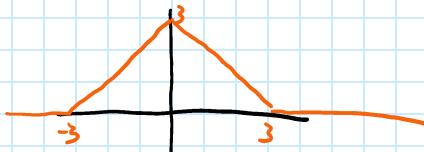
$$x_m(t) = u(t+2) + u(t+1) - 2u(t)$$

$$y_m(t) = u(t+2) - u(t-1)$$

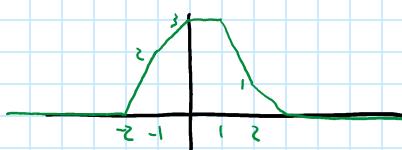
$$\begin{aligned}\phi_{xy} &= x(t) * x_m(t) = (2u(t) - u(t-1) - u(t-2)) * (u(t+2) + u(t+1) - 2u(t)) \\ &= 2r(t+2) + 2r(t+1) - 4r(t) \\ &\quad - r(t+1) - r(t) + 2r(t-1) \\ &= 2r(t+2) + r(t+1) - 6r(t) + r(t-1) + 2r(t-2)\end{aligned}$$



$$\begin{aligned}\phi_{yy} &= y(t) * y_m(t) = (u(t+1) - u(t-2)) * (u(t+2) - u(t-1)) \\ &= r(t+3) - r(t) - r(t) + r(t-3) = r(t+3) - 2r(t) + r(t-3)\end{aligned}$$



$$\begin{aligned}\phi_{xy} &= x(t) * y_m(t) = (2u(t) - u(t-1) - u(t-2)) * (u(t+2) - u(t-1)) \\ &= 2r(t+2) - r(t+1) - r(t) - 2r(t-1) + r(t-2) + r(t-1) \\ &= 2r(t+2) - r(t+1) - r(t) - 2r(t-1) + r(t-2) + r(t-1)\end{aligned}$$



$$MOC = \frac{(\phi_{xy, \text{max}})^2}{\phi_{xy}(0) \phi_{yy}(0)} = \frac{3^2}{5 \cdot 3} = \frac{9}{15} = \frac{3}{5}$$

3

$$1) \quad w_1 = 6 \quad w_2 = 15 \quad w_0 = 3 \\ h_1 = 2 \quad h_2 = 5$$

$$W[h] = \begin{cases} 5 & \frac{3}{2} \\ 2 & -1 \\ 0 & \frac{1}{2} \\ -2 & \frac{3}{2} \\ -5 & 0 \\ \text{other} & \end{cases}$$

$$2) \quad T=3 \quad w_0 = \frac{2\pi}{T} = \frac{2}{3}\pi$$

$$x(t) = 8\cos\left(\frac{14}{3}\pi t\right) - 10\sin\left(\frac{14}{3}\pi t\right) + 2\cos\left(\frac{4\pi}{3}t\right) + 2$$



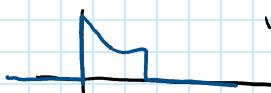
$$1a) \frac{1}{jw+4}$$

$$1b) te^{-4t}u(t-2) = (t-2+2)e^{-4(t-2+2)}u(t-2)$$

$$= (t-2)e^{-8}e^{-4(t-2)}u(t-2) + 2e^{-8}e^{-4(t-2)}u(t-2)$$

$$\rightarrow \frac{e^{-8}e^{-2jw}}{(jw+4)^2} + \frac{2e^{-8}e^{-2jw}}{(jw+4)}$$

$$1c) 2u(t) - r(t) + r(t-1) - u(t-2)$$



"TURNS OFF" so  $D(jw) = \frac{2}{jw} - \frac{1}{(jw)^2} + \frac{e^{jw}}{(jw)^2} - \frac{e^{-2jw}}{(jw)}$

$$1d) \sin\left(\frac{\pi}{2}t\right) \cdot (u(t-1) - u(t+1))$$

$$\frac{1}{2\pi} \left( \frac{\pi}{j} \delta(w - \pi/2) - \frac{\pi}{j} \delta(w + \pi/2) \right) * \left( \frac{2 \sin(w)}{w} \right)$$

$$\frac{1}{j} \left( \frac{\sin(w - \pi/2)}{w - \pi/2} - \frac{\sin(w + \pi/2)}{w + \pi/2} \right) = -j \left( \frac{\cos(w)}{w - \pi/2} - \frac{\cos(w)}{w + \pi/2} \right) = \frac{2jw(\cos(w))}{w^2 - (\frac{\pi}{2})^2}$$

$$2w) \frac{1}{jw+5} - \frac{e^{-5jw}}{jw+5} \rightarrow e^{-5t}u(t) - e^{-5(t-2)}u(t-2) = \frac{8jw\cos(w)}{4w^2 - \pi^2}$$

$$2x) \frac{5}{(jw)^2 + 8jw + 16} = \frac{5}{(jw+4)^2} \rightarrow 5te^{-4t}u(t)$$

$b^2 - 4ac = 64 - 16 = 0$   
repeat:

$$2y) \frac{12jw}{(jw)^2 + 7jw + 10} = \frac{12jw}{(jw+2)(jw+5)} = \frac{A}{jw+2} + \frac{B}{jw+5}$$

$b^2 - 4ac = 49 - 40 > 0$   
two real

$$A = \lim_{jw \rightarrow -2} \frac{12jw}{jw+5} = \frac{-24}{3} = -8 \quad B = \lim_{jw \rightarrow -5} \frac{12jw}{jw+2} = \frac{-60}{-3} = 20$$

$$w(t) = (-8e^{-2t} + 20e^{-5t})u(t)$$

$$2z) \frac{15jw + 20}{(jw)^2 + 6jw + 13} = \frac{A(jw+3) + B(z)}{(jw+3)^2 + (z)^2}$$

$b^2 - 4ac = 36 - 52 < 0$   
MOAT

$$A = 15, \quad 15jw + 4S + 2B = 15jw + 20$$

$$2B = -25$$

$$B = -12.5$$

$$(jw + \frac{6}{z})^2 = (jw + 3)^2 = (jw)^2 + 6(jw) + 9 + 4$$

$$z(t) = e^{-3t} (15\cos(2t) - 12.5\sin(2t))u(t)$$

$$X(jw) = \frac{3}{jw+1} - \frac{2}{jw+4} = \frac{3jw+12-2jw-2}{(jw+1)(jw+4)} = \frac{jw+10}{(jw+1)(jw+4)}$$

$$Y(jw) = \frac{2}{jw+2} - \frac{2}{jw+4} = \frac{2jw+8-2jw-4}{(jw+2)(jw+4)} = \frac{4}{(jw+2)(jw+4)}$$

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{\frac{4}{(jw+2)(jw+4)}}{\frac{(jw+1)(jw+4)}{(jw+1)(jw+10)}} = \frac{4(jw+1)}{(jw+2)(jw+10)}$$

$$2) H(jw) = \frac{A}{jw+2} + \frac{B}{jw+10} \quad A = \lim_{jw \rightarrow -2} \frac{4(jw+1)}{jw+10} = \frac{4(-1)}{8} = -\frac{1}{2} \quad B = \lim_{jw \rightarrow -10} \frac{4(jw+1)}{(jw+2)} = \frac{4(-9)}{(-8)} = \frac{36}{8} = \frac{9}{2}$$

$$h(t) = \left( -\frac{1}{2} e^{-2t} + \frac{9}{2} e^{-10t} \right) u(t)$$

$$3) X(jw) = \frac{1}{jw+1} \quad X(jw) H(jw) = \frac{4}{(jw+2)(jw+10)} = \frac{A}{jw+2} + \frac{B}{jw+10}$$

$$A = \lim_{jw \rightarrow -2} \frac{4}{jw+10} = \frac{4}{8} = \frac{1}{2} \quad B = \lim_{jw \rightarrow -10} \frac{4}{jw+2} = \frac{4}{-8} = -\frac{1}{2}$$

$$y(t) = \left( \frac{1}{2} e^{-2t} - \frac{1}{2} e^{-10t} \right) u(t)$$

$$4) X(jw) = \frac{1}{jw+2} \quad X(jw) H(jw) = \frac{4(jw+1)}{(jw+2)^2(jw+10)} = \frac{A}{(jw+2)^2} + \frac{B}{(jw+2)} + \frac{C}{(jw+10)}$$

$$A = \lim_{jw \rightarrow -2} \frac{4(jw+1)}{(jw+10)} = \frac{-4}{8} = -\frac{1}{2}$$

$$\begin{aligned} & A(jw+10) + B(jw+2)(jw+10) + (jw+2)^2 \\ & A(jw+10) + B((jw)^2 + 12jw + 20) + ((jw)^2 + 4(jw) + 4) \end{aligned}$$

$$C = \lim_{jw \rightarrow -10} \frac{4(jw+1)}{(jw+2)^2} = \frac{4(-9)}{(-8)^2} = \frac{-36}{64} = -\frac{9}{16}$$

$$(jw)^2: B+C=0 \quad B=-C$$

$$B = \frac{9}{16}$$

$$x(t) = \left( -\frac{1}{2} t e^{-2t} + \frac{9}{16} e^{-2t} - \frac{9}{16} e^{-10t} \right) u(t)$$

$$5) (jw+2)(jw+10)Y = 4(jw+1)X$$

$$((jw)^2 + 12jw + 20)Y = (4jw+4)X$$

$$\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 20y(t) = 4\frac{dx(t)}{dt} + 4x(t)$$