

$$(1) \quad X(t) = -u(t+3) + u(t+2) + r(t+2) - \frac{3}{2}r(t) + \frac{1}{2}r(t-2) - u(t-2)$$

(2) ENERGY (BOUNDED, FINITE DURATION)

RECALL:



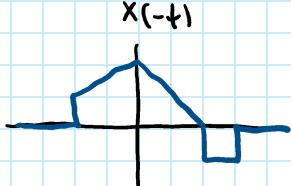
$$E_H = H^2 w \quad E_{\Delta} = \frac{1}{2} H^2 w \quad E_{\text{tri}} = \frac{1}{3} (H_1^2 + H_1 H_2 + H_2^2) w$$

$$\begin{aligned} E_{\infty} &= (-1)^2(1) + \frac{1}{2}(2)^2(2) + \frac{1}{3}(2^2 + 2 \cdot 1 + 1^2)(2) \\ &= 1 + \frac{8}{3} + \frac{14}{3} = 25/3 = 8\frac{1}{3} \end{aligned}$$

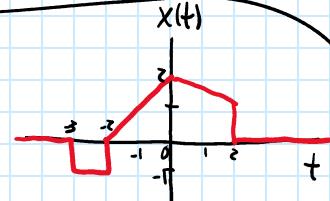
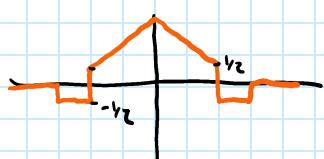
OR $\int_{-3}^{-2} (-1)^2 dt + \int_{-2}^0 (t+2)^2 dt + \int_0^2 (-\frac{t}{2}+2)^2 dt \quad (-\frac{t}{2}+2)^2 = \frac{t^2}{4} - 2t + 4$

$$\begin{aligned} &\left[t \right]_{-3}^{-2} + \left[\frac{t^3}{3} + 2t^2 + 4t \right]_{-2}^0 + \left[\frac{t^3}{12} - t^2 + 4t \right]_0^2 \\ &\left[-2 - -3 \right] + \left[0 - \left(-\frac{8}{3} + 8 - 8 \right) \right] + \left[\frac{8}{12} - 4 + 8 - 0 \right] \\ &1 + \frac{8}{3} + \frac{14}{3} = 8\frac{1}{3} \end{aligned}$$

I WAS GOING
TO ASK FOR
EVEN & ODD...



$$X_e = \frac{1}{2}(X(t) + X(-t))$$



$$X_o = \frac{1}{2}(X(t) - X(-t))$$



3)

Note SCALES



$$\begin{aligned} \text{OLD MAX-MIN} &= 3 \\ \text{NEW MAX-MIN} &= 9 \quad A = 3 \end{aligned}$$

$$\begin{aligned} \text{OLD DURATION} &= 5 \\ \text{NEW DURATION} &= 2.5 \quad |B| = 2 \end{aligned}$$

FLIPPED $B = -2$

DC SHIFT OF +1 : $D = +1$

OLD 0 MOVED TO -2

$$3(-2(t+2)) + 1$$

$$3(-2t - 4) + 1$$

2

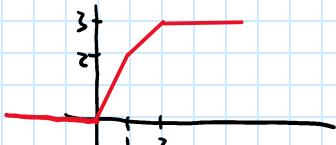
1a) $\cos(14\pi t) \cdot \sin(6\pi t) = \frac{1}{2} \sin(-8\pi t) + \frac{1}{2} \sin(20\pi t)$
 $w = 8\pi \quad w = 20\pi \quad w_0 = 4\pi \quad T = \frac{1}{2}$

1b) $\cos(4t) + \sin(4\pi t) \quad \frac{4}{4\pi} = \text{IRRATIONAL}$

1c) $\sum_{n=-\infty}^{\infty} u(t-1-3n) - u(t-2-3n) = \sum_{n=-\infty}^{\infty} g(t-T_n) \quad \underline{T=3}$
 PERIODIC

2k)  Energy = $(2)^2(1) + (1)^2(1) = 5 \quad \text{or} \quad \int_0^1 (2)^2 dt + \int_1^2 (1)^2 dt = [4t]_0^1 + [t]_1^2 = 4 - 0 + 2 - 1 = 5$

2l)



BOUNDED, INFINITE = POWER

$$\begin{aligned} P_\infty &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |m(t)|^2 dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \left(\int_0^{1/2} 4t^2 dt + \int_1^{T/2} (t+1)^2 dt + \int_2^{T/2} 3^2 dt \right) \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \left(\left[\frac{4t^3}{3} \right]_0^{1/2} + \left[\frac{(t+1)^3}{3} \right]_1^{T/2} + \left[9t \right]_2^{T/2} \right) \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \left(\left(\frac{1}{3} + \left(9 - \frac{8}{3} \right) + \left(\frac{9T}{2} - 18 \right) \right) \right) = \frac{9}{5} \end{aligned}$$

2m) PERIODIC: POWER

$$T = \pi$$

$$P_\infty = \frac{1}{T} \int_0^T |m(t)|^2 dt = \frac{1}{\pi} \int_0^\pi 25 \cos^2(2t) dt$$

$$= \frac{25}{\pi} \int_0^\pi \frac{1 + \cos(4t)}{2} dt = \frac{25}{\pi} \left[\frac{t}{2} + \frac{\sin(4t)}{8} \right]_0^\pi = \frac{25}{\pi} \left(\frac{\pi}{2} \right) = \frac{25}{2}$$

OR NOTE $P_\infty(A \cos(wt + \phi)) = \frac{A^2}{2}$

NOTE: 2n) was $n(t) = 5 \cos(2t) \cdot u(t)$

$$P_\infty(n(t)) = \frac{1}{2} P_\infty(m(t)) = \frac{25}{2}$$

IF $P_\infty(x(t)) = P$ for a periodic $x(t)$,

$$P_\infty(x(t)u(t-t_0)) = P/2$$

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

| System | Linear? | Time Inv.? | Stable? | Memoryless? | Causal? |
|--------------------------------|---------|------------|---------|-------------|---------|
| $y(t) = 2x(t) - 1$ | N | Y | Y | Y | Y |
| $y(t) = x(2t) - 1$ | N | N | Y | N | N |
| $y(t) = 2x(t - 1)$ | Y | Y | Y | N | Y |
| $y[n] = \frac{x[n]+1}{x[n]-1}$ | N | Y | N | Y | Y |
| $y[n] = \sum_{k=0}^5 x[n - k]$ | Y | Y | Y | N | Y |
| $y[n] = \sum_{k=0}^n x[n - k]$ | Y | N | N | N | N |

2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response ($h_i(t)$ or $h_i[n]$) or the step response ($s_{r,i}(t)$ or $s_{r,i}[n]$). You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

| System | Stable? | Memoryless? | Causal? |
|-------------------------|---------|-------------|---------|
| $h_1(t) = e^{-2 t }$ | Y | N | N |
| $h_2[n] = u[n - 1]$ | N | N | Y |
| $s_{r,3}(t) = 2u(t)$ | Y | Y | Y |
| $s_{r,4}[n] = u[n - 1]$ | Y | N | Y |

$$h_3(t) = 2\delta(t) \quad h_4[n] = \delta[n-1]$$

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

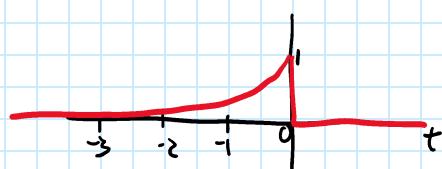
| System | Linear? | Time Inv.? | Stable? | Memoryless? | Causal? |
|--------------------------------|----------------------------|---------------------|---|-----------------------------------|-------------------------|
| $y(t) = 2x(t) - 1$ | FAILS $O \rightarrow O$ | Y | Y | Y | Y |
| $y(t) = x(2t) - 1$ | FAILS $O \rightarrow O$ | SCALES t | Y | $y(1)$ NEEDS $y(2)$ | $y(1)$ NEEDS $y(2)$ |
| $y(t) = 2x(t-1)$ | Y | Y | Y | $y(1)$ NEEDS $y(0)$ | Y |
| $y[n] = \frac{x[n]+1}{x[n]-1}$ | FAILS $O \rightarrow O$ | Y | $x[n]=1$ Blows up | Y | Y |
| $y[n] = \sum_{k=0}^5 x[n-k]$ | Y | Y | Y | $y(n)$ needs 5 different times | Y |
| $y[n] = \sum_{k=0}^n x[n-k]$ | Y | RELIES ON $x[0]$ | $x[n]=0 \rightarrow$ $\lim_{n \rightarrow \infty} y[n] \rightarrow \infty$ | $y(-1)$ needs $x[0]$ | $y(-1)$ needs $x[0]$ |

2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response ($h_i(t)$ or $h_i[n]$) or the step response ($s_{r,i}(t)$ or $s_{r,i}[n]$). You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

| System | Stable? | Memoryless? | Causal? |
|-----------------------|---|---------------------------------|------------------------------|
| $h_1(t) = e^{-2 t }$ | Y | $h(t) \neq 0$ for $t \neq 0$ | $h(t) \neq 0$ for $t < 0$ |
| $h_2[n] = u[n-1]$ | $\sum_{n=-\infty}^{\infty} h[n] \rightarrow \infty$ | $h(t) \neq 0$ for $t \neq 0$ | Y |
| $s_{r,3}(t) = 2u(t)$ | Y | Y | Y |
| $s_{r,4}[n] = u[n-1]$ | Y | $h[n] \neq 0$ for $n \neq 0$ | Y |

$$h_3(t) = 2\delta(t) \quad h_4[n] = \delta[n-1]$$

1) $x(t) = e^t u(-t)$

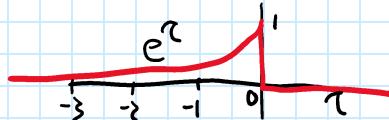


$$y(t) = u(t) - u(t-2)$$



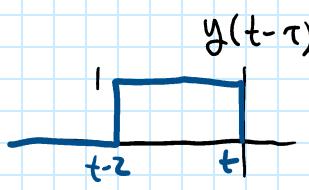
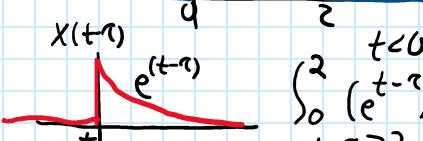
2) USE GRAPHICAL AL!

$$\int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$

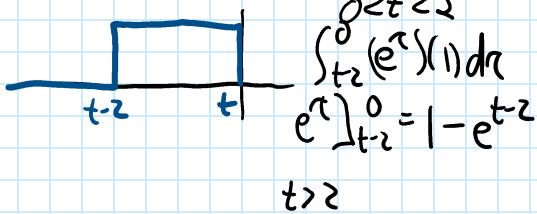


OR

$$\int_{-\infty}^{\infty} x(t-\tau) y(\tau) d\tau$$



$$= \int_{t-2}^t (e^\tau)(1) d\tau$$



$t > 2$

$$Z(t) = \begin{cases} e^t - e^{t-2} & t < 0 \\ 1 - e^{t-2} & 0 < t < 2 \\ 0 & t > 2 \end{cases}$$

$$\int_0^2 (e^{t-\tau})(1) d\tau$$

$$= -e^{t-\tau} \Big|_0^2 = -e^{t-2} + e^t$$

$$= e^t - e^{t-2}$$

$$\int_t^2 (e^{t-\tau})(1) d\tau$$

$$= -e^{t-\tau} \Big|_t^2 = -e^{t-2} + e^0$$

$$= 1 - e^{t-2}$$

$$\boxed{0}$$

$$Z(t) = \begin{cases} e^t - e^{t-2} & t < 0 \\ 1 - e^{t-2} & 0 < t < 2 \\ 0 & t > 2 \end{cases}$$

NOTE: 7 PEOPLE FILLED IN THE LETTERS FROM

- r - h - c - l

$$h(t) = r(t+1) - r(t) - u(t-2)$$

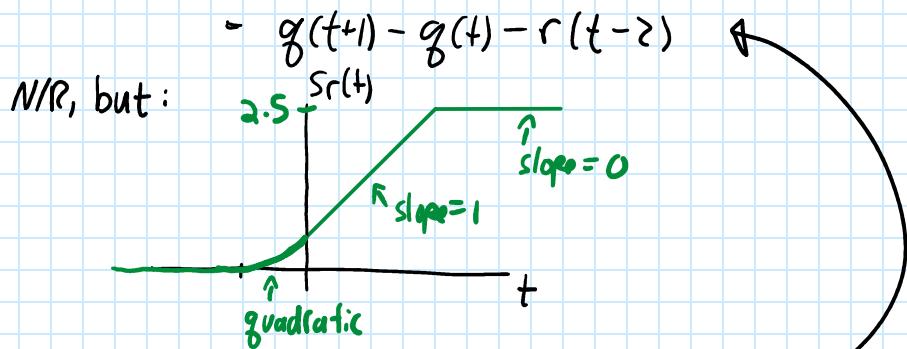
1)



$$2) \text{ YES, } \int_{-\infty}^{\infty} |h(t)| dt = 2.5 < \infty$$

3) NO, $h(t) \neq 0$ for all $t < 0$

$$4) S_r(t) = h(t) * u(t) = (r(t+1) - r(t) - u(t-2)) * u(t)$$



$$5) X_1(t) = u(t+1) - u(t-1) \quad y_1(t) = S_r(t+1) - S_r(t-1) \\ = g(t+2) - g(t+1) - r(t-1) \\ - g(t) + g(t-1) + r(t-3)$$

$$6) X_2(t) = r(t) \quad y_2(t) = r(t) * (r(t+1) - r(t) - u(t-2)) \\ = c(t+1) - c(t) - g(t-2)$$

**NOTE: BASICALLY
SECOND INTEGRAL OF $h(t)$**

$$h[n] = u[n] - \left(\frac{1}{3}\right)^n u[n]$$



2) NO $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |u[n] - \left(\frac{1}{3}\right)^n u[n]| = \sum_{n=0}^{\infty} \left(1 - \left(\frac{1}{3}\right)^n\right) \rightarrow \infty$

3) YES, $h[n] = 0$ FOR ALL $n < 0$

4) $h[n] * u[n] = \left(u[n] - \left(\frac{1}{3}\right)^n u[n]\right) * u[n]$

$$u[n] * u[n] = (n+1) u[n]$$

$$\left(-\left(\frac{1}{3}\right)^n u[n]\right) * u[n] = -\frac{\left(1 - \left(\frac{1}{3}\right)^{n+1}\right)}{\left(1 - \frac{1}{3}\right)} u[n]$$

$$= (n+1) u[n] - \frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n+1}\right) u[n]$$

or: $\sum_{k=-\infty}^{\infty} u[k] u[n-k] = u[n] \sum_{k=0}^n 1 = (n+1) u[n]$

$$\sum_{k=-\infty}^{\infty} u[n-k] \left(-\left(\frac{1}{3}\right)^k u[k]\right) = -u[n] \sum_{k=0}^n \left(\frac{1}{3}\right)^k = -u[n] \left(\frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \frac{1}{3}}\right)$$

$$\text{so } (n+1) u[n] - \frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n+1}\right) u[n]$$

5) $(u[n] - \left(\frac{1}{3}\right)^n u[n]) * \left(\frac{1}{z}\right)^n u[n]$

$$\frac{\left(\frac{1}{z}\right)^{n+1} - 1}{\left(\frac{1}{z}\right) - 1} u[n] - \frac{\left(\frac{1}{z}\right)^{n+1} - \left(\frac{1}{3}\right)^{n+1}}{\frac{1}{z} - \frac{1}{3}} u[n] = \left(-2\left(\frac{1}{z}\right)^{n+1} - 1\right) - 6\left(\left(\frac{1}{z}\right)^{n+1} - \left(\frac{1}{3}\right)^{n+1}\right) u[n]$$

$$= \left(6\left(\frac{1}{3}\right)^{n+1} - 8\left(\frac{1}{z}\right)^{n+1} + 2\right) u[n]$$

or: $\sum_{k=-\infty}^{\infty} (u[k] - \left(\frac{1}{3}\right)^k u[k]) \left(\frac{1}{z}\right)^{n-k} u[n-k]$

$$= u[n] \sum_{k=0}^n \left(\frac{1}{z}\right)^k - u[n] \sum_{k=0}^{\infty} \left(\frac{1}{z}\right)^k \left(\frac{1}{3}\right)^k = \left(\frac{1}{z}\right)^n \left(\frac{1 - z^{n+1}}{1 - z}\right) u[n] - \left(\frac{1}{z}\right)^n \left(\frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \frac{1}{3}}\right) u[n]$$

$$= -\left(\frac{1}{z}\right)^n \left(1 - z^{n+1}\right) u[n] - 3\left(\frac{1}{z}\right)^n \left(1 - \left(\frac{1}{3}\right)^{n+1}\right) u[n] = \left(-\left(\frac{1}{z}\right)^n + 2 - 3\left(\frac{1}{z}\right)^n + 2\left(\frac{1}{3}\right)^n\right) u[n] = \left(2\left(\frac{1}{3}\right)^n - 4\left(\frac{1}{z}\right)^n + 2\right) u[n]$$

VARIOUS
FORMS
OF
THESE.

SHORTCUTS

SUMMATIONS

$$= -\left(\frac{1}{2}\right)^n \left(1 - 2^{n+1}\right) u[n] - 3\left(\frac{1}{2}\right)^n \left(1 - \left(\frac{2}{3}\right)^{n+1}\right) u[n] = \left(-\left(\frac{1}{2}\right)^n + 2 - 3\left(\frac{1}{2}\right)^n + 2\left(\frac{1}{3}\right)^n\right) u[n] = \left(2\left(\frac{1}{3}\right)^n - 4\left(\frac{1}{2}\right)^n + 2\right) u[n]$$