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𝔈𝔡𝔪𝔲𝔫𝔡 𝔗. 𝔓𝔯𝔞𝔱𝔱, 𝔍𝔯. 𝔖𝔠𝔥𝔬𝔬𝔩 𝔬𝔣 𝔈𝔫𝔤𝔦𝔫𝔢𝔢𝔯𝔦𝔫𝔤

ECE 280 Fall 2021 Test II

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Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

    Only write on one side of any given page and **please be sure that your name** *and* **NetID are** *clearly* **written at the top of** *every* **page**. If you need more space for a particular problem or want to show more work, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page and submit those extra pages in problem-order **after** all pre-printed pages of the test. Also, in the space set aside for solutions to the problem, write a note that says "see extra page."

    Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**) and put them in the box with the top left corner of the test going into the back left corner of the folder. You must turn in all the original pages of the test even if all you wrote on them is your name and NetID. You do *not* need to staple your test - just make sure your name and NetID are on every page!

    Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or the convolution integral. Also, unless otherwise specified:

- The · symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $h(t)$ is the impulse response and both $s_\mathrm{r}(t)$ and $y_\mathrm{step}(t)$ represent the step response
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$
- $c(t)$ is the "unit" cubic $\frac{1}{6}t^3 \cdot u(t)$
- $r_{xy}(t) = x(-t) * y(t)$ is the cross-correlation function

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Problem I: [20 *pts.*] But I repeat myself...

1. Determine the non-zero Fourier Series coefficients $W[k]$ and the fundamental frequency ω_0 for the periodic signal:

$$w(t) = 2 + 3\cos(9t) \cdot \sin(4t)$$

2. Determine the time-domain representation for a periodic signal $x(t)$ with a period $T = 2$ s and the following non-zero Fourier Series coefficients:

$$X[k] = \begin{cases} k = 5, & 3 + j2 \\ k = 3, & -4 - j5 \\ k = 0, & 7 \\ k = -3 & -4 + j5 \\ k = -5 & 3 - j2 \end{cases}$$

3. The signal $x(t)$ from above is used as the input to an LTI system with an impulse response of:

$$h(t) = 10\frac{\sin(11t)}{\pi t}$$

Find the response $y(t)$ for this system with $x(t)$ as an input.

4. Sketch a few periods of the periodic signal $z(t)$ given below, then determine the fundamental frequency ω_0 and an expression for the Fourier Series coefficients $Z[k]$ of:

$$z(t) = \sum_{d=-\infty}^{\infty} \left(2u(t - 1 - 5d) - u(t - 3 - 5d) - u(t - 4 - 5d)\right)$$

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Extra work for Problem I

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Problem II: [25 *pts.*] ↔

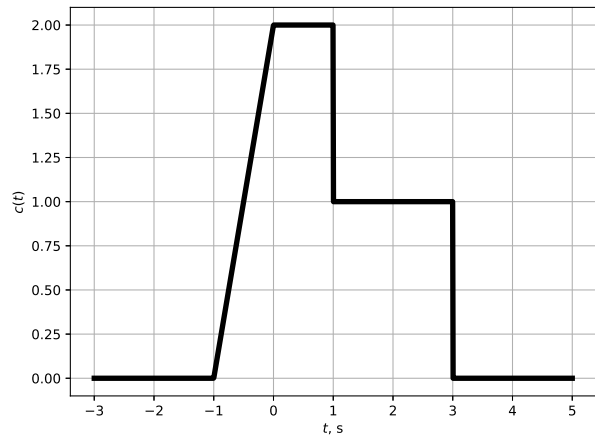1. Determine the Fourier Transform for the following signals:

(a) $a(t) = e^{-3t} u(t-1)$

(b) $b(t) = 4e^{-5t} \left(\cos(6t) + \sin(7t) \right) u(t)$

(note that the frequencies are different!)
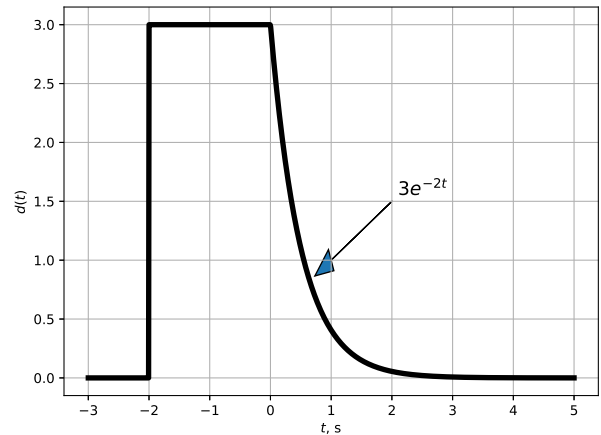
(c) $c(t)$:

$$c(t) = \begin{cases} t < -1, & 0 \\ -1 \le t < 0, & 2t+2 \\ 0 \le t < 1, & 2 \\ 1 \le t < 3, & 1 \\ t \ge 3, & 0 \end{cases}$$

(d) $d(t)$ (be careful!):

$$d(t) = \begin{cases} t < -2, & 0 \\ -2 \le t < 0, & 3 \\ t \ge 0, & 3e^{-2t} \end{cases}$$

2. Determine the inverse Fourier Transform for the following signals:

(w) $W(j\omega) = \frac{1 - e^{-3j\omega}}{j\omega + 4}$

(x) $X(j\omega) = \frac{60j\omega}{(j\omega)^2 + 4j\omega + 3}$

(y) $Y(j\omega) = \frac{60j\omega}{(j\omega)^2 + 4j\omega + 4}$

(z) $Z(j\omega) = \frac{60j\omega}{(j\omega)^2 + 4j\omega + 5}$

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Extra work for Problem II

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More extra work for Problem II

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Problem III: [20 *pts.*] System Analysis

An LTI system has an input signal $x(t)$, an output signal $y(t)$, and an impulse response

$$h(t) = e^{-2t}u(t)$$

1. Is the system stable?

2. Is the system causal?

3. What is the transfer function for the system, $H(j\omega)$?

4. What is the step response of the system, $s_r(t) = y_{\text{step}}(t)$?

5. Clearly using Fourier Series and/or inverse Fourier Series and/or phasor analysis, find the AC steady-state response of the system to an input $x_1(t) = 1 + 2\cos(2t)$? You may leave square roots in your answer but you may not leave imaginary numbers in your answer!

6. Clearly using Fourier Transforms and/or inverse Fourier Transforms, find the response of the system to an input $x_2(t) = e^{-2t}u(t)$.

7. Clearly using Fourier Transforms and/or inverse Fourier Transforms, find the response of the system to an input $x_3(t) = e^{-3t}u(t)$.

8. Write a differential equation that models this system, relating $y(t)$ and its derivatives to $x(t)$ and its derivatives.

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Extra work for Problem III

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Problem IV: [**25** *pts.*] **Sampling and Reconstruction**

Assume you have the following signals:

$$w(t) = \sin(100t) + \cos(200t) \qquad x(t) = u(t + 0.05) + u(t - 0.05) \qquad y(t) = \frac{\sin 150t}{\pi t}$$

1. Which of the three signals is band-limited? That is, for which signals is there some frequency ω_M above which the signal has no energy or power? If a signal is band-limited, also give its band-limit. You may want to sketch the Fourier Transform though that is not explicitly required.

2. Which of the following signals (formed by products) are band-limited? If a signal is band-limited, also give its band-limit:

   (a) $a(t) = w(t) \cdot x(t)$
   (b) $b(t) = w(t) \cdot y(t)$
   (c) $c(t) = x(t) \cdot y(t)$

3. Which of the following signals (formed by convolutions) are band-limited? If a signal is band-limited, also give its band-limit:

   (d) $d(t) = w(t) * x(t)$
   (e) $e(t) = w(t) * y(t)$
   (f) $f(t) = x(t) * y(t)$

4. Imagine that $w(t)$ is sampled using an impulse train sampler with a function of

   $$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_S)$$

   meaning a sampling period of T_S (or sampling frequency ω_S) and recovered using an ideal low-pass filter $H(j\omega)$ with a gain of T_S and a cutoff frequency ω_{co}.

   (a) Make a properly labeled sketch of $p(t)$ in the time domain, $P(j\omega)$ in the frequency domain, and $H(j\omega)$ in the frequency domain. Be sure to label your graphs using T_S, ω_S, and ω_{co} as appropriate.

   (b) Draw a block diagram showing the system starting with the original signal, creating the sampled signal (call it $w_p(t)$), and recovering the signal from samples using an ideal low-pass filter (call that $w_r(t)$).

   (c) What does the sampling period need to be in order to have a shot at perfectly reconstructing $w(t)$?

   (d) Assuming you have a sampling frequency w_S that will allow perfect reconstruction, What is the relationship between the band-limit ω_M of the signal (hint - go back and check part 1 to make sure you said $w(t)$ was band-limited!), the sampling frequency w_S, and the cutoff frequency of the low-pass filter ω_{co} to perfectly recover the original signal?

   (e) If $\omega_S = 300$ rad/s and $\omega_{co} = 250$ rad/s, what is the recovered signal $w_r(t)$

   (f) If $\omega_S = 400$ rad/s and $\omega_{co} = 250$ rad/s, what is the recovered signal $w_r(t)$

   (g) If $\omega_S = 500$ rad/s and $\omega_{co} = 250$ rad/s, what is the recovered signal $w_r(t)$

   (h) If $\omega_S = 600$ rad/s and $\omega_{co} = 150$ rad/s, what is the recovered signal $w_r(t)$

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Extra work for Problem IV

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More extra work for Problem IV

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Problem V: [**10** *pts.*] **Communication Systems**

Write the correct term(s) from the list given below the image that depicts that term. Note that some images may have multiple appropriate terms and also that some terms are not pictured at all! You can either write the terms or term numbers below the images. Put a line through the terms you use at least once - any remaining terms are thus not represented in the images.

1. (Full) Amplitude Modulation
2. Asynchronous Demodulation
3. Double Sideband - Suppressed Carrier Modulation
4. Envelope Detection
5. Frequency Division Multiplexing
6. Frequency Modulation
7. Synchronous Demodulation
8. Turboencabulation