Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280 Fall 2021 Test I

Name (please print): NetID (please print):

Submitting your work for a grade implies agreement with the following: In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to communicate with anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did communicate with another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure that your name *and* **NetID are** *clearly* **written at the top of** *every* **page**. If you need more space for a particular problem or want to show more work, put that work on its own piece of paper, clearly write your name, NetID, and the problem number (in either Arabic or Roman numerals) at the **top center** of that page and submit those extra pages in problem-order **after** all pre-printed pages of the test. Also, in the box for the problem, write a note that says "see extra page."

Carefully stack the test pages in order (with any additional pages properly labeled and **after all the original test pages**) and put them in the box with the top left corner of the test going into the back left corner of the folder. You must turn in all the original pages of the test even if all you wrote on them is your name and NetID. You do *not* need to staple your test - just make sure your name and NetID are on every page!

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or the convolution integral. Also, unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $h(t)$ is the impulse response and both $s_{\mathrm{r}}(t)$ and $y_{\mathrm{step}}(t)$ represent the step response
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$
- $c(t)$ is the "unit" cubic $\frac{1}{6}t^3 \cdot u(t)$
- $r_{xy}(t) = x(-t) * y(t)$ is the cross-correlation function

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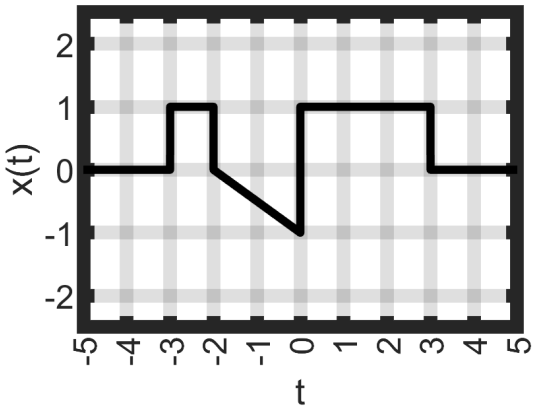
Problem I: [15 *pts.*] Signals I

1. Given the following graph of $x(t)$, and noting that $x(t) = 0$ for all $|t| > 3$,



- Write an equation for the signal using singularity functions:
  $x(t) =$

- Does $x(t)$ represent an energy signal, a power signal, or neither? If it is an energy signal, also give its total energy E_∞; if it is a power signal, calculate its overall average power P_∞.

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- Given $x(t)$, make a sketch of the even part of $x(t)$ and a sketch of the odd part of $x(t)$. Be sure to label your sketches.

- Given $x(t)$, accurately sketch the transformed signal

$$y(t) = 4x(-3t + 6) + 1$$

Be sure to put labels and values on your axes.

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Problem II: [10 *pts.*] Signals 2

1. For each of the following signals, determine if they are periodic or not. If they are periodic, state the period:
   (a) $a(t) = \cos(6\pi t) + \sin(12\pi t) + \cos(15\pi t + 12°)$
   (b) $b(t) = \sin(7t) \cdot \cos(17t)$
   (c) $c(t) = (\cos(7t) \cdot \cos(9t)) + \cos(11t)$

2. For each of the following signals, determine if they are energy signals, power signals, or neither. If a signal is an energy signal, also give its total energy E_∞; if a signal is a power signal, calculate its overall average power P_∞.
   (k) $k(t) = r(t+1) - 2r(t) + r(t-1)$
   (l) $l(t) = \sum_{n=-\infty}^{\infty} k(t-5n)$ where $k(t)$ is given above.
   (m) $m(t) = (1 - e^{-t}) \cdot u(t)$

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Problem III: [25 *pts.*] System Classifications

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
$y(t) = \frac{2}{x(t)+2}$					
$y(t) = x(t-1)$					
$y(t) = \int_{t-3}^{t-1} x(\tau+1)\ d\tau$					
$y(t) = x(2t) \cdot \cos(t)$					
$y(t) = x(t) + \cos(2t)$					

2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response $h_i(t)$ or the step response $s_{\mathrm{r,i}}(t)$. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Stable?	Memoryless?	Causal?
$h_1(t) = (1 - e^{-t}) \cdot u(t)$			
$h_2(t) = u(t+1) - u(t-1)$			
$s_{\mathrm{r},3}(t) = (1 - e^{-t}) \cdot u(t)$			
$s_{\mathrm{r},4}(t) = \sin(t) \cdot u(t)$			

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Problem IV: [25 *pts.*] Basic Convolution and Correlation

1. Write an **integral** formula to calculate the convolution of generic signals $x(t)$ and $y(t)$:

   $x(t) * y(t) =$

2. Write an **integral** formula to calculate the cross-correlation function of generic signals $x(t)$ and $y(t)$:

   $r_{xy}(t) =$

3. Given the following specific functions:

$$x(t) = u(t+1) - r(t) + r(t-1) \qquad\qquad y(t) = u(t+1) - u(t-2)$$

   - Make labeled sketches of $x(t)$, $y(t)$, $x_m(t) = x(-t)$, $y_m(t) = y(-t)$ and, for the last two, write formulas for them as functions of right-facing singularity functions.

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Solve the following convolutions and correlations but do not make sketches of them - remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page:

- $A(t) = x(t) * x(t)$
- $B(t) = x(t) * y(t)$
- $C(t) = r_{xx}(t)$
- $D(t) = r_{yy}(t)$
- $E(t) = r_{xy}(t)$
- Determine the measure of correlation between $x(t)$ and $y(t)$.

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Problem V: [25 *pts.*] System Analysis

A linear, time-invariant system S has an impulse response of:

$$h(t) = u(t) - 3u(t-2) + 2u(t-3)$$

1. Make a sketch of $h(t)$.
2. Is the system stable? Why do you believe that to be the case?
3. Is the system causal? Why do you believe that to be the case?
4. Determine an expression for the step response of the system, $s_{\mathrm{r}}(t)$ (a.k.a. $y_{\mathrm{step}}(t)$)

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5. Determine an expression for the output of this system to the input

$$x_1(t) = u(t) - u(t-2)$$

and call this output $y_1(t)$. Remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page.

6. Determine an expression for the output of this system to the input

$$x_2(t) = e^{-t} \cdot u(t)$$

and call this output $y_2(t)$. Remember, you *cannot* leave unevaluated integrals or convolutions but you can use the singularity functions defined on the cover page.