Problem 1

Monday, November 15, 2021

1) $w(t) = 2 + 3\cos(9t)\cdot\sin(4t) = 2 + \frac{3}{2}\left(\sin(-5t) + \sin(13t)\right)$

$= 2 - \frac{3}{2}\sin(5t) + \frac{3}{2}\sin(13t)$

$w_0 = 1 \qquad W[k] = \begin{cases} k=13 & 3/4j \\ k=5 & -3/4j \\ k=0 & 2 \\ k=-5 & 3/4j \\ k=-13 & -3/4j \end{cases}$

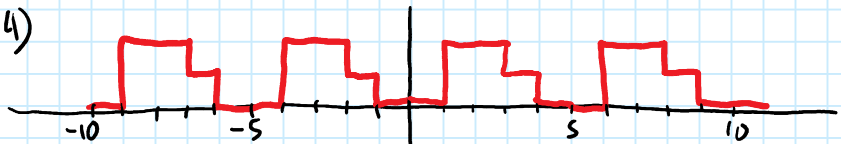
2) $T = 2 \quad w_0 = \frac{2\pi}{T} = \pi$

$x(t) = 6\cos(5\pi t) - 4\sin(5\pi t) - 8\cos(3\pi t) + 10\sin(3\pi t) + 7$

3) $H(jw) = 10\left(u(w+11) - u(w-11)\right) \to$ LPF, $w_{co} = 11$; $\quad 3\pi < 11, \quad 5\pi > 11$, so

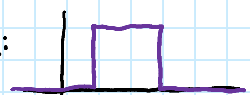
$y(t) = -80\cos(3\pi t) + 100\sin(3\pi t) + 70$

4)



$T = 5 \quad w_0 = \frac{2\pi}{5}$

opt 1:



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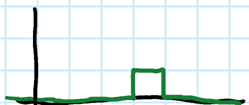
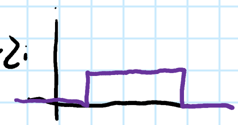


width = 2
height = 2
center = 2

width = 1
height = 1
center = 7/2

$Z[k] = (2)\frac{(2)}{5}\,\text{sinc}\left(k\cdot\frac{2}{5}\right)e^{-jk\frac{2\pi}{5}\cdot 2} + (1)\frac{(1)}{5}\,\text{sinc}\left(k\cdot\frac{1}{5}\right)e^{-jk\frac{2\pi}{5}\cdot\frac{7}{2}}$

opt 2:



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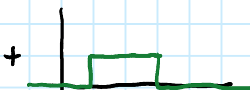


width = 3
height = 1
center = 5/2

width = 2
height = 1
center = 2

$\frac{(1)(3)}{(5)}\,\text{sinc}\left(k\cdot\frac{3}{5}\right)e^{-jk\frac{2\pi}{5}\cdot\frac{5}{2}} + \frac{(1)(2)}{(5)}\,\text{sinc}\left(k\cdot\frac{2}{5}\right)e^{-jk\frac{2\pi}{5}\cdot 2}$

Problem 2

1) a) $e^{-3t}u(t-1) = e^{-3(t-1+1)}u(t-1) = e^{-3}e^{-3(t-1)}u(t-1) \rightarrow \dfrac{e^{-3}e^{-j\omega}}{j\omega+3}$

b) $4e^{-5t}(\cos(6t)+\sin(7t))u(t) \rightarrow \dfrac{4(j\omega+5)}{(j\omega+5)^2+(6)^2} + \dfrac{4(7)}{(j\omega+5)^2+(7)^2}$

c) $2r(t+1) - 2r(t) - u(t-1) - u(t-3)$ FINITE DURATION, SO

$C(j\omega) = \dfrac{2e^{j\omega}}{(j\omega)^2} - \dfrac{2}{(j\omega)^2} - \dfrac{e^{-j\omega}}{j\omega} - \dfrac{e^{-j3\omega}}{j\omega}$

d) $3u(t-2) - 3u(t) + 3e^{-2t}u(t)$ FINITE DURATION FOR PULSE, SO

$D(j\omega) = \dfrac{3}{j\omega}e^{j2\omega} - \dfrac{3}{j\omega} + \dfrac{3}{j\omega+2}$

2 w) $\dfrac{1-e^{-3j\omega}}{j\omega+4} = e^{-4t}u(t) - e^{-4(t-3)}u(t-3)$

x) $4^2 - 4\cdot 3 > 0 \rightarrow \dfrac{60j\omega}{(j\omega+1)(j\omega+3)} = \dfrac{A}{j\omega+1} + \dfrac{B}{j\omega+3}$

$A = \lim_{j\omega \to -1} \dfrac{60j\omega}{j\omega+3} = \dfrac{-60}{2} = -30$

$B = \lim_{j\omega \to -3} \dfrac{60j\omega}{j\omega+1} \quad \dfrac{-180}{-2} = 90$

$x(t) = -30e^{-t}u(t) + 90e^{-3t}u(t)$

y) $4^2 - 4\cdot 4 = 0 \rightarrow \dfrac{60j\omega}{(j\omega+2)^2} = \dfrac{A}{(j\omega+2)^2} + \dfrac{B}{(j\omega+2)} = \dfrac{A + B(j\omega+2)}{(j\omega+2)^2}$

$A = \lim_{j\omega \to -2} 60j\omega = -120 \quad B = 60$ FROM $60j\omega = Bj\omega$

$y(t) = -120te^{-2t}u(t) + 60e^{-2t}u(t)$

alt: $g = \mathcal{F}^{-1}\left\{\dfrac{1}{(j\omega+2)^2}\right\} \quad g = te^{-2t}u(t)$

$y = 60\dfrac{dg}{dt} = 60e^{-2t}u(t) - 120te^{-2t}u(t) + \cancel{60te^{-2t}\delta(t)}$ 0

z) $4^2 - 4\cdot 5 < 0 \rightarrow \dfrac{60j\omega}{(j\omega+2)^2+(1)^2} = \dfrac{A(j\omega+2) + B(1)}{(j\omega+2)^2+(1)^2}$

$A = 60$ FROM $j\omega$, $2A + B = 0$ $B = -120$ FROM $(j\omega)^0$

$z(t) = e^{-2t}(60\cos(t) - 120\sin(t))u(t)$

Problem 3

1) $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$ yes

2) $h(t) = 0,\ t < 0$ yes

3) $H(j\omega) = \frac{1}{j\omega + 2}$

4) opt. 1: $s_r = \int_{-\infty}^{t} h(\tau)\, d\tau$

$= \int_{-\infty}^{t} e^{-2\tau} (\tau)\, d\tau$

$= u(t) \int_{0}^{t} e^{-2\tau}\, d$

$= u(t) \left[\frac{e^{-2\tau}}{-2} \right]_0^t = u(t) \left(\frac{1 - e^{-2t}}{2} \right)$

opt 2: $S_r(j\omega) = \frac{H(j\omega)}{j\omega} = \frac{1}{j\omega(j\omega + 2)}$

$= \frac{A}{j\omega} + \frac{B}{j\omega + 2}$

$A = \lim_{j\omega \to 0} \frac{1}{j\omega + 2} = \frac{1}{2}$ $\quad B = \lim_{j\omega \to -2} \frac{1}{j\omega} = \frac{-1}{2}$

$\boxed{s_r(t) = \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t)}$

5) $x_1(t) = 1 + 2\cos(2t)$ $\quad w_0 = 2$ $\quad X[k] = \begin{cases} k = +1 & 1 \\ k = 0 & 1 \\ k = -1 & 1 \end{cases}$

$H(j0) = \frac{1}{2}$ $\quad H(j2) = \frac{1}{j2 + 2} = \frac{2 - j2}{8} = \frac{2\sqrt{2}}{8} \angle -45^\circ$ $\quad H(-j2) = H^*(j2) = \frac{2 + j2}{8} = \frac{2\sqrt{2}}{8} \angle 45^\circ$

$Y_1[k] = \begin{cases} k = 1 & \frac{1}{4} - \frac{j}{4} = & \frac{\sqrt{2}}{4} \angle -45^\circ \\ k = 0 & 1/2 & \\ k = -1 & \frac{1}{4} + \frac{j}{4} & \frac{\sqrt{2}}{4} \angle 45^\circ \end{cases}$ $\quad y_1(t) = \frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t) + \frac{1}{2}$

(PHASORS: $w = 0$ $\quad X = 1$ $\quad H = 1/2$ $\quad Y = 1/2$ $\quad y = 1/2$

$w = 2$ $\quad X = 2\angle 0^\circ$ $\quad H = \frac{\sqrt{2}}{4} \angle 45^\circ$ $\quad Y = \frac{\sqrt{2}}{2} \angle -45^\circ$ $\quad y = \frac{\sqrt{2}}{2} \cos(2t - 45^\circ)$

6) $X_2(j\omega) = \frac{1}{j\omega + 2}$ $\quad Y_2(j\omega) = \frac{1}{(j\omega + 2)^2}$ $\quad y_2(t) = t e^{-2t} u(t)$

7) $X_3(j\omega) = \frac{1}{j\omega + 3}$ $\quad Y_3(j\omega) = \frac{1}{(j\omega + 2)(j\omega + 3)} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 3}$ $\quad A = \lim_{j\omega = -2} \frac{1}{j\omega + 3} = 1$ $\quad B = \lim_{j\omega = -3} \frac{1}{j\omega + 2} = -1$

$y_3(t) = e^{-2t} u(t) - e^{-3t} u(t)$

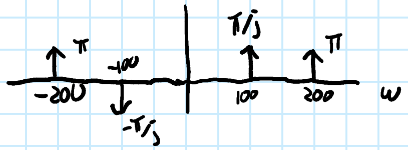
8) $\frac{Y}{X} = \frac{1}{j\omega + 2}$ $\quad (j\omega + 2) Y = X$

$\frac{dy}{dt} + 2y = x$

Problem 4

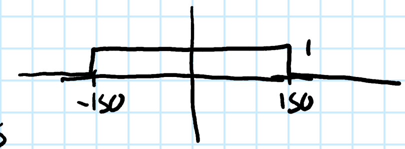
1) $w(t)$ BAND-LIMITED AT 200 rad/s $x(t)$ NOT BAND-LIMITED $y(t)$ BAND-LIMITED TO 150 rad/s

$W(j\omega)$



$X(j\omega) = \dfrac{2\sin(0.05\,\omega)}{\omega}$

$Y(j\omega)$



2) a) $\frac{1}{2\pi}(W * X)$ not band limited

b) $\frac{1}{2\pi}(W * Y)$ band limited to 350 rad/s

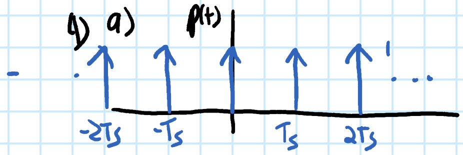
c) $\frac{1}{2\pi}(X * Y)$ not band limited

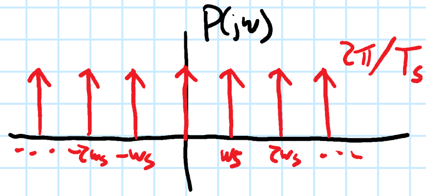
3) d) $W \cdot X$ band limited to 200 rad/s

$W \cdot Y$ band limited to $\underline{100}$ rad/s

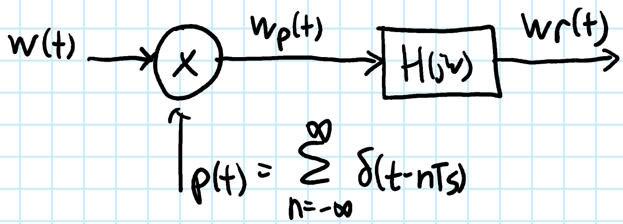
$X \cdot Y$ band limited to 150 rad/s

4) a)





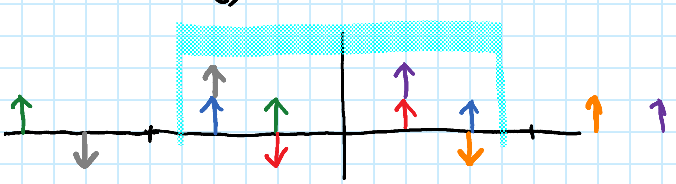
b)



c) $\omega_s > 2\omega_M$ so > 400 rad/s and thus $\boxed{T_s < \frac{\pi}{200}}$

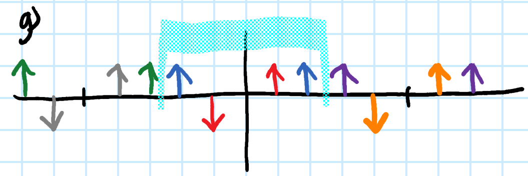
d) $\omega_M < \omega_{co} < \omega_s - \omega_M$

e)



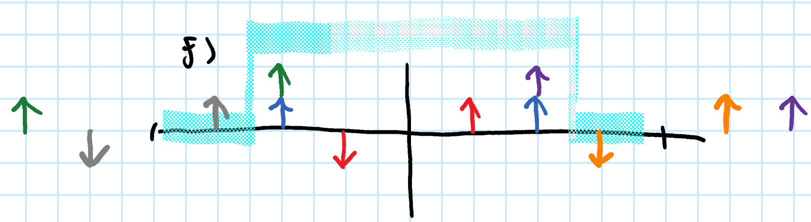
$\sin(100t) + \cos(100t) - \sin(200t) + \cos(200t)$

g)



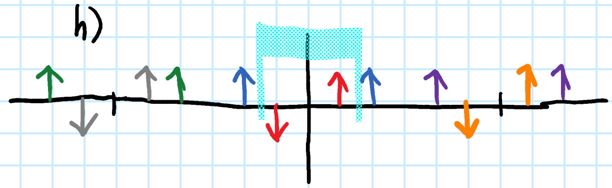
$\sin(100t) + \cos(200t)$

f)



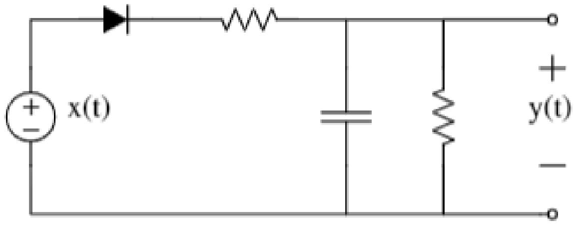
$\sin(100t) + 2\cos(200t)$

h)

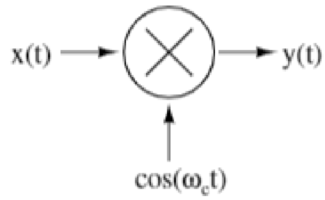


$\sin(100t)$

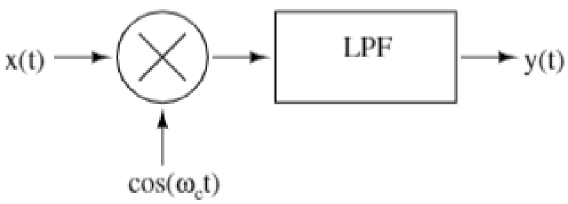
Problem 5



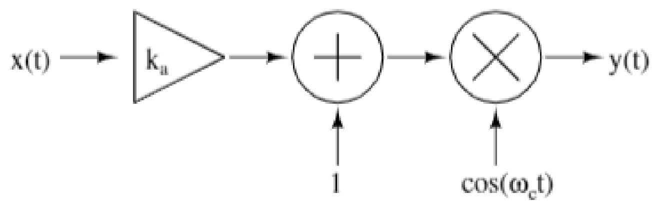
ASYNCHRONOUS DEMOD. OR ENVELOPE DETECTION



DSB-SC MOD.



SYNCHRONOUS DEMOD.



(FULL) AM

NO: FDM, FM, OR TURBOENCABULATION!