

# ECE 280 Test 1

Monday, October 11, 2021

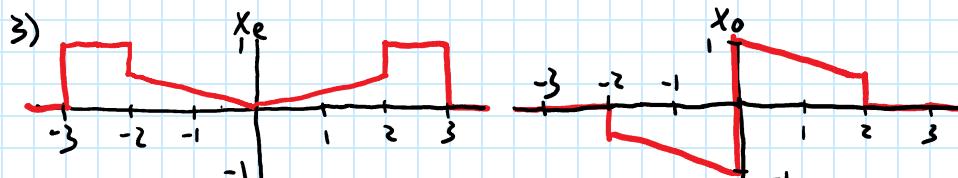
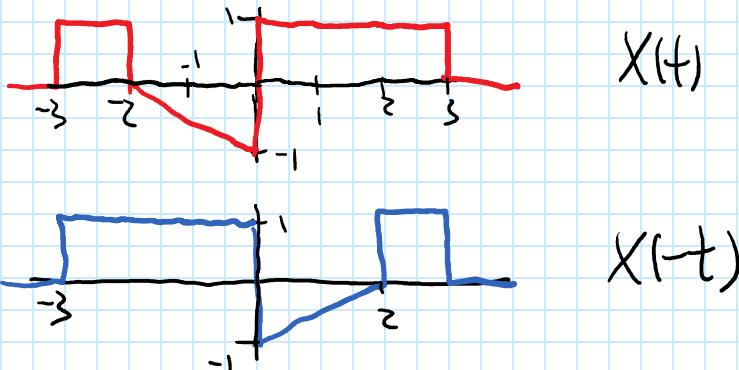
1)  $x(t) = u(t+3) - u(t+2) - 0.5 u(t-2) + 2 u(t) + 0.5 u(t) - u(t-3)$

2) ENERGY (BOUNDED & FINITE DURATION)

$$E(-3, -2) = H^2 W = (1)^2 (1) = 1$$

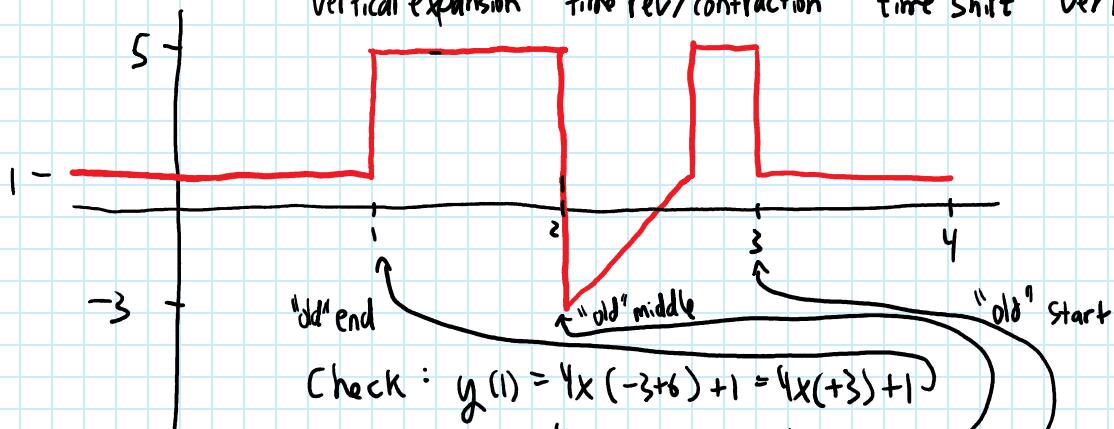
$$E(-2, 0) = \sum_{k=-3}^{-1} (1)^2 (2) = \frac{2}{3} \quad E = 1 + \frac{2}{3} + 3 = 4 \frac{2}{3} = \frac{14}{3}$$

$$E(0, 3) = H^2 W = (1)^2 (3) = 3$$



4)  $4x(-3t+6)+1 = 4x(-3(t-2))+1$

vertical expansion      time rev/contraction      time shift      vertical shift



Check:  $y(1) = 4x(-3+6)+1 = 4x(+3)+1$

$y(2) = 4x(-6+6)+1 = 4x(0)+1$

$y(3) = 4x(-9+6)+1 = 4x(-3)+1$

## Problem 2

a)  $\cos(6\pi t) + \sin(12\pi t) + \cos(15\pi t + 12^\circ)$

$$\omega = 6\pi, 12\pi, 15\pi \quad GCF = 3\pi, T = \frac{2\pi}{3\pi} = \frac{2}{3}$$

PERIODIC

b)  $\sin(7t) \cdot \cos(17t)$  means  $\omega = 17 \pm 7 = 10, 24$  COMMENSURABLE; PERIODIC

$$GCF = 2, T = \frac{2\pi}{2} = \pi$$

c)  $\cos(7t) \cdot \cos(9t)$  means  $\omega = 9 \pm 7 = 2, 16$  Also  $\omega = 11$  COMMENSURABLE; PERIODIC

$$GCF = 1, T = \frac{2\pi}{1} = 2\pi$$

2 k)



$$\text{ENERGY} = \frac{1}{3}(1)^2(1) + \frac{1}{3}(1)^2(1) = \frac{2}{3}$$

$$\int_{-T/2}^{T/2} |f(t)|^2 dt$$

l) PERIODIC & BOUNDED  $\Rightarrow$  PERIODIC

$$P_\infty = \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt = \frac{1}{T} E_{\text{one period}} = \frac{2/3}{5} = \frac{2}{15}$$

m)



INFINITE DURATION, DOES NOT DECAY  $\rightarrow$  NOT ENERGY

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} T h(1 - 2e^{-t} + e^{-2t}) dt$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \left[ t - \frac{2e^{-t}}{-1} + \frac{e^{-2t}}{-2} \right]_0^{T/2}$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \left[ \frac{T}{2} + 2e^{-T/2} - 2e^{-T} - 0 - 2e^0 + 2e^0 \right]$$

$$= \frac{1}{2}$$

### Problem 3

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
$y(t) = \frac{2}{x(t)+2}$	N	Y	N	Y	Y
$y(t) = x(t-1)$	Y	Y	Y	N	Y
$y(t) = \int_{t-3}^{t-1} x(\tau+1) d\tau$	Y	Y	Y	N	Y
$y(t) = x(2t) \cdot \cos(t)$	Y	N	Y	N	N
$y(t) = x(t) + \cos(2t)$	N	N	Y	Y	Y

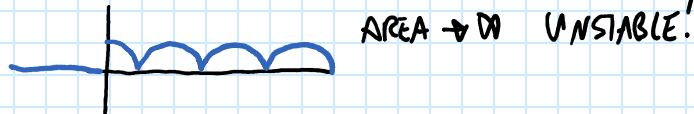
2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response  $h_i(t)$  or the step response  $s_{r,i}(t)$ . You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Stable?	Memoryless?	Causal?
$h_1(t) = (1 - e^{-t}) \cdot u(t)$	N	N	Y
$h_2(t) = u(t+1) - u(t-1)$	Y	N	N
$s_{r,3}(t) = (1 - e^{-t}) \cdot u(t)$	Y	N	Y
$s_{r,4}(t) = \sin(t) \cdot u(t)$	N	N	Y

$$1) \int_{-\infty}^{\infty} |(1 - e^{-t})u(t)| dt = \int_0^{\infty} (1 - e^{-t}) dt = [t + e^{-t}]_0^{\infty} = \underline{\underline{\underline{\underline{\underline{0+0-(0+1)}}}}}$$

$$3) h_{r,3} = \frac{ds_{r,3}}{dt} = e^{-t} u(t) \quad \int_{-\infty}^{\infty} |e^{-t} u(t)| dt = \int_0^{\infty} e^{-t} dt = [-e^{-t}]_0^{\infty} = 0 - -1 = 1 \checkmark$$

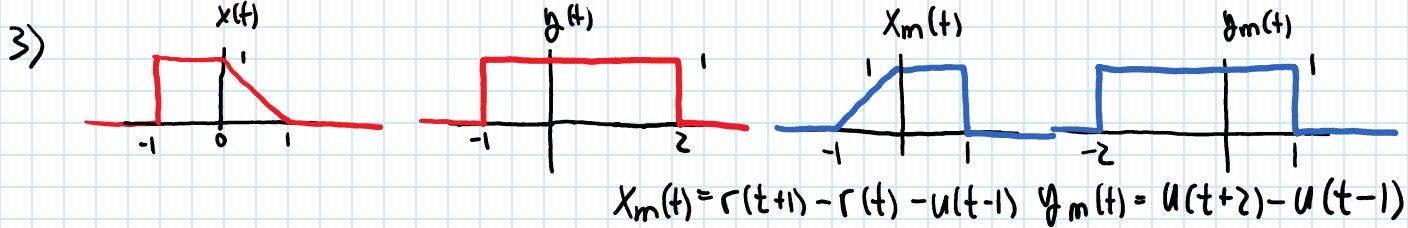
$$4) h_{r,4} = \frac{ds_{r,4}}{dt} = \frac{d}{dt} (\sin(t) u(t)) = (\cos(t) u(t) + \underbrace{\sin(t) \delta(t)}_0 \text{ since at } t=0, \sin(t)=0) \\ \int_{-\infty}^{\infty} |\cos(t) u(t)| dt = \int_0^{\infty} |\cos(t)| dt$$



## Problem 4

$$1) X(t) * y(t) = \int_{-\infty}^{\infty} X(\tau) y(t-\tau) d\tau \text{ OR } \int_{-\infty}^{\infty} X(t-\tau) y(\tau) d\tau$$

$$2) r_{xy}(t) = \int_{-\infty}^{\infty} X(\tau) y(t+\tau) d\tau \text{ OR } \int_{-\infty}^{\infty} X(\tau-t) y(\tau) d\tau$$



$$A = (u(t+1) - r(t) + r(t-1)) * (u(t+1) - r(t) + r(t-1)) = \\ r(t+2) - g(t+1) + g(t) \\ - g(t+1) + c(t) - c(t-1) \\ + g(t) - c(t-1) + c(t-2)$$

$$= r(t+2) - 2g(t+1) + c(t) + 2g(t) - 2c(t-1) + c(t-2)$$

$$B = (u(t+1) - r(t) + r(t-1)) * (u(t+1) - u(t-2)) \\ r(t+2) - r(t-1) - g(t+1) + g(t-2) + g(t) - g(t-3)$$

$$C = X_m(t) * X(t) = (r(t+1) - r(t) - u(t-1)) * (u(t+1) - r(t) + r(t-1)) \\ g(t+2) - c(t+1) + c(t) \\ - g(t+1) + c(t) - c(t-1) \\ - r(t) + g(t-1) - g(t-2)$$

$$= g(t+2) - c(t+1) - g(t+1) + 2c(t) - r(t) \\ - c(t-1) - g(t-1) - g(t-2)$$

$$D = r_{xy}(t) = y_m(t) * y(t) = (u(t+2) - u(t-1)) * (u(t+1) - u(t-2)) \\ = r(t+3) - r(t) - r(t) + r(t-3) = r(t+3) - 2r(t) + r(t-3)$$

$$E = X_m(t) * y(t) = (r(t+1) - r(t) - u(t-1)) * (u(t+1) - u(t-2)) \\ g(t+2) - g(t-1) - g(t+1) + g(t-2) - r(t) + r(t-3)$$

$$MOC_{xy} = \overline{(r_{xy,max})^2} \leftarrow \text{MAX AREA OF PRODUCT WHEN } X \text{ FULLY WITHIN } y \text{ IS } \boxed{\frac{1}{2}} = 1 + \frac{1}{2} = \frac{3}{2}$$

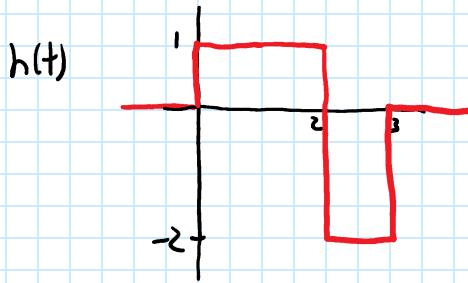
$$\overbrace{r_{xx}(0)r_{yy}(0)}^{\text{ENERGY IN } X = H^2W + H\frac{H^2W}{3} = (1)^2(1) + \frac{(1)^2(1)}{3} = \frac{4}{3}}$$

$$MOC = \frac{\left(\frac{3}{2}\right)^2}{\left(\frac{4}{3}\right)(3)} = \frac{\left(\frac{9}{4}\right)}{\left(\frac{4}{3}\right)} = \frac{9}{16}$$

$$\text{ENERGY IN } y = H^2W = (1)^2(3) = 3$$

# Problem 5

1)



2) YES  $\int_{-\infty}^{\infty} |h(t)| dt = 4$  FINITE

3) YES  $h(t)=0$  ALL  $t < 0$

4)  $(u(t) - 3u(t-2) + 2u(t-3)) * u(t) = r(t) - 3r(t-2) + 2r(t-3)$   $\Rightarrow$   
 OR:  $\int_{-\infty}^t h(\tau) d\tau = \int_{-\infty}^t (u(\tau) - 3u(\tau-2) + 2u(\tau-3)) d\tau$   
 $u(t) \left( \int_0^t d\tau - 3u(t-2) \int_2^t d\tau + 2u(t-3) \int_3^t d\tau \right)$   
 $= t u(t) - 3(t-2) u(t-2) + 2(t-3) u(t-3)$

5)  $y_1 = h(t) * x_1(t)$   
 $= (u(t) - 3u(t-2) + 2u(t-3)) * (u(t) - u(t-2))$   
 $= \cancel{r(t)} - \cancel{r(t-2)} - \cancel{3r(t-2)} + \cancel{3r(t-4)} + \cancel{2r(t-3)} - \cancel{2r(t-5)}$

OR  $y_1(t) = s_r(t) - s_r(t-2)$   
 $= \cancel{r(t)} - \cancel{3r(t-2)} + \cancel{2r(t-3)} - (\cancel{r(t-2)} - \cancel{3r(t-4)} + \cancel{2r(t-5)})$

6)  $y_2 = h(t) * x_2(t)$   
 $= (u(t) - 3u(t-2) + 2u(t-3)) * (e^{-t} u(t))$   
 $= u(t) * e^{-t} u(t) - 3u(t-2) * e^{-t} u(t) + 2u(t-3) * e^{-t} u(t)$   
 $u(t) * e^{-t} u(t) = \int_{-\infty}^{\infty} u(t-\tau) e^{-\tau} u(\tau) d\tau$   
 $\uparrow \quad \uparrow \quad \uparrow$   
 $\tau < t \quad \tau > 0 \quad 0 < \tau < t$   
 $u(t) \int_0^t e^{-\tau} u(\tau) d\tau = u(t) [-e^{-\tau}]_0^t = u(t)(-e^{-t} + 1) = (1 - e^{-t})u(t)$

THE OTHER TWO TERMS ARE SCALED, SHIFTED VERSIONS OF THIS;

$$y_2 = (1 - e^{-t})u(t) - 3(1 - e^{-(t-2)})u(t-2) + 2(1 - e^{-(t-3)})u(t-3)$$

### Problem 3 - extra

1. For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

	System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
a	$y(t) = \frac{2}{x(t)+2}$	N	Y	N	Y	Y
b	$y(t) = x(t-1)$	Y	Y	Y	N	Y
c	$y(t) = \int_{t-3}^{t-1} x(\tau+1) d\tau$	Y	Y	Y	N	Y
d	$y(t) = x(2t) \cdot \cos(t)$	Y	N	Y	N	N
e	$y(t) = x(t) + \cos(2t)$	N	N	Y	Y	Y

2. Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response  $h_i(t)$  or the step response  $s_{r,i}(t)$ . You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Stable?	Memoryless?	Causal?
$h_1(t) = (1 - e^{-t}) \cdot u(t)$	N	N	Y
$h_2(t) = u(t+1) - u(t-1)$	Y	N	N
$s_{r,3}(t) = (1 - e^{-t}) \cdot u(t)$	Y	N	Y
$s_{r,4}(t) = \sin(t) \cdot u(t)$	N	N	Y

- a)  $x(t)$  in denom - nonlinear,  $x(t) = -2 \Rightarrow y(t) = \infty$  - unstable
- b)  $y(t)$  depends on past - memory
- c)  $y(t)$  depends on  $x(t-2)$  to  $x(t)$  once limits go in; causal but w/memory
- d)  $\cos(t)$  outside args of  $x$  or  $y$  - time varying,  $x(2t)$  in future if  $t < 0$  non-causal, memory
- e) additive element - nonlinear;  $\cos(2t)$  outside of args of  $x$  or  $y$  - time varying