Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280L Fall 2013

Test II

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Name (please print)______________________________

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:______________________________

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

For this test, you should not leave unevaluated convolution sums/integrals. Unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$

Name (please print):
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Problem I: [25 pts.] Fourier Series

(1) Determine the fundamental frequency and Fourier series coefficients for each of the following signals:

(a) $a(t) = \cos(3t) \cdot \sin(t)$

(b) $b(t) = u(\cos(\pi t)) + u(\sin(\pi t))$ - you will want to draw it first.

(2) Determine the time-domain representation for each of the following signals given the period or fundamental frequency and Fourier series coefficients:

(x) $T_0 = \frac{1}{4}$, $X[k] = \begin{cases} 3-4j & k=4 \\ j & k=2 \\ 2 & k=1 \\ 2 & k=-1 \\ -j & k=-2 \\ 3+4j & k=-4 \end{cases}$

(y) $\omega_0 = 4\pi$, $Y[k] = \left(e^{-jk\pi/2}\right)\left(\frac{\sin\left(\frac{\pi k}{4}\right)}{\pi k}\right)$

(z) $X[k] \cdot Y[k]$

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Problem II: [25 pts.] Fourier Transforms

(1) Find the Fourier transform for the following signals:

(a) $a(t) = \cos(3t) \cdot e^{-2t}u(t)$

(b) $b(t) = 5\left(1 - e^{-4t}\right) \cdot u(t+2)$

(2) Find the inverse Fourier transform for the following:

(x) $X(j\omega) = \sum_{k=-2}^{2} j^k \delta(\omega - 3k)$

(y) $Y(j\omega) = \frac{(j\omega)^2+6j\omega-10}{(j\omega)^2+7j\omega+6}$

(z) $Z(j\omega) = \frac{e^{-j\omega}-e^{-2j\omega}}{j\omega+4}$

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Problem III: [25 pts.] System Analysis

(1) Find the transfer function $H_1(j\omega)$ and impulse response $h_1(t)$ for a system if a given input $x_1(t)$ yields a measured output $y_1(t)$ as follows:

$$x_1(t) = e^{-2t}u(t) \qquad\qquad y_1(t) = e^{-3(t-1)}u(t-1)$$

(2) Determine the output $y_2(t)$ of a linear, time-invariant system that has a step response given by $s_{r2}(t)$ if the input is $x_2(t)$:

$$s_{r2} = \left(1 - e^{-4t}\right)u(t) \qquad\qquad x_2(t) = \left(u(t) - u(t-1)\right)e^{-t}$$

(3) Determine a differential equation that models a system if that system has an impulse response $h_3(t)$ of:

$$h_3(t) = \left(e^{-5t} + te^{-5t}\right)u(t)$$

(4) A generic right-sided exponential $x_4(t) = e^{-at}u(t), a > 0$ is put through a generic low pass filter $H_4(j\omega) = \frac{Kb}{j\omega+b}, b > 0$. Determine the generic form of the output from this system, $y_4(t)$. *Note:* there may be a special case to consider.[1]

[1] And by "may be," I mean "is."

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Problem IV: [25 pts.] Applications

For this problem, assume you are always using the same message signal $m(t)$:

$$m(t) = \text{sinc}(100t) = \frac{\sin(100\pi t)}{100\pi t}$$

First, find sketch the magnitude spectrum of $m(t)$, $M(j\omega)$. Be sure to label your axes. Next, determine if the frequency domain of $m(t)$ band-limited. If you believe it is, indicate the band-limit. If you do not believe it is, you may need to adjust your belief system.

(1) SAMPLING: Draw a block diagram showing a system that uses impulse sampling at a rate T_s to sample $m(t)$. Determine the minimum sampling rate that would be required in order to have any chance of perfectly recovering the message signal. Draw the magnitude spectrum of the output of this system if the sampling rate is 500 rad/s. Be sure to label your axes and include units where appropriate. *Note:* 500 rad/s may or may not be a sampling rate that allows for full recovery.

(2) RECOVERY: Assuming the signal is sampled with an appropriately high sampling rate, how would you recover the original signal from the sampled signal? Clearly indicate any parameters of the system you propose, either in the time domain or in the frequency domain. The magnitude of the recovered signal must exactly match the magnitude of the original, so be sure to show how you are accomplishing that as well.

(3) FULL AMPLITUDE MODULATION: Sketch a block diagram for full amplitude modulation going from message signal $m(t)$ to transmitted signal $s(t)$, then sketch the magnitude of the frequency spectrum which would result if $m(t)$ were sent through a Full AM modulator with a carrier amplitude A_c of 3, a carrier frequency that is four times the band-limit of $m(t)$, and an amplitude sensitivity factor of 0.25 (that is, 25% modulation). Be sure to label your axes and indicate important values. The drawing does not need to be perfectly to scale as long as the important points are properly labeled.

(4) FULL AMPLITUDE DEMODULATION: Draw the general circuit used to demodulate Full AM signals. What is this circuit called?

(5) DSB-SC MODULATION: Sketch a block diagram for DSB-SC modulation going from message signal $m(t)$ to transmitted signal $s(t)$, then sketch the magnitude of the frequency spectrum which would result if $m(t)$ were sent through a DSB-SC modulator with a carrier amplitude of 3 and a carrier frequency that is four times the band-limit of $m(t)$. Be sure to label your axes and indicate important values.

(6) DSB-SC DEMODULATION: Sketch the block diagram for the system that you would use to recover the DSB-SC modulated signal above. The recovered signal must have the same amplitude as the original message $m(t)$. Be sure to clearly indicate either the precise impulse response or transfer function of any system blocks based upon $m(t)$ and the DSB-SC modulation scenario given above. You may either provide mathematical expressions or labeled magnitude sketches for these. Also, you only need to provide either the impulse response *or* the transfer function for any block.