Duke University
Edmund T. Pratt, Jr. School of Engineering

ECE 280L Fall 2013
Test I
Michael R. Gustafson II

Name (please print)_____

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature:_____

Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Notes

If you need to use convolution to solve a problem, you must evaluate the convolution. Your answers cannot be left in terms of convolution or the convolution integral. Also, unless otherwise specified:

- The $\cdot$ symbol means multiplication
- The * symbol means convolution
- $\delta(t)$ is the unit impulse function
- $u(t)$ is the unit step
- $r(t)$ is the unit ramp $t \cdot u(t)$
- $q(t)$ is the "unit" quadratic $\frac{1}{2}t^2 \cdot u(t)$

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Problem I: [15 *pts.*] Signal Classifications

(1) State whether each of the signals below is a power (P) or energy (E) signal and circle the appropriate letter. If the signal is a power signal, calculate the average power. If the signal is an energy signal, calculate the total energy.

- (P or E): $a(t) = e^{-5|t|}$

- (P or E): $b(t) = 3 + e^{-5|t|}$

- (P or E): $c(t) = \sum_{k=-\infty}^{\infty} (u(t-4k) - u(t-4k-1)) \cdot (t-4k)$

(2) State whether each of the signals below is periodic or not and circle the appropriate statement. If the signal is periodic, state the period.

- (P or NotP): $d(t) = \cos^2(t)$

- (P or NotP): $e(t) = \cos(t^2)$

- (P or NotP): $f(t) = \cos(4\pi t) + \sin(10\pi t)$

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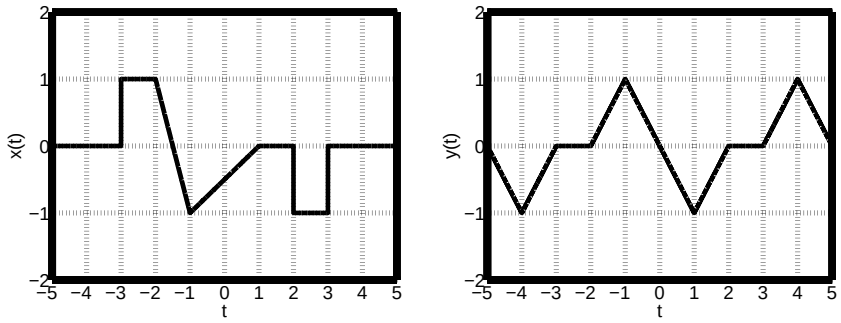
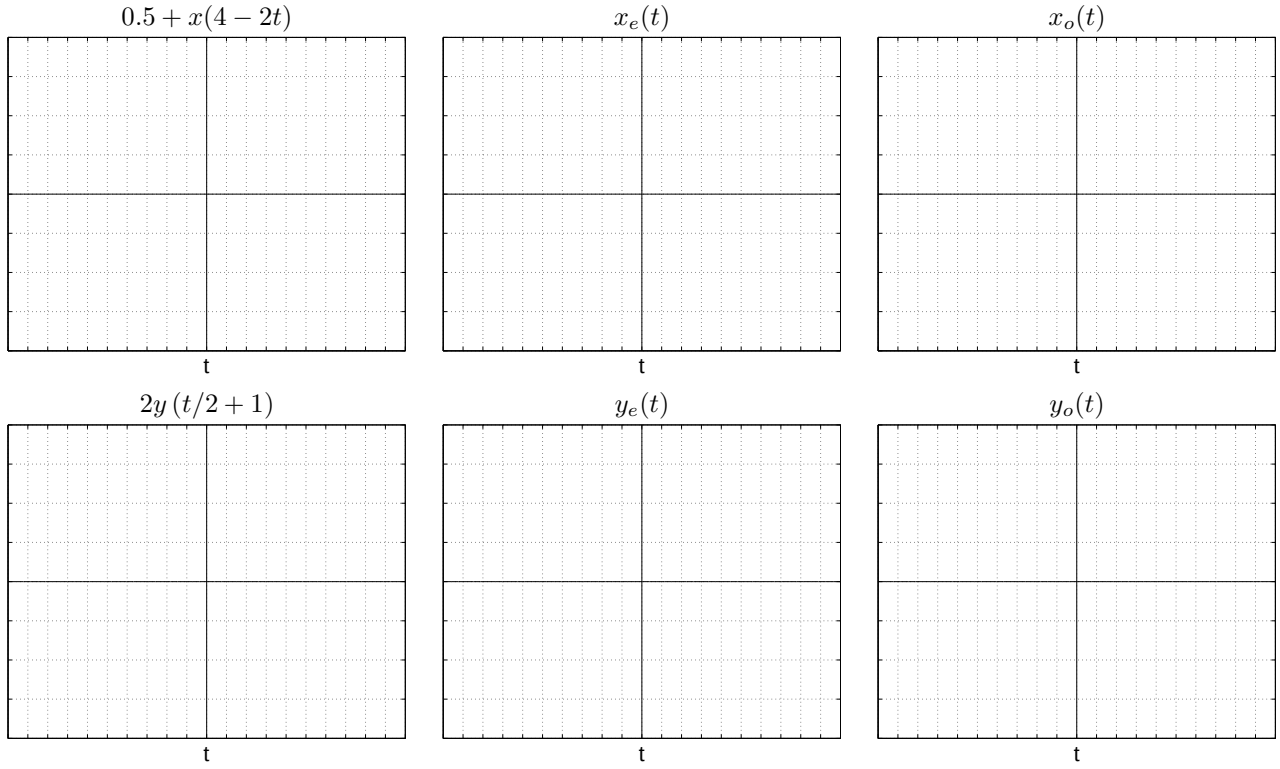
Problem II: [20 *pts.*] Signal Construction and Deconstruction

(1) Given each of the signal graphs below, write an equation for the signal using singularity functions. *Carefully note* the independent and dependent axis divisions - they do not have the same scales! Also: $y(t)$ is periodic; state the period.



- $x(t) =$
- $y(t) =$
- Period of $y(t)$:

(2) Given the signals above, graph the transformed versions below. Note the second and third columns are for the even and odd parts, respectively. Be sure to put values on your axes - you need to be sure to show all of the transformed version of $x(t)$ and at least one period of the transformed versions of $y(t)$. You do not have to use the same scales as the original.



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Problem III: [25 *pts.*] System Classifications

(1) For the following system equations, determine if the system represented is linear, time-invariant, stable, memoryless, and/or causal. You may show any work on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Linear?	Time Inv.?	Stable?	Memoryless?	Causal?
$y(t) = \tan\left(x\left(t\right)\right)$					
$y(t) = \sin(t+1) \cdot x(t-1)$					
$y(t) = 3x\left(\frac{t}{2}\right)$					
$y(t) = \frac{t+1}{x(t)}$					
$y(t) = \int_{-\infty}^{t+1} x(\tau)\ d\tau$					

(2) Assuming the following systems are each linear and time invariant, determine if the system represented is stable, memoryless, and/or causal based on the impulse response $h_i(t)$ or the step response $s_{r,i}(t)$. You may show any work below or on an additional piece of paper, but clearly indicate which system and system property you are working with.

System	Stable?	Memoryless?	Causal?
$h_1(t) = e^{-t} \cdot u(t)$			
$h_2(t) = \cos(t) \cdot u(t+\pi)$			
$s_{r,3}(t) = 5u(t)$			
$s_{r,4}(t) = r(t-1) - r(t-2)$			

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Problem IV: [20 *pts.*] Basic Convolution and Correlation

(1) Write an integral formula to calculate the convolution of signals $x(t)$ and $y(t)$:

$x(t) * y(t) =$

(2) Write an integral formula to calculate the correlation function of signals $x(t)$ and $y(t)$:

$\phi_{xy}(t) =$

(3) Write a formula to calculate the correlation function of signals $x(t)$ and $y(t)$ using convolution:

$\phi_{xy}(t) =$

(4) Given the following functions:

$$x(t) = u(t) - u(t-2) \qquad y(t) = r(t) - r(t-1) - u(t-1) \qquad z(t) = u(t-3)$$

make sketches of each, then determine formulas for **and sketch** the following - your sketches need to have sufficient labels to clearly indicate values and be drawn in such a way to clearly indicate shapes. Put your *final answer* and your sketch in the space below; put any work on a separate piece of paper.

- $a(t) = x(t) * x(t)$
- $b(t) = x(t) * y(t)$
- $c(t) = x(t) * z(t)$
- $d(t) = \phi_{xx}(t)$
- $e(t) = \phi_{xz}(t)$

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Problem V: [20 *pts.*] System Analysis

A linear, time-invariant system has an impulse response of:

$$h(t) = e^{-t}u(t) + e^{-(t-2)}u(t-2)$$

and an input signal:

$$x_1(t) = u(t+1)$$

is applied to the system.

(1) Is the system stable? Why do you believe that to be the case?

(2) Is the system causal? Why do you believe that to be the case?

(3) Determine an expression for the output of this system to the input $x_1(t)$ given above - call this output $y_1(t)$. You must evaluate any required integrals but are not required to simplify any algebra.

(4) Determine an expression for the output of the system if the input is changed to:

$$x_2(t) = u(t-1) - u(t-4)$$

Call this output $y_2(t)$. You must evaluate any required integrals but are not required to simplify any algebra. *Hint:* there is an easier way to do this and a harder way to do this based on what you already know.

(5) Determine an expression for the output of the system if the input is changed to:

$$x_3(t) = e^{-2t}u(t)$$

Call this output $y_3(t)$. You must evaluate any required integrals but are not required to simplify any algebra. *Hint:* there is only a harder way to do this.