

ECE 280 Fall 2013 Test B

Note Title

I)

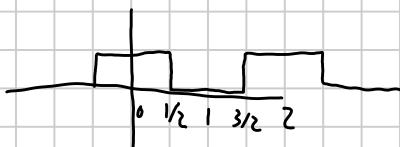
$$a) \cos(3t) \sin(t) = \frac{1}{2} (\sin(-2t) + \sin(4t)) = -\frac{1}{2} \sin(2t) + \frac{1}{2} \sin(4t)$$

$$\omega_0 = 2$$

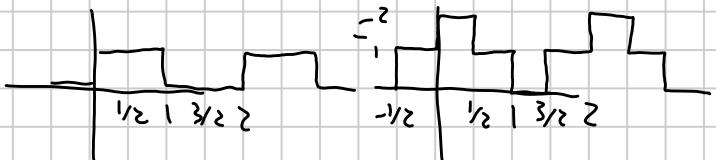
$$A[k] = \begin{cases} k=2 & \frac{1}{4}j \\ k=1 & -\frac{1}{4}j \\ k=-1 & \frac{1}{4}j \\ k=-2 & -\frac{1}{4}j \\ \text{otherwise} & 0 \end{cases}$$

These can be obtained directly or by using Euler to get $-\frac{1}{2} \left(\frac{e^{j2t} - e^{-j2t}}{2j} \right) + \frac{1}{2} \left(\frac{e^{j4t} - e^{-j4t}}{2j} \right)$

$$b) u(\cos(\pi t)) + u(\sin(\pi t))$$



$$T=2 \quad \omega_0=\pi \quad W=1$$



$$T=2 \quad \omega_0=\pi \quad W=1 \quad A_t = 1/2$$

$$B_1[k] = \frac{1}{2} \operatorname{sinc}\left(\frac{1}{2}k\right)$$

$$B_1[0] = 1/2$$

$$B_2[k] = \frac{1}{2} \operatorname{sinc}\left(\frac{1}{2}k\right) e^{-jk\pi/2}$$

$$B_2[0] = 1/2$$

$$B[k] = \begin{cases} k=0 & 1 \\ \text{otherwise} & \frac{1}{2} \operatorname{sinc}\left(\frac{1}{2}k\right) \left(1 + e^{-jk\pi/2}\right) \end{cases}$$

$$\text{or } \left(\frac{3}{4} \operatorname{sinc}\left(\frac{3}{4}k\right) + \frac{1}{4} \operatorname{sinc}\left(\frac{1}{4}k\right) \right) e^{-jk\pi/4}$$

$$\text{on } 1 + (-j)^k \quad \text{on } 1 + (j)^{-k}$$

$$2) \quad \omega_0 = 8\pi$$

$$x(t) = 6 \cos(32\pi t) + 8 \sin(32\pi t) - 2 \sin(16\pi t) + 4 \cos(8\pi t)$$

$$g) \quad jk\omega_0 t_0 = jk4\pi t_0 = jk\pi/2 \quad t_0 = 1/8 \text{ sec.}$$

$$\frac{\sin(\pi k/4)}{\pi k} = \frac{\sin(\pi k/4)}{4 \pi k/4} = \frac{1}{4} \operatorname{sinc}\left(\frac{k}{4}\right)$$

$$\hookrightarrow T = \frac{1}{2}, \quad \frac{W}{T} = \frac{1}{4} \quad \text{so } W = \frac{1}{8}$$

$$\text{Shift right by } 1/8 \text{ sec} \quad - \quad \sum_{k=-\infty}^{\infty} u(t - \frac{1}{16} - \frac{k}{2}) - u(t - \frac{3}{16} - \frac{k}{2}) \quad | \quad \begin{matrix} 1/16 & 3/16 & 9/16 & 11/16 & \dots \end{matrix} \quad t$$

Alternatively: $y(t) = \begin{cases} |t - 1/8| < 1/16 \\ 1/16 < |t - 1/8| < 1/4 \end{cases}$

$$y(t + n/2) = y(t)$$

z) $\omega_{0,x} = 8\pi, \omega_{0,y} = 4\pi \Rightarrow v/w \text{ i.t.D. } \omega_0 = 4\pi$

k	$k\omega_0$	$X[k]$	$Y[k]$	$Z[k]$
8	32π	$3 - 4j$	0	0
4	16π	j	0	0
2	8π	2	$-1/2\pi$	$-1/2\pi$
-2	-8π	2	$-1/2\pi$	$-1/2\pi$
-4	-16π	$-j$	0	0
-8	-32π	$3 + 4j$	0	0

$$Z(t) = -\frac{1}{\pi} \cos(8\pi t)$$

II)

$$a) \cos(3t) \cdot e^{-2t} u(t)$$

$$\frac{1}{2\pi} \left(\pi(\delta(w-3) + \delta(w+3)) \star \frac{1}{jw+z} \right) = \frac{1}{2} \left(\frac{1}{jw-3+z} + \frac{1}{jw+3+z} \right)$$

$$A(jw) = \frac{jw+z}{(jw+2)^2 + 3^2}$$

b) $b(t) = 5(1 - e^{-4t}) u(t+2)$ must get shifts the same

$$= 5u(t+2) - 5e^{-4t} u(t+2)$$

$$= 5u(t+2) - 5e^{-4(t+2-2)} u(t+2)$$

$$= 5u(t+2) - 5e^8 e^{-4(t+2)} u(t+2)$$

$$B(jw) = \frac{5e^{j2w}}{jw} + 5\pi\delta(w) - \frac{5e^8 e^{j2w}}{jw+4}$$

2) $X(jw) = -\delta(w-6) + j\delta(w-3) + \delta(w) - j\delta(w+3) - \delta(w+6)$

$$X(t) = -\frac{1}{\pi} \cos(6t) - \frac{1}{\pi} \sin(3t) + \frac{1}{2\pi}$$

$$Y(jw) = \frac{(jw)^2 + 6(jw) - 10}{(jw)^2 + 7(jw) + 6} = \frac{(jw)^2 + 6(jw) + jw - 10 + 16 - jw - 16}{(jw)^2 + 7(jw) + 6}$$

$$Y(jw) = \left| -\frac{(jw+16)}{(jw)^2 + 7(jw) + 6} \right| = \left| -\left(\frac{-2}{jw+6} + \frac{3}{jw+1} \right) \right|$$

must reduce
order of
numerator.

$$y(t) = \delta(t) + 2e^{-6t} u(t) - 3e^{-t} u(t)$$

$$Z(jw) = \frac{e^{-jw} - e^{-2jw}}{jw+4}$$

$$z(t) = \delta(t-1) \star e^{-4t} u(t) - \delta(t-2) \star e^{-4t} u(t)$$

$$= e^{-4(t-1)} u(t-1) - e^{-4(t-2)} u(t-2)$$

$$\text{IV) 1)} \quad X_1(jw) = \frac{1}{jw+2} \quad Y_1(jw) = \frac{e^{-jw}}{jw+3}$$

$$H_1 = \frac{Y_1}{X_1} = e^{-jw} \frac{(jw+2)}{(jw+3)} = \bar{e}^{jw} \frac{(jw+2+1-1)}{(jw+3)} = \bar{e}^{jw} \left(1 - \frac{1}{jw+3}\right)$$

$$h_1(t) = \delta(t-1) \Rightarrow (\delta(t) - \bar{e}^{-3t} u(t))$$

$$= \delta(t-1) - \bar{e}^{-3(t-1)} u(t-1)$$

$$2) \quad S_{r2}(jw) = \frac{1}{jw} + \pi \delta(w) - \frac{1}{jw+4} \rightarrow \text{on } h(t) = \frac{dS_{r2}(t)}{dt}$$

$$X_2(jw) = \frac{1}{jw+1} - \frac{\bar{e}^1 e^{-jw}}{jw+1} = \underbrace{(1 - e^{-4t})}_{=0 \text{ if } t=0, \text{ so}} \delta(t) + 4 \bar{e}^{-4t} u(t)$$

$$h(t) = 4 \bar{e}^{-4t} u(t)$$

$$H_2(jw) = jw S_{r2}(jw) = |1 + jw \cancel{\pi \delta(w)}| - \frac{jw}{jw+4}$$

and
H(jw)

$$= 1 - \frac{(jw+4-4)}{(jw+4)} = 1 - 1 + \frac{4}{jw+4} = \frac{4}{jw+4}$$

$$Y_2(jw) = X_2 H_2 = \frac{(4 - 4 \bar{e}^1 e^{-jw})}{(jw+1)(jw+4)}$$

$$= \left(\frac{4/3}{jw+1} - \frac{4/3}{jw+4} \right) (1 - \bar{e}^1 e^{-jw})$$

$$y_2(t) = \frac{4}{3} e^{-t} u(t) - \frac{4}{3} e^{-4t} u(t) - \frac{4}{3} \bar{e}^1 \bar{e}^{-(t-1)} u(t-1) + \frac{4}{3} \bar{e}^1 \bar{e}^{-(t-1)} u(t-1)$$

$$3) \quad H_3(jw) = \frac{1}{jw+5} + \frac{1}{(jw+5)^2} = \frac{jw+6}{(jw+5)^2} = \frac{jw+6}{(jw)^2 + 10jw + 25} = \frac{Y}{X}$$

$$(jw)^2 + 10jw + 25) Y = (jw+6) X$$

$$\frac{d^2y}{dt^2} + 10 \frac{dy}{dt} + 25y = \frac{dx}{dt} + 6x$$

$$4) \quad X_4(jw) = \frac{1}{jw+a} \quad H_4(jw) = \frac{Kb}{jw+b} \quad Y_4 = \frac{Kb}{(jw+a)(jw+b)}$$

$$Y_4 = \frac{Kb/b-a}{jw+a} - \frac{Kb/b-a}{jw+b} \quad y_4 = \frac{Kb}{b-a} \bar{e}^{-at} u(t) - \frac{Kb}{b-a} \bar{e}^{-bt} u(t)$$

$$\text{if } a \neq b; \text{ if } a=b, \quad Y_4 = \frac{Ka}{(jw+a)^2} \quad y_4(t) = Kae^{-at} u(t)$$

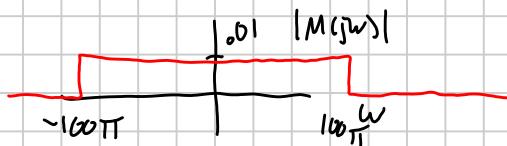
IV)

$$m(t) = \frac{\sin(100\pi t)}{100\pi t} = \frac{1}{100} \frac{\sin(100\pi t)}{\pi t}$$

$$W = 100\pi$$

Ampitude = .01

$$M(j\omega) = \begin{cases} .01 & |\omega| \leq 100\pi \\ 0 & \text{otherwise} \end{cases}$$



band limited $\omega_{max} = 100\pi$

1)



$$\omega_{max,m} = 100\pi$$

$$\omega > 2 \cdot 100\pi = 200\pi$$

$$T_{s,min} = \frac{2\pi}{\omega} = \frac{1}{100}$$

if $w_s = 500$ rad/sec, $T_s = \frac{\pi}{250}$

magnitude = $\frac{.01}{T_s}$ or $\frac{5}{2\pi}$

$s(j\omega)$

magnitude = $2(\frac{5}{2\pi})$
= $5/\pi$



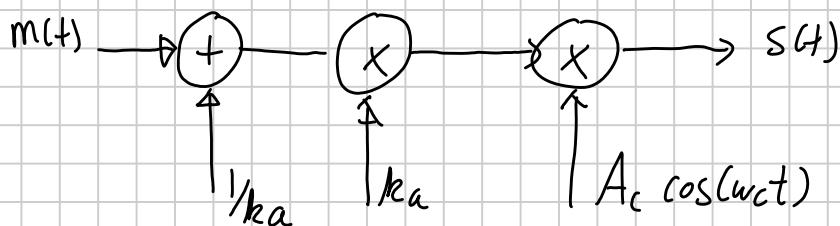
2) LPF, cutoff between $w_s - 100\pi$ rad/s, Gain of $T_s = \frac{\pi}{250}$

to get to original



$$100\pi < w_c < w_s - 100\pi$$

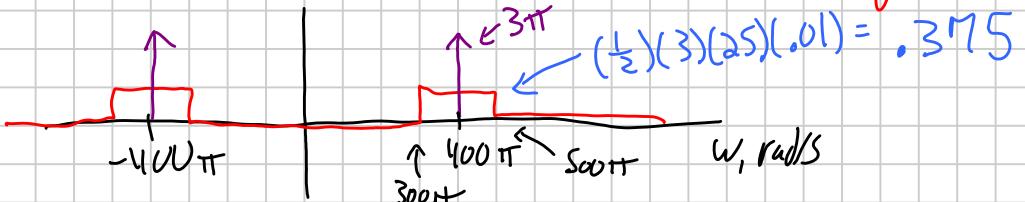
3)

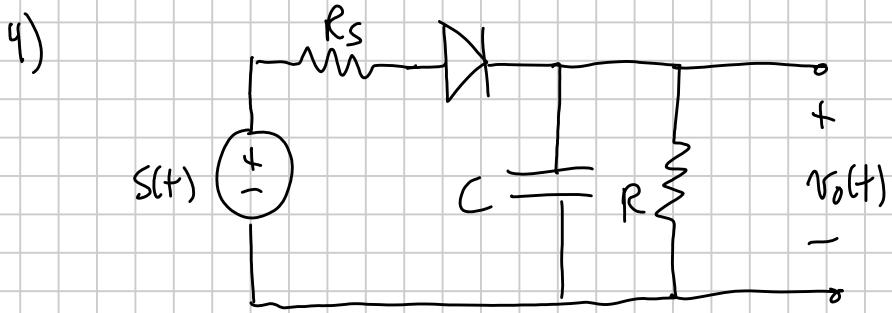


NOTE:
25% modulation
requires $k_a = 25000$

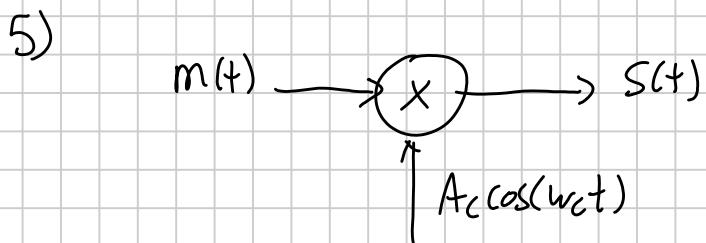
Specific Example: $k_a = 25$, $A_c = 3$, $\omega_c = 400$ rad/s

Not to scale!





Envelope detector



Specific Example: $A_c = 3$, $w_c = 400 \text{ rad/s}$

