

Duke University  
Edmund T. Pratt, Jr. School of Engineering

ECE 141 Spring 2010  
Test I  
Michael R. Gustafson II

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Name (please print) \_\_\_\_\_

In keeping with the Community Standard, I have neither provided nor received any assistance on this test. I understand if it is later determined that I gave or received assistance, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class. I also understand that I am not allowed to speak to anyone except the instructor about any aspect of this test until the instructor announces it is allowed. I understand if it is later determined that I did speak to another person about the test before the instructor said it was allowed, I will be brought before the Undergraduate Conduct Board and, if found responsible for academic dishonesty or academic contempt, fail the class.

Signature: \_\_\_\_\_

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## Instructions

First - please turn **off** any cell phones or other annoyance-producing devices. Vibrate mode is not enough - your device needs to be in a mode where it will make no sounds during the course of the test, including the vibrate buzz or those acknowledging receipt of a text or voicemail.

Please be sure to put each problem on its own page or pages - do *not* write answers to more than one problem on any piece of paper and do not use the back of a problem for work on a *different* problem. You will be turning in each of the problems independently. This cover page should be stapled to the front of Problem 1.

Make sure that your name *and* NET ID are *clearly* written at the top of *every* page, just in case problem parts come loose in the shuffle. Make sure that the work you are submitting for an answer is clearly marked as such. Finally, when turning in the test, individually staple all the work for each problem and place each problem's work in the appropriate folder.

Note that there may be people taking the test after you, so you are not allowed to talk about the test - even to people outside of this class - until I send along the OK. This includes talking about the specific problem types, how long it took you, how hard you thought it was - really anything. Please maintain the integrity of this test.

Name (please print):

Community Standard (print ACPUB ID):

### Problem I: [20 pts.] Mathematical Underpinnings

(a) System 1:

A single-input, single-output LTI system has a transfer function:

$$\frac{\mathcal{Y}(s)}{\mathcal{X}(s)} = \frac{1}{s^2 + 5s + 6}$$

- (1) Determine the impulse response of the system (note: this is a function of time).
- (2) Determine the step response of the system (note: this is also a function of time).
- (3) Given the transfer function, write the differential equation for  $y(t)$  and its derivatives in terms of  $x(t)$  and its derivatives.
- (4) Assuming  $y(0) = 8$  and  $\dot{y}(0) = -7$ , find  $y(t)$  if  $x(t) = e^{-4t}u(t)$ . You may use any valid method of your choosing.

(b) System 2:

A single-input, single-output LTI system has a transfer function:

$$\mathcal{G}(s) = \frac{\mathcal{C}(s)}{\mathcal{R}(s)}$$

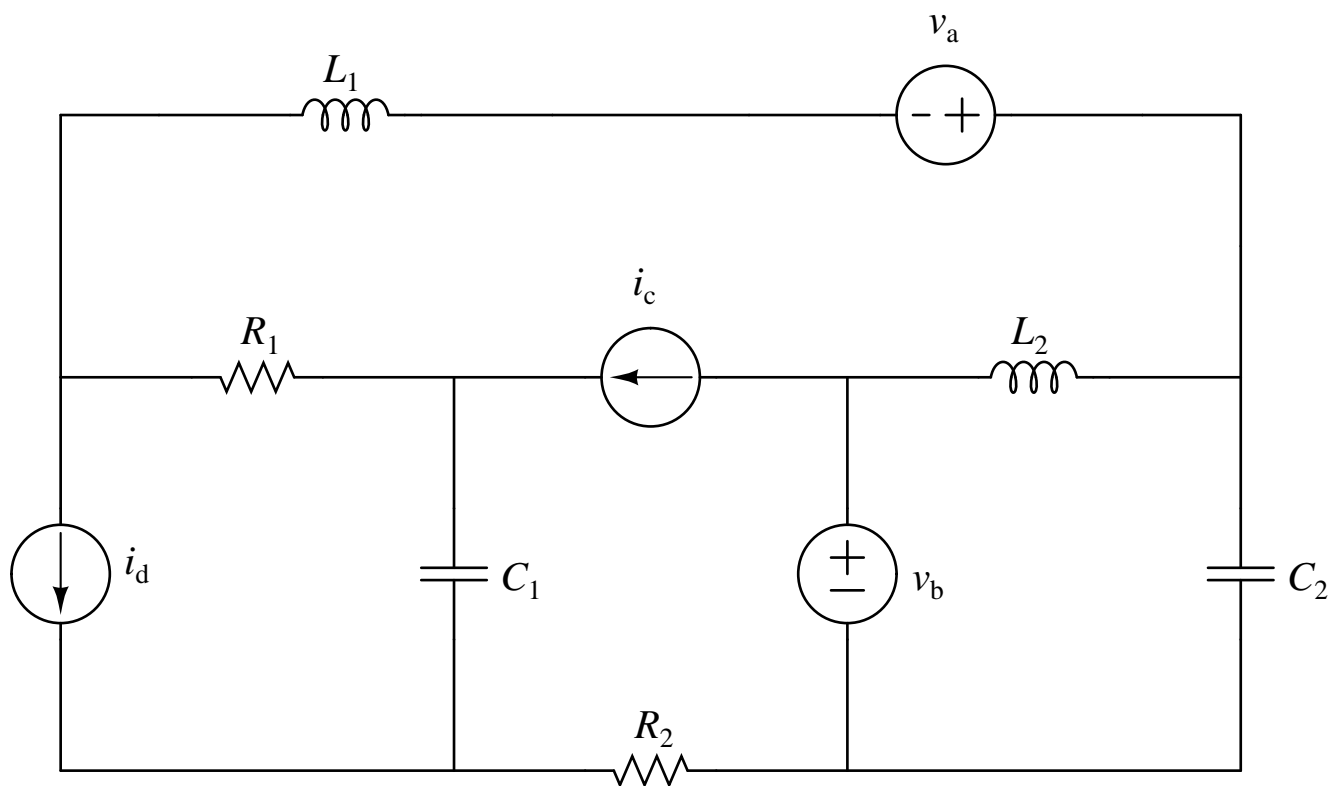
When the system receives an input of  $r(t) = \cos(3t)u(t)$ , the measured response is

$$c(t) = \frac{(\cos(3t) + 2 \sin(3t) - 1)}{5} e^{-6t} u(t)$$

- (1) Find the transfer function  $\mathcal{G}(s)$ . Present your answer as a simplified ratio of polynomials.
- (2) Re-write the system function as a differential equation for  $c(t)$  and its derivatives in terms of  $r(t)$  and its derivatives.

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Given the following electrical system:



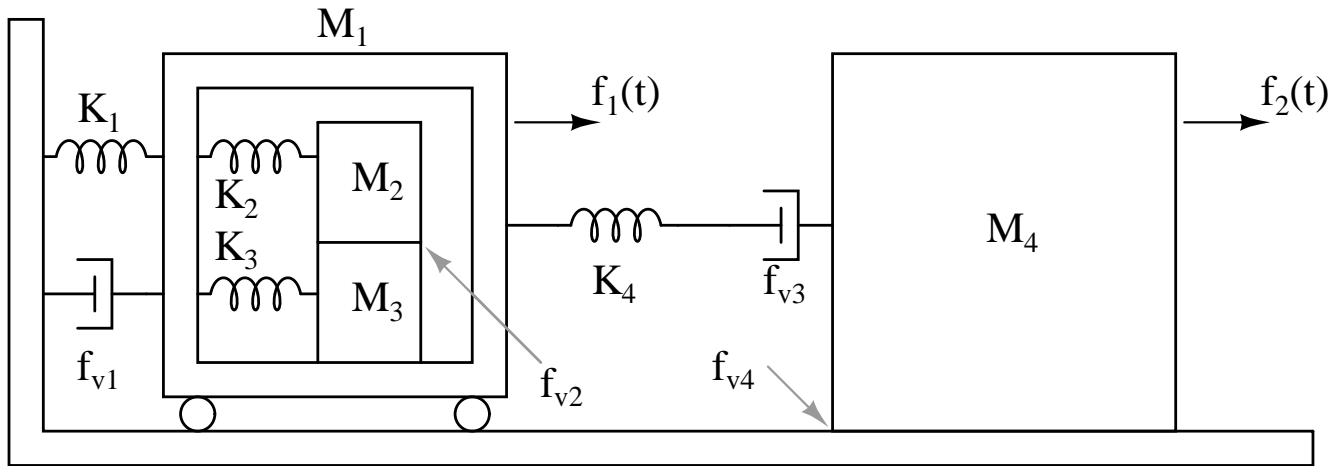
and assuming that the values of the passive elements and of the independent sources are known, *clearly* demonstrate the use of the Mesh Current Method in the frequency domain to label unknowns for the circuit and to determine a complete set of equations that could be used to solve for these unknowns. List the set of unknowns you believe your equations will find. You do not need to arrange the equations in matrix format.

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### Problem III: [20 pts.] Translational Systems

Given the following translational system:



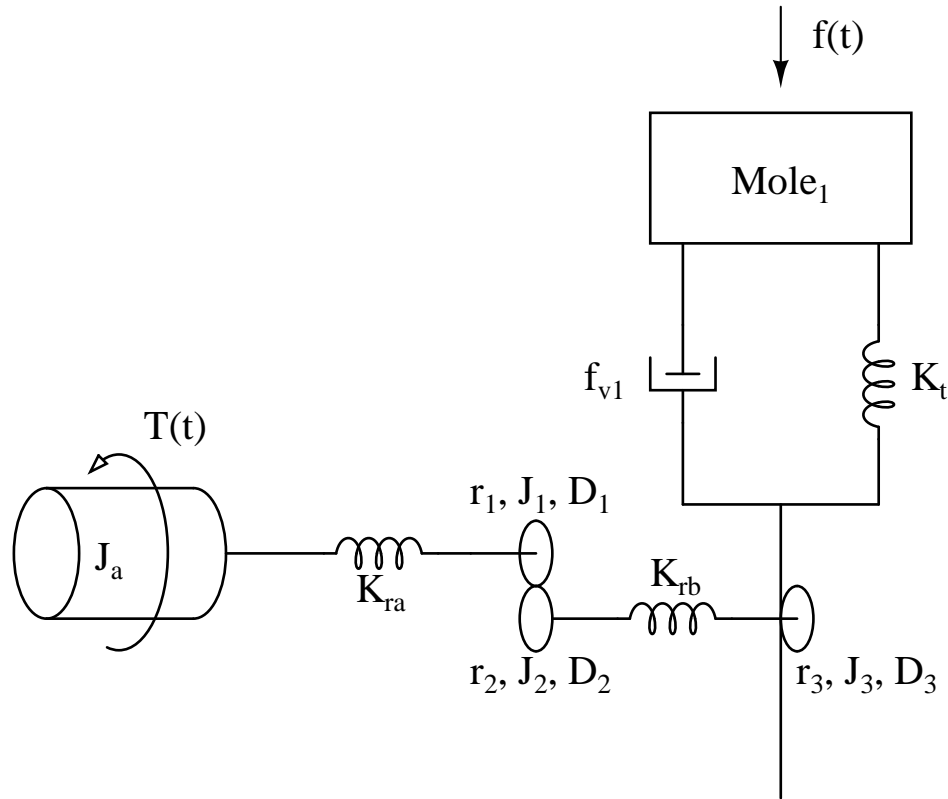
and assuming that the values of the passive elements and of the external forces are known, *clearly* determine the equations of motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are clearly labeled on the diagram. You do not need to arrange the equations in matrix format.

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### Problem IV: [20 pts.] Translational and Rotational Systems - “Whack-A-Mole”

The following system:



is one model of the famous “Whack-A-Mole” game. Torque  $T(t)$  is used to provide the effort needed to lift the mole up through a hole in the playing board. The rotational and translational shafts have stiffnesses represented by the various springs and the gears are all supported by housings which, among other things, provide damping. Force  $F$  represents the force applied by the mallet when a player attempts to whack said mole. Damper  $f_{v1}$  is in place to cushion the blow.

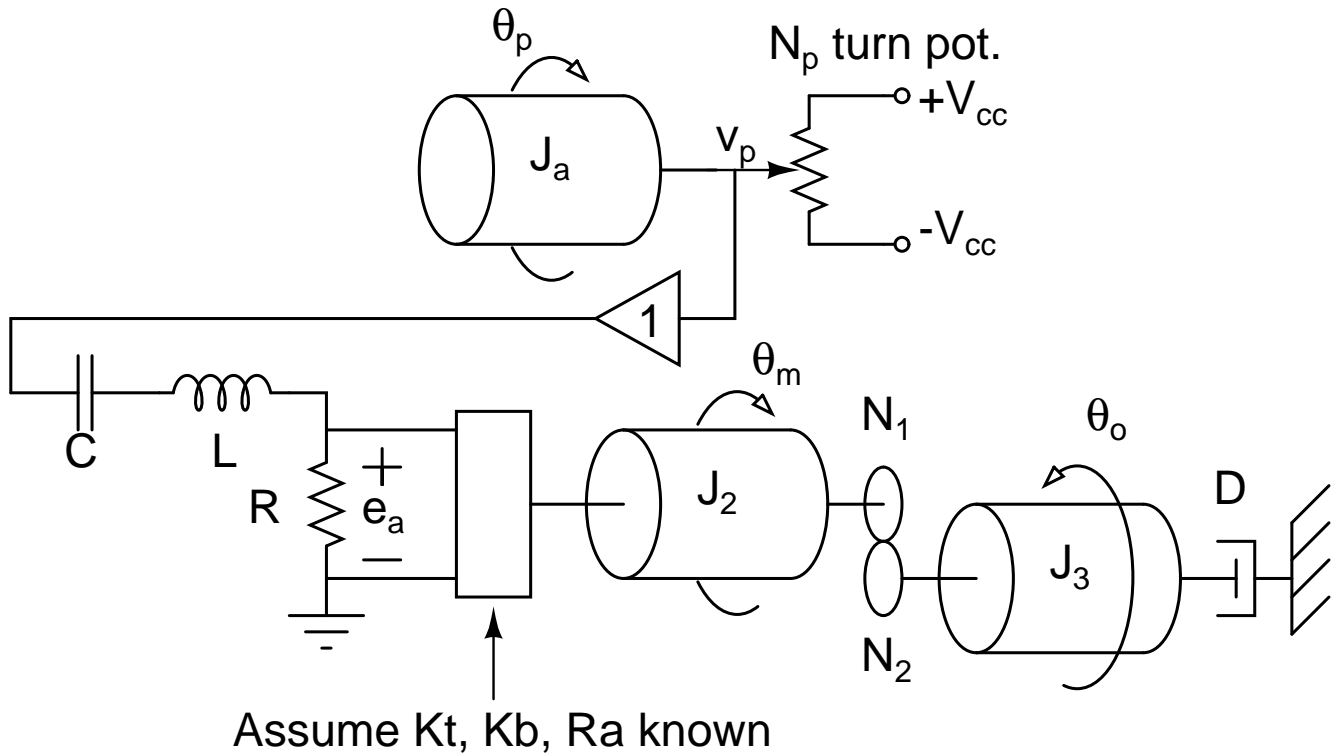
- (a) Clearly draw the system from the perspective of the applied torque. Be sure to label each independent angle.
- (b) Assuming that the values of the passive elements and of the external forces and torques are known, *clearly* determine the equations of (angular) motion for the system in the frequency domain. List the set of unknowns you believe your equations use and be sure they are clearly labeled on the diagram. You do not need to arrange the equations in matrix format.
- (c) Clearly draw the system from the perspective of the mole. Be sure to label each independent translational position.

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### Problem V: [20 pts.] Integrated System

Given the following system:



determine the following transfer functions:

- (1)  $\mathcal{G}_1(s) = \frac{V_p(s)}{\Theta_p(s)}$  - assume that a positive angle  $\theta_p(t)$  yields a positive voltage.
- (2)  $\mathcal{G}_2(s) = \frac{E_a(s)}{V_p(s)}$  - note that the buffer amplifier ensures that voltage  $v_p$  is applied across the circuit.
- (3)  $\mathcal{G}_3(s) = \frac{\Theta_m(s)}{E_a(s)}$  - assume that a positive  $e_a(t)$  yields a positive  $\theta_m(t)$  as drawn.
- (4)  $\mathcal{G}_4(s) = \frac{\Theta_o(s)}{\Theta_m(s)}$