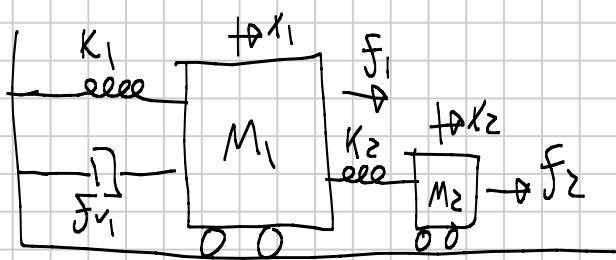


ECE 141 | Spring 2010 Test P

Note Title

I)



States: $x_1 \quad \dot{x}_1 \quad x_2 \quad \dot{x}_2$ so $z = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$

1) $\frac{d}{dt} x_1 = \dot{x}_1 \text{ so } \frac{d}{dt} z_1 = z_2$

2) $M_1: (M_1 s^2 + f_{v1}s + (K_1 + K_2))x_1 - K_2 x_2 = f_1$

$$M_1 \frac{d^2}{dt^2} x_1 + f_{v1} \frac{d}{dt} x_1 + (K_1 + K_2)x_1 - K_2 x_2 = f_1$$

$$M_1 \ddot{z}_1 + f_{v1} \dot{z}_1 + (K_1 + K_2)z_1 - K_2 z_2 = f_1$$

so $\frac{d}{dt} z_1 = -\frac{(K_1 + K_2)}{M_1} z_1 - \frac{f_{v1}}{M_1} z_2 + \frac{K_2}{M_1} z_3 + \frac{1}{M_1} f_1$

3) $\frac{d}{dt} x_2 = \dot{x}_2 \text{ so } \frac{d}{dt} z_2 = z_3$

4) $M_2: (M_2 s^2 + K_2)x_2 - K_2 x_1 = f_2$

$$M_2 \frac{d^2}{dt^2} x_2 + K_2 x_2 - K_2 x_1 = f_2$$

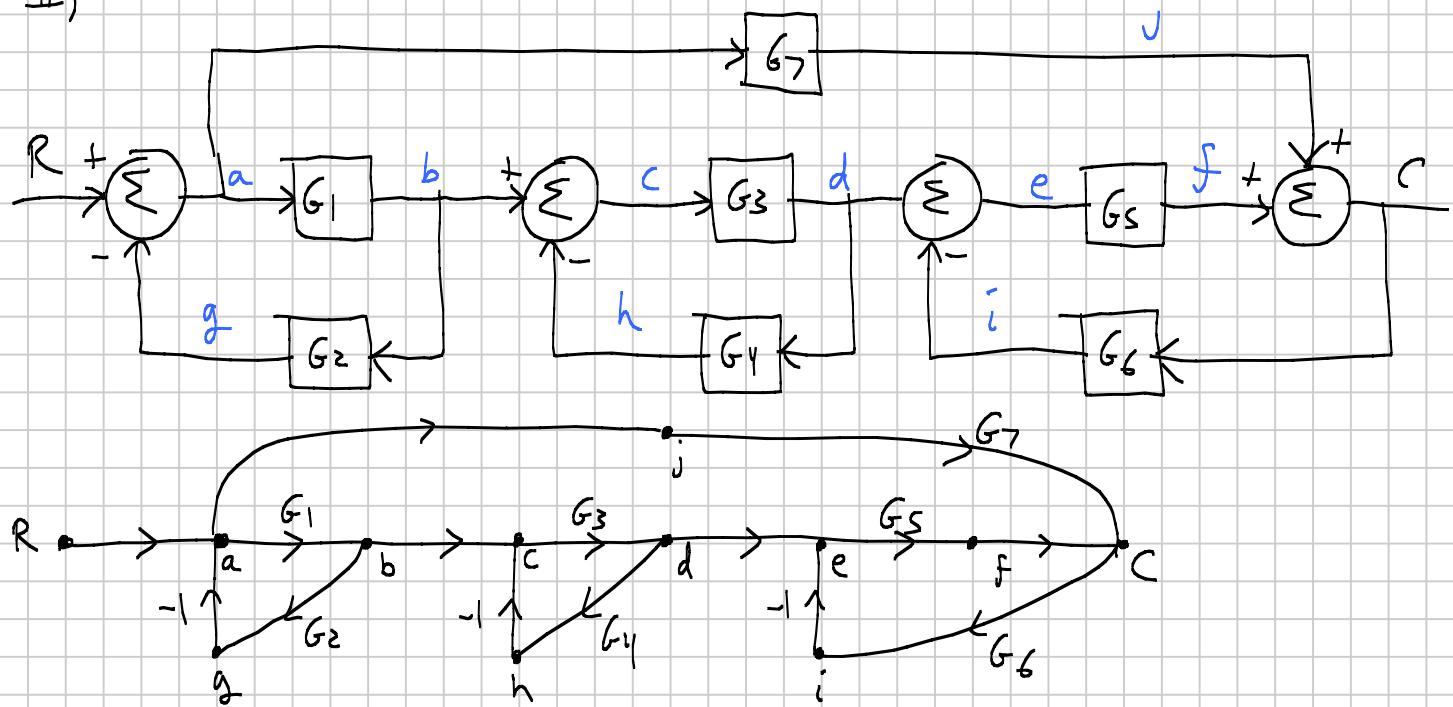
$$M_2 \ddot{z}_2 + K_2 z_3 - K_2 z_1 = f_2$$

so $\frac{d}{dt} z_2 = \frac{K_2}{M_2} z_1 - \frac{K_2}{M_2} z_3 + \frac{1}{M_2} f_2$

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(K_1 + K_2)}{M_1} & -\frac{f_{v1}}{M_1} & \frac{K_2}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{M_2} & 0 & -\frac{K_2}{M_2} & 0 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix}}_B \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}}_C z + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_D u$$

II)



$$T_1 : RabcdefC = G_1 G_3 G_5 \quad L_1 : abga = -G_1 G_2$$

$$T_2 : Ra\bar{c}jC = G_7 \quad L_2 : cdhc = -G_3 G_4$$

$$L_3 : efCi\bar{e} = -G_5 G_6$$

$$\Delta = 1 - L_1 - L_2 - L_3 + L_1 L_2 + L_1 L_3 + L_2 L_3 - L_1 L_2 L_3$$

$$\Delta_1 = 1 \quad (T_1 \text{ touches all loops})$$

$$\Delta_2 = 1 - L_2 \quad (T_2 \text{ touches loops 1 and 3})$$

$$T = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

(III)

$$T = \frac{C}{R} = \frac{K(s^2 + 10s + 24)}{s^3 + 18s^2 + K(s^2 + 10s + 24)}$$

a) $s^3 \quad | \quad 10K$

$$s^2 \quad 18+K \quad 24K$$

$$s^1 \quad \frac{\begin{vmatrix} 1 & 10K \\ 18+K & 24K \end{vmatrix}}{-18(K+18)} = \frac{24K - 180K - 10K^2}{-(18+K)} = \frac{10K^2 + 156K}{K+18}$$

$$s^0 \quad 24K$$

b) From s^1 , $K > -18$

From s^0 , $K > 0$ so, $K > 0$ so far

From s^1 : $\frac{10K^2 + 156K}{K+18} > 0$ if $K > 0$, denom is positive, so

$$10K^2 + 156K > 0$$

$$10(K)(K + 15.6) > 0 \quad \text{so } K > 0 \text{ or } K < -15.6$$

so $K > 0$

can't be!

c) Marginal stability requires none of all zeros;

so none all zero if $K=0$ or -15.6 which are meaningless and unstable; this system cannot be marginally stable!

d) $G_{\text{eq}} = \frac{T}{1-T} = \frac{K(s^2 + 10s + 24)}{s^3 + 18s^2} = \frac{K(s^2 + 10s + 24)}{s^2(s + 18)}$

Type II System

e) K_a for Type II = $\lim_{s \rightarrow 0} s^2 G_{\text{eq}} = \frac{24K}{18} = \frac{4K}{3}$

$$\rho_{\text{ss,para}} = \frac{1}{K_a} = \frac{3}{4K} = \frac{1}{3} \quad \text{so} \quad K = \frac{9}{4}$$

i.e. input of $\frac{1}{2}t^2 u(t)$

$$\text{IV) } T = \frac{C}{R} = \frac{K}{s^4 + 8s^3 + 19s^2 + 12s + K}$$

$$\text{a) } s^4 \quad | \quad 1 \quad 19 \quad K$$

$$s^3 \quad 8 \quad 12$$

$$s^2 \quad \frac{12 - 18}{-8} = \frac{140 - 35}{8} \cancel{35} \quad \cancel{K} 2K$$

$$s^1 \quad \frac{16K - 420}{-8s} = \frac{420 - 16K}{8s} \cancel{105 - 4K} \quad \text{multiply by } \frac{+35}{4}$$

$$s^0 \quad \cancel{2K} \quad K \quad \text{multiply by } \frac{+1}{2}$$

optional scaling
multiply by 2

$$\text{b) } s^0: K > 0$$

$$s^1: 105 - 4K > 0 \quad \text{so } K < 26.25$$

Stable if $0 < K < 26.25$

c) $K=0$ trivial case --- $K=26.25$ makes s^1 row all 0,

$$s^2 \text{ row: } 35 \quad s^2 + 2(26.25) = 0; \quad s = \pm j \sqrt{\frac{52.5}{35}} = \pm j \sqrt{1.5}$$

$w_{osc} = 1.22 \text{ rad/s}$

$$\text{d) } G_{\text{eq}} = \frac{T}{1-T} = \frac{K}{s^4 + 8s^3 + 19s^2 + 12s} = \frac{K}{s(s^3 + 8s^2 + 19s + 12)}$$

$\xrightarrow{\text{Type I}}$

$$\text{e) } K_p = \lim_{s \rightarrow 0} G_{\text{eq}} = \infty \quad \text{f) } e_{\text{stop}} = \frac{1}{1+K_p} = 0$$

$$K_v = \lim_{s \rightarrow 0} sG_{\text{eq}} = \frac{K}{12} \quad e_{\text{ramp}} = \frac{1}{K_v} = \frac{12}{K}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G_{\text{eq}} = 0 \quad e_{\text{pwm}} = \frac{1}{K_a} = \infty$$

$$IV) \quad G = \frac{s+100}{(s+5)(s+20)} \quad G_{\text{eq}} = \frac{K(s+100)}{(s+5)(s+20)} = \frac{K(s+100)}{s^2 + 25s + 100}$$

$$a) \quad T = \frac{G}{1+G} = \frac{K(s+100)}{s^2 + (25+K)s + 100 + 100K}$$

$$s^2 \quad | \quad 100 + 100K$$

$$s^1 \quad 25 + K$$

$$s^0 \quad \cancel{100+100K} \quad K+1 \quad \text{multiply by } \frac{+1}{100}$$

$$b) \quad s^1: \quad K > -25 \\ s^0: \quad K > -1 \quad \text{so} \quad K > -1$$

$$c) \quad K = -1 \quad \text{yields all 0 } s^0 \text{ row; } 24s = 0 \text{ so } w = 0 \text{ hen}$$

$K = -25$ not stable

$$d) \quad G_{\text{eq}} = \frac{K(s+100)}{s^2 + 25s + 100} \quad \text{so Type } \emptyset$$

$$e) \quad \text{denom}(T) = s^2 + (25+K)s + (100+100K) = s^2 + 2\zeta w_n s + w_n^2$$

$$w_n = \sqrt{100 + 100K}$$

$$\zeta = \frac{25+K}{2w_n} = \frac{25+K}{2\sqrt{100+100K}} = .9$$

$$\text{Square both sides: } \frac{K^2 + 50K + 625}{400K + 400} = .81$$

$$K^2 + 50K + 625 = 324K + 324$$

$$K^2 - 274K + 301 = 0$$

$$K = \frac{274 \pm \sqrt{75076 - 1204}}{2}$$

$$K = \frac{274 \pm 271.8}{2} = 272.9 \text{ or } 1.1$$

Both are stable and OK!

$$K = 272.9$$

1-1

$$\omega_n = \sqrt{100+100K} = 165.5 \text{ rad/sec}$$

14.5 rad/sec

e1) $\zeta = .9$

.9 by design

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 172.14 \text{ rad/sec}$$

6.32 rad/sec

e2) $T_s = \frac{4}{\zeta \omega_n} = 0.027 \text{ sec}$

0.3065 sec

e3) NOT VERY!

REAL PARTS OF DOM. POLES AT
- $\zeta \omega_n = -148.95$, FURTHER
FOR ORIGIN THAN ZERO AT
-100!

CLOSE!

REAL PARTS OF DOM. POLES AT
- $\zeta \omega_n = -5.69$, MUCH CLOSER
THAN -100!

f) Type 0 so $K_p = \lim_{s \rightarrow 0} G_{eg} = \frac{K \cdot 100}{100} = K$

$$e_{\text{step}} = \frac{1}{1+K_p} = \frac{1}{1+K} = \frac{1}{3} \quad K=2 \quad \text{stable here so OK}$$

f2) $\omega_n = \sqrt{100+100K} = \sqrt{300} = 17.32 \text{ rad/sec}$

f1) $\zeta = \frac{25+K}{2\omega_n} = \frac{27}{34.64} = 0.78$

f3) Underdamped time $0 < \zeta < 1$

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