

ECE 141 SPRING 2010 TEST 1

Note Title

Problem I

a) $\frac{y}{x} = \frac{1}{s^2 + 5s + 6} = \frac{1}{(s+2)(s+3)}$

i) $h(t) = \mathcal{L}^{-1}\left\{\frac{y}{x}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+2} + \frac{-1}{s+3}\right\}$

$$h(t) = (e^{-2t} - e^{-3t}) u(t)$$

a) Transfer Function:

$$\rightarrow sr(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \frac{y}{x}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{1/6}{s} + \frac{-1/2}{s+2} + \frac{1/3}{s+3}\right\}$$

$$sr(t) = \left(\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}\right) u(t)$$

$$\begin{aligned} \underline{\text{or}} \rightarrow sr(t) &= \int_{-\infty}^t h(\tau) d\tau \\ &= \int_{-\infty}^t (e^{-2\tau} - e^{-3\tau}) u(\tau) d\tau \\ &= u(t) \int_0^t (e^{-2\tau} - e^{-3\tau}) d\tau \end{aligned}$$

$$= u(t) \left[\frac{e^{-2\tau}}{-2} - \frac{e^{-3\tau}}{-3} \right]_0^t$$

$$= u(t) \left(\frac{e^{-2t}}{-2} - \frac{e^{-3t}}{-3} - \frac{1}{-2} + \frac{1}{-3} \right)$$

$$= u(t) \left(\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} \right)$$

3) $(s^2 + 5s + 6) y = x$

$$\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = x$$

4) Two ways:

$$\rightarrow y_h = Ae^{st}, \quad s^2 + 5s + 6 = 0 \\ s = -2, -3$$

$$y_h = Ae^{-2t} + Be^{-3t}$$

$$y_p = Ce^{-4t} \text{ so}$$

$$16(Ce^{-4t}) + 5(-4Ce^{-4t}) + 6(Ce^{-4t}) = e^{-4t}$$

$$2Ce^{-4t} = e^{-4t} \quad C = 1/2$$

$$y = y_p + y_h = Ae^{-2t} + Be^{-3t} + \frac{1}{2}e^{-4t}$$

$$\dot{y} = -2Ae^{-2t} - 3Be^{-3t} - 2e^{-4t}$$

$$y(0) = A + B + \frac{1}{2} = 8$$

$$\dot{y}(0) = -2A - 3B - 2 = -7$$

$$2y(0) + \dot{y}(0): \quad -B - 1 = 9 \quad B = -10 \\ y(0): \quad A = 7.5 - B \quad A = 17.5$$

$$y = (17.5e^{-2t} - 10e^{-3t} + 0.5e^{-4t}) u(t)$$

or use Laplace

$$\rightarrow (s^2 Y - sg(0) - \dot{y}(0)) + 5(sY - y(0)) + 6Y = X$$

$$(s^2 + 5s + 6)Y = X + sg(0) + \dot{y}(0) + 5y(0)$$

$$(s^2 + 5s + 6)Y = \frac{1}{s+4} + 8s + 33 = \frac{8s^2 + 65s + 133}{s+4}$$

$$Y = \frac{8s^2 + 65s + 133}{(s+2)(s+3)(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = \lim_{s \rightarrow -2} \frac{8s^2 + 65s + 133}{(s+2)(s+3)(s+4)} = \frac{3s}{2} = 17.5$$

$$B = \lim_{s \rightarrow -3} \frac{8s^2 + 65s + 133}{(s+2)(s+3)(s+4)} = -\frac{10}{1} = -10$$

$$C = \lim_{s \rightarrow -4} \frac{8s^2 + 65s + 133}{(s+2)(s+3)} = \frac{1}{2} = 0.5$$

$$y = (17.5e^{-2t} - 10e^{-3t} + 0.5e^{-4t}) u(t)$$

(b)

$$1) \quad G = \frac{C}{R}$$

$$R = \frac{1}{2} \left\{ \cos(3t) u(t) \right\} = \frac{s}{s^2 + 9}$$

$$C = \frac{1}{2} \left\{ \frac{\cos(3t) + 2\sin(3t)}{s} - 1 \right\} e^{-6t} u(t) = \frac{1}{5} \frac{(s+6) + 2(3)}{(s+6)^2 + (3)^2} - \frac{1}{5} \frac{1}{s+6}$$

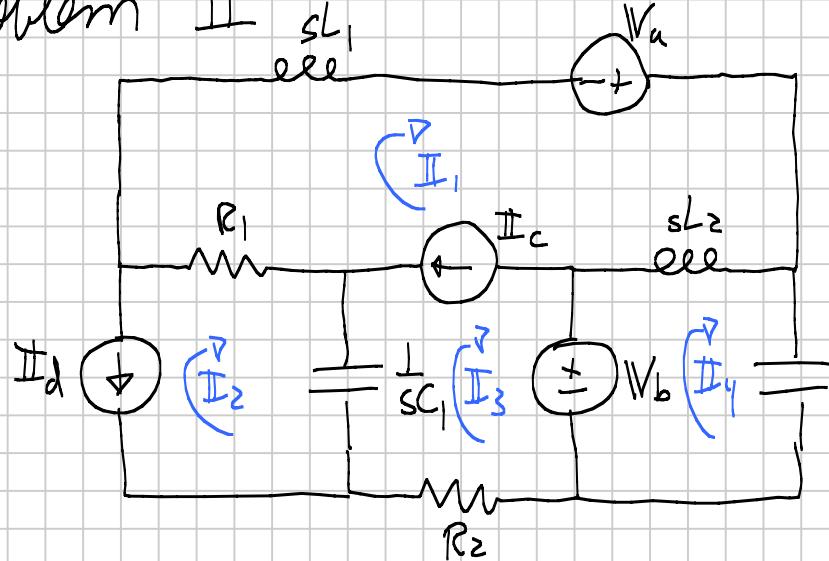
$$C = \frac{1}{5} \frac{(s+12)(s+6) - (s^2 + 12s + 45)}{(s+6)^2 + (3)^2} (s+6)$$

$$C = \frac{1}{5} \frac{s^2 + 18s + 72 - s^2 - 12s - 45}{(s^2 + 12s + 45)(s+6)} = \frac{1}{5} \frac{6s + 27}{s^3 + 18s^2 + 117s + 270}$$

$$\frac{C}{R} = \frac{1}{5} \frac{6s + 27}{s^3 + 18s^2 + 117s + 270} \quad \frac{s^2 + 9}{s} = \frac{1}{5} \frac{6s^3 + 27s^2 + 54s + 243}{s^4 + 18s^3 + 117s^2 + 270s}$$

$$\frac{d^4}{dt^4} C + 18 \frac{d^3}{dt^3} C + 117 \frac{d^2}{dt^2} C + 270 \frac{d}{dt} C = \frac{6}{5} \frac{d^3}{dt^3} r + \frac{27}{5} \frac{d^2}{dt^2} r + \frac{54}{5} \frac{d}{dt} r + \frac{243}{5} r$$

Problem II



Note: voltage drop across current source is unknown; use KVL on path not including $\frac{1}{sC_2}$ a current source!

$$\text{Current source equations: } I_2 = -I_d$$

$$I_1 - I_3 = I_c$$

KVL equations

$$\text{KVL, } I_4: -V_b + sL_2(I_4 - I_1) + \frac{1}{sC_2}(I_4) = 0$$

$$\text{KVL, } sL_{13}: \frac{1}{sC_1}(I_3 - I_2) + R_1(I_1 - I_2) + sL_1(I_1) - V_a + sL_2(I_1 - I_4) + \dots \\ V_b + R_2(I_3) = 0$$

Will find I_1, I_2, I_3, I_4

Problem III : assuming $X_1 - X_4$ are mass positions and X_5 is between K_4 and F_{V_3} :

$$1) (M_1 s^2 + F_{V_1} s + (K_1 + K_2 + K_3 + K_4))X_1 - (K_2)X_2 - (K_3)X_3 - (0)X_4 - (K_4)X_5 = F_1(s)$$

$$2) -(K_2)X_1 + (M_2 s^2 + F_{V_2} s + K_2)X_2 - (F_{V_2} s)X_3 - (0)X_4 - (0)X_5 = 0$$

$$3) -(K_3)X_1 - (F_{V_2} s)X_2 + (M_3 s^2 + F_{V_2} s + K_3)X_3 - (0)X_4 - (0)X_5 = 0$$

$$4) -(0)X_1 - (0)X_2 - (0)X_3 + (M_4 s^2 + (F_{V_3} + F_{V_4})s)X_4 - (F_{V_3} s)X_5 = F_2(s)$$

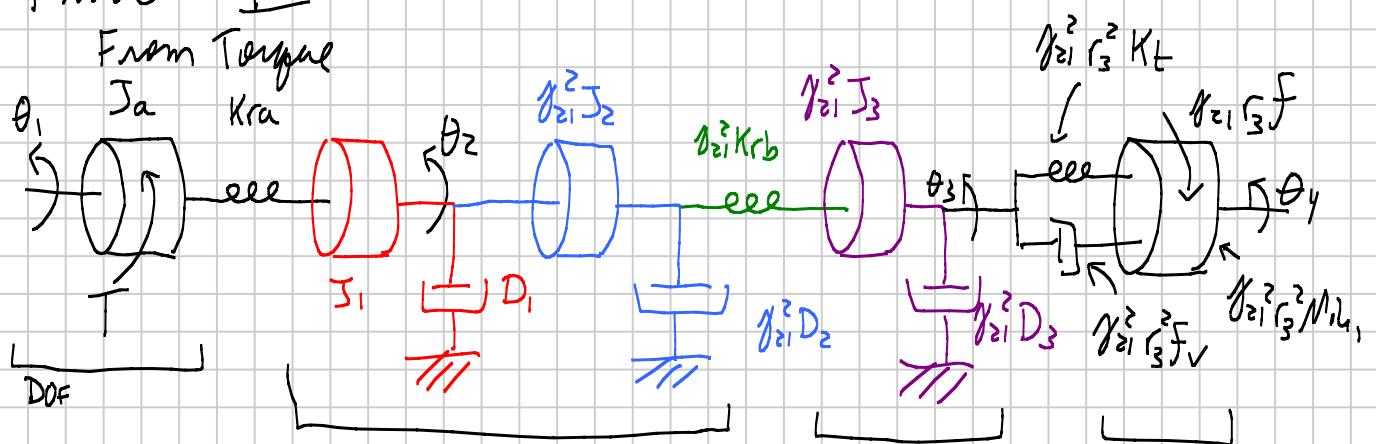
$$5) -(K_4)X_1 - (0)X_2 - (0)X_3 - (F_{V_3} s)X_4 + (F_{V_3} s + K_4)X_5 = 0$$

Can do matrix version by inspection

$$\left[\begin{array}{ccccc} M_1 s^2 + F_{V_1} s + K_1 + K_2 + K_3 + K_4 & -K_2 & -K_3 & 0 & -K_4 \\ -K_2 & M_2 s^2 + F_{V_2} s + K_2 & -F_{V_2} s & 0 & 0 \\ -K_3 & -F_{V_2} s & M_3 s^2 + F_{V_2} s + K_3 & 0 & 0 \\ 0 & 0 & 0 & M_4 s^2 + (F_{V_3} + F_{V_4})s & -F_{V_3} s \\ -K_4 & 0 & 0 & -F_{V_3} s & F_{V_3} s + K_4 \end{array} \right] \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} = \begin{pmatrix} F_1 \\ 0 \\ 0 \\ F_2 \\ 0 \end{pmatrix}$$

Problem IV

From Torque



Using $\theta_1 \ \theta_2 \ \theta_3 \ \theta_4$

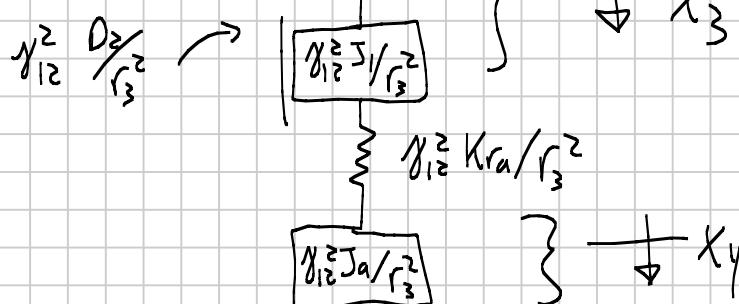
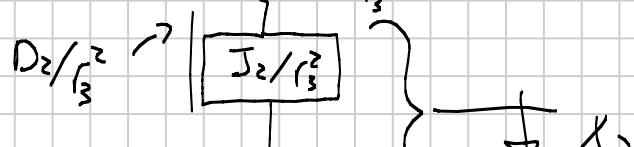
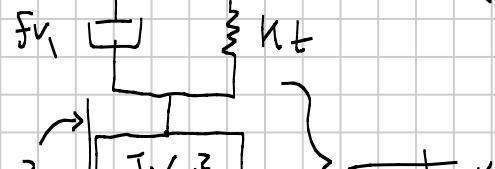
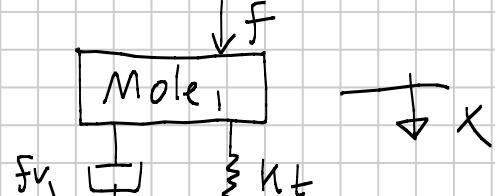
$$1) (J_a s^2 + K_r a) \theta_1 - (K_r a) \theta_2 = T$$

$$2) -(K_r a) \theta_1 + ((J_1 + \gamma_{21}^2 J_2) s^2 + (D_1 + \gamma_{21}^2 D_2) s + (K_r a + \gamma_{21}^2 K_r b)) \theta_2 - (\gamma_{21}^2 K_r b) \theta_3 = 0$$

$$3) -(\gamma_{21}^2 K_r b) \theta_2 + (\gamma_{21}^2 J_3 s^2 + (\gamma_{21}^2 D_3 + \gamma_{21}^2 r^2 f_v) s + (\gamma_{21}^2 K_r b + \gamma_{21}^2 r^2 K_t)) \theta_3 - (\gamma_{21}^2 r_3^2 f_v s + \gamma_{21}^2 r_3^2 K_t) \theta_4 = 0$$

$$4) -(\gamma_{21}^2 r_3^2 f_v s + \gamma_{21}^2 r_3^2 K_t) \theta_3 + (\gamma_{21}^2 r_3^2 M_{inv} s^2 + \gamma_{21}^2 r_3^2 f_v s + \gamma_{21}^2 r_3^2 K_t) \theta_4 = -\gamma_{21}^2 r_3 F$$

From the mole:



$$\uparrow \frac{T \gamma_{12}}{r_3}$$

Problem IV

$$a) \frac{V_p}{\Theta_p} = \frac{V_{cc}}{N_p/2 + v_{ins}} \times \frac{\pi r_m}{2\pi \text{ rad}} = \frac{V_{cc}}{\pi N_p} \boxed{}$$

$$b) \frac{E_a}{V_p} = \frac{R}{sL + R + \frac{1}{sC}} = \frac{sCR}{s^2LC + SCR + 1} \boxed{}$$

$$c) \frac{\Theta_m}{E_a} = \frac{\frac{1}{J_m} \frac{K_t}{R_a}}{s(s + \frac{1}{J_m}(D_m + \frac{K_t K_b}{R_a}))}$$

$$J_m = J_2 + \gamma_{z1}^2 J_s \quad D_m = \gamma_{z1}^2 D$$

$$d) N_1 \Theta_m = N_2 \Theta_o \quad \text{so} \quad \frac{\Theta_o}{\Theta_m} = \frac{N_1}{N_2} = \gamma_{z1} = \frac{1}{\gamma_{z2}}$$